

COMPARATIVE STUDY OF FUNDAMENTAL AND SECOND HARMONIC ICRF WAVE  
PROPAGATION AND DAMPING AT HIGH DENSITY IN THE ALCATOR TOKAMAK

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ABSTRACT

This thesis presents a comparative study of the theoretical foundations and experimental results of the ICRF heating experiment on the Alcator A tokamak at the MIT Plasma Fusion Center and Francis Bitter National Magnet Laboratory. Due to the versatility of the high power apparatus, the fast magnetosonic branch is used with  $\omega_o = 1, 2, 3, 4 \omega_{ci}$ , unlike most other ICRF experiments. Unusually high magnetic field ( $B_o = 40-80$  kG), plasma density ( $n_e = 10^{13} - 5 \times 10^{14}/\text{cm}^3$ ), generator frequency ( $f_o = 90-200$  MHz) and transmitter power, with shielded and unshielded antennas, are the key parameters of the experiment. This wide parameter range allows a direct comparison between fundamental and second harmonic regimes, and shielded and unshielded antennas, our prime goals. The real and imaginary parts of the parallel and perpendicular wave numbers are measured with extensive magnetic probe diagnostics for a spectrum of plasma parameters and compared with theory. Qualitative and quantitative evaluations of the wave structure and scaling laws are derived analytically in simple geometries and computed numerically for realistic plasma parameters and profiles. General figures of merit, such as radiation resistance and quality factor, are also derived and compared with the experiment. Secondary effects of the high power wave launching, such as changes in plasma current, density,  $Z_{eff}$ , energetic neutral flux, soft X-rays, neutron flux, and impurities are also discussed. Most important, a general synthesis of the many engineering, physics, and experimental problems and conclusions of the Alcator A ICRF program are inspected in detail. Finally, the derived and experimentally determined scaling laws and engineering constraints are used to estimate the ICRF requirements, advantages, and potential pitfalls of the next generations of experiments on the Alcator tokamaks.

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To My Parents

Joseph and Françoise

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## I. INTRODUCTION

### I-1. Motivation

Since the early 1950's, extensive work in the area of plasma physics has been undertaken to achieve controlled thermonuclear fusion, which may be the ultimate source of energy for the next century<sup>1</sup>. There are two main steps to demonstrate scientific feasibility of controlled thermonuclear fusion power. The first is to achieve sufficient levels of the product of plasma density  $n$ , and energy confinement time  $\tau$ , known as Lawson's criterion  $n\tau = 10^{14} \text{ sec/cm}^3$ . Many schemes for attaining these high levels of  $n\tau$  have been proposed, an example being the Alcator tokamak<sup>2</sup>. Lawson's criterion is expected to be achieved within the next decade of large scale experiments.

The second step, which certainly has not been achieved, is to attain the formidable temperatures required for fusion ( $\approx 10^8 \text{ }^\circ\text{K}$ ). Again, several schemes have been proposed and tried with some success. Ohmic heating of the plasma by intense currents has achieved consistently high temperatures in tokamaks<sup>3</sup>, but fails in the thermonuclear regime since the plasma resistivity<sup>1</sup> drops as  $T_e^{-3/2}$ . Injection of high energy neutral particles<sup>5</sup> has so far been successful in low density plasmas, but may be impractical in a high density reactor because of the low efficiency of the extremely high energy beams required for good penetration to the plasma center<sup>4</sup>. For many years, radio frequency heating of these plasmas has appeared attractive. A wide spectrum of wave frequencies is available at high power and good efficiency with current technology. Several specific frequency regimes have been tested, in particular, low frequency ( $\approx 10^6 \text{ Hz}$ )

Alfvén heating<sup>6</sup>, medium frequency ( $\approx 3 \times 10^7$  Hz) Ion Cyclotron Range of Frequencies heating (ICRF)<sup>7</sup>, high frequency ( $\approx 10^9$  Hz) lower hybrid<sup>8</sup>, and ultra high frequency ( $\approx 3 \times 10^{10}$  Hz) electron cyclotron heating<sup>9</sup>. Each regime has its specific theoretical and practical advantages, disadvantages and, especially, unknowns.

For the moment, the Ion Cyclotron Range of Frequencies is the prime candidate because of the relatively good agreement between theory and experiment, and compatibility with engineering constraints<sup>10</sup>. Even within the Ion Cyclotron Range of Frequencies, there are fundamentally different regimes of wave propagation and absorption, which can be broadly classified in terms of harmonic number of the ion gyrofrequency.

For the present work, we will investigate ICRF heating in a high density tokamak, and attempt to identify the most promising wave launching structures, wave frequency, and plasma parameters for efficient heating<sup>21</sup>.

Almost all recent ICRF work has been in one particular regime: the minority fundamental ion gyrofrequency regime<sup>11,12</sup>. Furthermore, the conditions have always been in tokamak plasmas in similar regimes,  $n_e \approx 3 \times 10^{13}/\text{cm}^3$ ,  $B_0 \approx 20$  kg,  $f_0 \approx 25$  MHz, all being significantly short of the high field and high density thermonuclear regime. The Alcator ICRF experiment<sup>13</sup> in these respects is different since it is a high field ( $\approx 60$  kG), high central current density ( $\approx 1,500$  amps/cm<sup>2</sup>), high density ( $\approx 3 \times 10^{14}/\text{cm}^3$ ) and high frequency ( $f_{ci} = 100$  MHz) experiment<sup>14,15</sup>. Fortunately, for comparison, the experiment can also be run at moderately low fields, current density, plasma density, and frequency. The wide range of parameters and their proximity to the reactor regime<sup>16</sup> make Alcator an almost ideal test machine for RF heating.

## I-2. Methodology and Overview

The main goals of this ICRF program are to identify the proper parameter regime which produces efficient heating. The single most important parameter is the harmonic number, i.e., fundamental, second harmonic or even higher harmonics of the ion gyrofrequency. The harmonic number, coupled with working gas and toroidal field, fully determine the transmitter chain and antenna operating frequency, a non-trivial amount of expensive and complicated hardware to be adjusted to a particular regime. A prime objective is to be able to change the parameters of the experiment and still satisfy  $\omega_0 = n \frac{eB}{M}$  for different harmonic numbers  $n$ . Two frequencies, 183.5, 92 MHz, four gases,  $H^1$ ,  $D^2$ ,  $He^3$ ,  $He^4$  and toroidal fields from 30-90 kg enable  $n$  to be changed over a wide range ( $n = 1, 2, 4, 8$ ) in different parts of parameter space.

The second issue is the selection of proper launching structure. This is an especially sensitive topic in the fusion RF community, and considerable controversy exists over fundamental issues, in particular, whether or not the antenna should be shielded<sup>17</sup>. Many other secondary but nevertheless essential issues are the choice of location, insulation, and method of RF feeding of the antenna<sup>18</sup>. As far as achieving this second goal, the Alcator experiment has a severe handicap; almost total lack of space for insertion, access, and location of antenna near the plasma<sup>2</sup>. However, several all metal shielded and unshielded antennas (Section III-2) were built and tested, and experimental results showed small but acceptable loading resistances of a few ohms<sup>13</sup>. An innovative all metal antenna and matching system with the vacuum breaks at a point where there is little reactive power is also proposed (Section III-2-6), and could be a great

step forward in Alcator ICRF antenna technology and performance.

To achieve the above goals, a clear understanding of the basic physics is necessary as well as a quantitative evaluation of the theory (Chapter IV). First, a more or less complete review of the current theory<sup>7,19-22</sup> is warranted to ensure that the approximations are still valid at high field, high frequency, and in particular, high density.

The physics of the problem may be divided broadly into three areas<sup>7, 89</sup>; wave launching and propagation, wave damping, and the physics of heating. There are three basic aspects of the wave propagation problem:

- (1) the dielectric tensor, wave equation, dispersion relation and polarization<sup>7,45</sup>;
- (2) the homogeneous and inhomogeneous plasma cylindrical wave field solutions<sup>21,22,45,46</sup>; and
- (3) stochastic mode stacking<sup>24</sup>.

The above aspects are first treated in conventional rigorous analytical ways with approximations justified quantitatively in the Alcator regime<sup>3</sup>. Successively more complicated models (Sections IV- 1-7) are developed starting with cold uniform infinite plasma with a single ion species and zero electron mass. The final theory includes hot plasma effects to first order, finite electron mass, two ion species, cylindrical geometry,  $1/R$  toroidal field, and inhomogeneous density profile<sup>21,22,26,27</sup>. A tangible physical explanation of the mathematical results is given whenever possible.

Second, the wave fields are numerically evaluated with realistic plasma parameters and profiles<sup>28,29</sup> and compared with experimental observations. A multitude of important measurable quantities is also calculated in the

same manner. Two prime examples are antenna loading resistance and RF signals from magnetic probes distributed around the torus.

The second facet of the problem is wave damping. The many wave damping mechanisms<sup>7,21,26,30,31</sup> are derived in the high density regime, and again we try to draw simple, intuitive, qualitative understanding as well as a quantitative evaluation and comparison with the experiment. Damping depends on the theoretical and experimental results of wave propagation. Damping may also significantly modify the propagation picture, especially on sensitive parameters such as radiation resistance<sup>32</sup>.

The last theoretical aspect is the physics of heating, and we will concentrate on the parameters that directly affect the plasma diagnostics used for monitoring heating, or the parameters that are likely to modify the wave propagation or damping. A few good examples are the ion distribution function as inferred from the fast neutral spectrum monitored by charge exchange analyzer<sup>3,33-38</sup>, neutron flux<sup>39</sup>, plasma current and density<sup>73</sup>, soft  $\chi$ -ray radiation<sup>39</sup>, edge temperature, and density as measured by Langmuir probe<sup>29</sup>.

The actual experiment is in the form of multi-dimensional scans to map out the different functions of the multitude of RF and plasma parameter combinations. This results in experimental scaling laws<sup>40</sup> that are compared with theory. A good example of this is that, experimentally, antenna loading resistance  $R_p$  linearly increases with electron density<sup>13</sup>. Loading resistance also increases with mass density, i.e.,  $R_p$  is larger in deuterium than in hydrogen, but on the other hand,  $\text{He}^3$  or  $\text{He}^4$  loading resistance is very small<sup>41,42</sup> (Chapter II). Until now, there has been no theory that satisfactorily explains these effects, and they have not been seen in any other

experiment. Plausible new explanations are discussed in detail, especially in light of "Stochastic Mode Stacking" (Section IV-6), a new concept developed in this work that refers to simultaneous excitation of a number of randomly phased toroidal eigenmodes, which may profoundly affect the wave structure at high density<sup>23-25,93</sup>.

The conclusion to this work will consist of a tentative synthesis of the many interacting processes and parameters of the experiment, as well as recommendations for further investigation.

### I-3. Summary of Related Work

The relatively new Ion Cyclotron Range of Frequencies (ICRF) regime<sup>43</sup> has evolved out of the Ion Cyclotron Resonant Heating (ICRH) scheme in the older magnetic confinement devices like the stellarator<sup>44</sup>. Much of the theory for wave excitation and propagation was then summarized in two books by T.H. Stix<sup>45</sup> and Allis et al.<sup>46</sup>, and the formulation and notation are still widely accepted.

The fundamental difference between ICRH and ICRF is that they each use a different branch of the wave dispersion relation<sup>7</sup>. ICRH used the slow branch, also called the ion cyclotron wave, at frequencies near or below the ion cyclotron frequency. The new (1970)<sup>43</sup> ICRF uses the fast branch, also called the fast wave or fast magnetosonic wave, and the wave is usually carried by a majority ion component at some low harmonic number. The ion cyclotron wave is evanescent under these conditions<sup>7</sup>.

Many national laboratories and universities throughout the world are involved in ICRF, but the most important are located in the U.S. (Princeton University, Princeton, NJ), France (TFR, Fontenay-aux-Roses), and Japan (Japan Atomic Energy Institute Tokai, Ibaraki). Princeton has been in the field of RF heating for about two decades, and has developed both ICRH and ICRF through the Model C Stellarator<sup>44</sup>, the Adiabatic Toroidal Compressor (ATC)<sup>11</sup>, and finally, the Princeton Large Torus (PLT)<sup>12</sup>. Much of the literature is by T.H. Stix<sup>7, 93, 45</sup>, F.J. Paoloni<sup>19, 47</sup>, F.W. Perkins<sup>48</sup>, J.C. Hosea<sup>49-50</sup>, P.L. Colestock<sup>51</sup>, and D.G. Swanson<sup>20</sup>. The TFR group is newer, but has been producing consistently good experimental data and theory, and is headed by J. Adams<sup>21, 52, 54</sup> and J. Jaquinot<sup>22, 53, 55</sup>. Several other institutions<sup>56</sup> also have published good work, particularly the

University of Wisconsin (Madison, WI)<sup>16,59,26,27</sup>, but their confinement device's plasma parameters are usually remote from the high density regime<sup>57</sup>.

For this work, three review publications broadly define current theory and technology, and contain the foundations of this program:

- The Theory of Plasma Waves, T.H. Stix, McGraw-Hill, New York (1962).
- "Fast Wave Heating of a Two Component Plasma," T.H. Stix, Nuclear Fusion 15, 737 (1975).
- "Eigenmode Field Structure of the Fast Magnetosonic Wave in a Tokamak and Loading Impedence of Coupling Structures," J. Adams and J. Jaquinot, EUR-CEA-FC-886 (April, 1977).

State of the art in ICRF is well summarized in a number of papers presented at the 8th International Conference on plasma physics<sup>101-106</sup> (Brussels, July, 1980), and the Fourth Topical Conference on RF Heating in plasma<sup>107-115</sup> (University of Texas, February, 1981).



## II. EXPERIMENTAL RESULTS

### II-1. Available Parameter Space and Principal New Results

#### II-1.1. Available parameter space

The experiment was usually run in the form of multi-dimensional parameter scans. The basic methodology was to map out the various plasma and wave processes by independently varying each parameter at a time over as wide a range as possible, with as many other different parameter combinations as possible, while monitoring all the diagnostics. The main parameters that could be independently varied were:

- (1) RF power from .1 W to 100 kW
- (2) Toroidal field from 40 to 80 kG
- (3) Resonance layer position from 30 to 70 cm
- (4) Plasma density from  $10^{13}$  to  $5 \times 10^{14}/\text{cm}^3$
- (5) Wave frequency 90, 180 (50, 360) MHz
- (6) RF spectrum width up to 2 MHz
- (7) Plasma current from 50 to 250 kA
- (8) Working gas,  $\text{H}^1$ ,  $\text{D}^2$ ,  $\text{He}^3$ ,  $\text{He}^4$
- (9) Minority concentration from 1 to 100%
- (10) Plasma radius from 9 to 10 cm
- (11) Antenna loop area from 5 to 25  $\text{cm}^2$
- (12) Probe radial position from 9 to 13 cm
- (13) Shielded or unshielded antenna
- (14) Antenna phasing  $m = 0$  or  $m = 1$

This represents considerable amounts of machine time, and efficient, judicious choices of parameter scans are necessary. Theory and gradual increase in experience guide these choices to enable formulation of reasonably sound scaling laws, maximization of machine time, and better understanding of the many interacting physical processes. Nevertheless, only a fraction of all parameter space was explored.

Figure 1 shows a plot of resonant frequency with respect to magnetic field for typical Alcator, TFR and PLT regimes and different hydrogen and helium isotopes. As we stressed in the introduction, Alcator stands out as very different from other tokamaks in many directions of parameter space. We also note from Figure 1 how several resonant regimes may be present at the same time. This effect is further emphasized in Figure 2 and Table 1 where we note how partially ionized impurities may scan the whole width of the plasma and cause edge heating (large concentrations of impurities are most likely confined to the plasma edge).

Table 1

Normalized cyclotron frequencies and refractive indices

gas ( $n_e$ , $B_0$ = constant)	$\omega_c \propto \frac{Z}{\mu}$	$n_A \propto \frac{\sqrt{\mu}}{Z}$
H <sup>1</sup>	1.	1.
D <sup>2</sup>	.5	1.41
He <sup>3</sup>	.667	.866
He <sup>4</sup>	.5	1.
Fully Ionized Impurities	$\sim .5$	$\sim \left[ \frac{2}{\sqrt{\mu}} \right]$
Partially Ionized Impurities	$\frac{1}{\mu}$ to .5	$\left[ \frac{2}{\sqrt{\mu}} \text{ to } \sqrt{\mu} \right]$

# ICRF Wave Regime In ALCATOR A

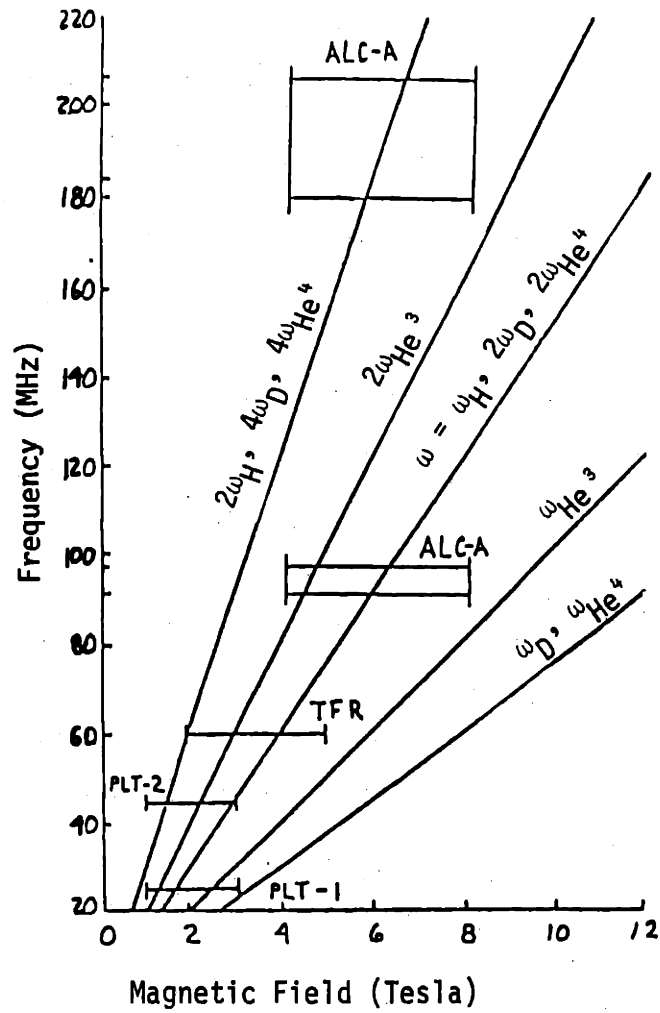


Figure 1

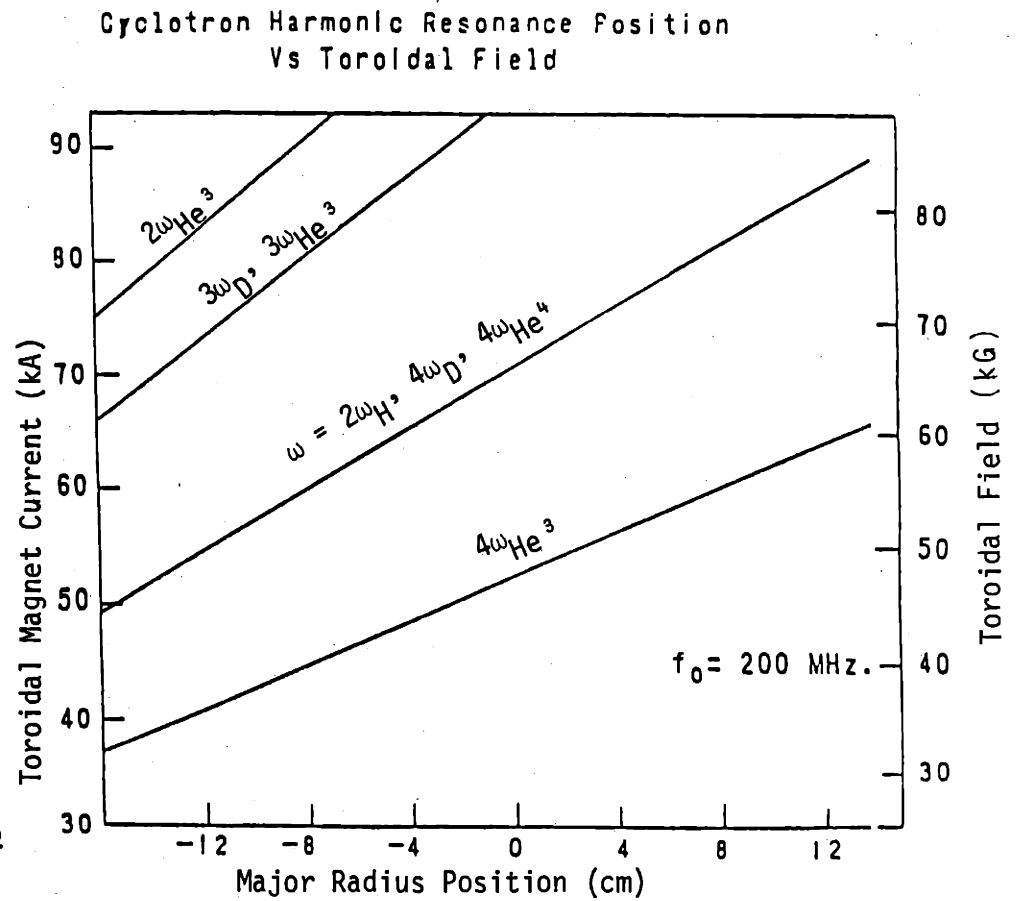


Figure 2

## II-1.2. Principal new experimental results

The most important products of this research are sound experimental results that preferably can be explained by theory. To date, there are many important questions that need to be answered, most of which are still unexplored in the high field, high frequency, and high density regime.

The first question is whether or not the minority or second harmonic regime should be used. This is a far reaching question and needs much consideration, many parameter scans and is, of course, answered with best heating and engineering compatibility.

The second question is whether the antenna should be shielded or not. This is answered for Alcator A by comparing the performance of the  $A_1$ ,  $A_2$ , and  $A_4$  antenna over a wide range of experimental conditions.

Third, also of utmost importance, and closely related to the first two questions, is the effect of density, working gas, frequency, antenna loop area, shielding, RF power and wave Q on antenna loading. Also correlations between  $R_p$ ,  $H_\alpha$  light, impurities, disruptions, X-rays, etc., are important and not well understood.

Fourth, what are the predominant wave damping mechanisms, near-field power losses, and power deposition profiles? This can be answered by changing the most critical parameters of particular processes, and noting changes in field structure and heating efficiency. A prime example is the effect of the resonant layer position on antenna loading, RF probe signals, and the charge exchange neutral spectra. Wall losses can be directly calculated from probe signals. Near-field losses could be affected by antenna shielding, etc.

Fifth, what is the wave field structure? In particular, what modes are predominantly excited, and how are they affected by other parameters such as toroidal field, ion mass, and electron density. Is the cold plasma dispersion relation valid? What is the role of surface waves? This set of questions will be answered by detailed study of the RF probe signal phase measurements and comparison with theory.

Sixth, is what is the role of stochastic mode stacking? How many modes are excited simultaneously? What are their Q's? Does it explain the increase in radiation resistance and the saturation of probe signals with density? This will be tentatively answered by extensive simulations and comparison with experimental values of  $R_R$  and probe signals and phases. Finally, does large  $k_{||}$  mean small  $k_{\perp}$ , making second harmonical damping small, leading to a natural selection of undamped modes of high Q and  $R_R$ ?

All these questions, and many more omitted here, are not fully answered, but much light is shed on the basic issues. Tables 1 to 4 summarize some of the basic experimental results that will be discussed in this chapter. The results are broadly categorized as wave propagation (Table 1), wave coupling (Table 2), wave damping (Table 3) and heating (Table 4). In this chapter, voltages and currents will always be referred to by their RMS value ( $\frac{\text{Peak to Peak}}{2\sqrt{2}}$ ).

Table 1

Wave Propagation Measurements from Magnetic Probes Around Torus

- Large number of high Q, closely packed eigenmodes
- Apparent mode cutoff below  $n_e \approx 10^{13}/\text{cm}^3$
- No significant decrease in amplitude in the toroidal direction
- Magnitude compatible with radiation resistance and stochastic mode stacking
- $B_\theta$ ,  $B_r$ ,  $B_z$ , at edge, all have roughly same magnitude ( 3 axis probes in ceramic thimble)
- 20 cm long,  $k_\parallel$  array shows nonuniform wave structure along torus (short wavelength stochastic mode stacking)
- Large apparent  $k_\parallel \approx .5 \text{ cm}^{-1}$ , hence significant evanescent region at the edge
- $k_\parallel$  fringes show positive going sawteeth
- Electrostatic probe may not contain same phase information (tentative)
- Small  $k_\theta$  from relative phases between probes at different poloidal positions

Table 2

Antenna Loading Resistance

- Three antennas tested

A<sub>1</sub> electrostatically shielded

A<sub>2</sub> bare metal, slotted limiters each side

A<sub>4</sub> electrostatically shielded, slotted limiters each side

- R<sub>R</sub> increases linearly with density
- R<sub>R</sub> essentially independent of I<sub>p</sub>, B<sub>0</sub>, RF power
- R<sub>R</sub> roughly increases as  $\omega_0^2$
- R<sub>R</sub> roughly unaffected by Faraday shield (taking into account antenna geometry)
- At 180 MHz, H<sub>2</sub> and D<sub>2</sub> show similar resistances (D<sub>2</sub> slightly larger than H<sub>2</sub>) and He exhibits a much smaller resistance
- R<sub>R</sub> sensitive to plasma position (R<sub>R</sub> decreases at end of shot when plasma moves inward)
- R<sub>R</sub> and H<sub>α</sub> light have similar time evolution
- Multipactor regime is very wide in the new large ceramic breaks (5W - 500W)
- R<sub>R</sub> is roughly proportional to antenna length squared
- R<sub>R</sub> is roughly independent of antenna phasing (m = 0,1)



Table 3

Comparison of Fundamental and Second Harmonic Regimes

$$(n_e = 3 \times 10^{14} \text{ cm}^{-3}, B_0 = 60 \text{ kG}, I_p = 150 \text{ kA})$$

	Experimental Regime	90 - 100 MHz ( $\omega_{CH}$ , $2\omega_{CD}$ ) Minority $n_H/n_D \approx .05$	180 - 200 MHz ( $2\omega_{CH}$ , $4\omega_{CD}$ ) H, D, and H & D
Typical Radiation Resistance	Shielded ( $A_1$ ) Unshielded ( $A_2$ ) Shielded ( $A_4$ )	$\approx 0.5 \Omega$ $\approx 2 \Omega$ $\approx 2 \Omega$	2 - 3 $\Omega$ 7 - 10 $\Omega$ 5 - 8 $\Omega$
Magnetic Field Dependence	Radiation Resistance	<ul style="list-style-type: none"> <li>- Background component unchanged</li> <li>- Eigenmode structure disappears with layer inside plasma</li> </ul>	Independent of toroidal field (40 - 80 kG)
	Probe signals	Modes remain, but with reduced amplitude when layer is at plasma center	Probe signal decreases slightly when layer is near probe position (preliminary)

Table 4

Medium Power Heating Experiments

High Density:

Typical Parameters	$\bar{n}_e = 2 - 3 \times 10^{14} \text{ cm}^{-3}$ , $I_p = 150 \text{ kA}$ $B_0 = 60 \text{ kG}$ , 50% H, 50% D (also 100% H or D) $f_0 = 180 \text{ MHz} = 2F_{CH} = 4F_{CD}$ $P = 30 - 100 \text{ kW}$
Results	<ul style="list-style-type: none"> <li>- Energetic H &amp; D tails (&gt;5 Kev) with short decay times (&lt;100 <math>\mu\text{s}</math>)</li> <li>- 30 - 50% increase in soft X-rays</li> <li>- Bolometric measurement indicates an increase in (fast particles/radiation) flux proportional to the RF input power</li> <li>- 30% increase in <math>H_{\alpha}</math> light both at the antenna and around the torus</li> <li>- Some evidence of light impurities (extreme UV spectra)</li> <li>- Small dip in neutron production</li> <li>- No significant change in <math>I_p</math>, <math>V_{loop}</math>, hard X-rays, <math>\bar{n}_e</math></li> <li>- With carbon limiter:             <ul style="list-style-type: none"> <li>- 10% density increase</li> <li>- carbon deposit on antenna</li> <li>- no consistent increase in energetic neutrals</li> </ul> </li> </ul>

Low Density:

- $\bar{n}_e = 5 \times 10^{13} \text{ cm}^{-3}$ , deuterium ( $4 \omega_{cD}$ )
- Very energetic ion tail formation (both early and at peak of discharge)

## II-2. Low Power Wave Measurements

### II-2.1. High Q eigenmodes

In every case examined, except those at high density in the minority regime, high Q closely packed eigenmodes were observed. An apparent mode cutoff was also observed at about  $10^{13}/\text{cm}^3$ . Density measurements below  $3 \times 10^{13}/\text{cm}^3$  were very crude since the resolution of the alcohol interferometer was about  $10^{13}/\text{cm}^3$ . Particle recycling usually keeps the density above  $2 \times 10^{13}/\text{cm}^3$ . As expected, cutoff was much easier to reach at 90 MHz than at 200 MHz because  $k_A \propto \omega_0$ .

Figure 1 shows the plasma current (~150 kA), electron line average density ( $10^{14} \text{ cm}^{-1}/\text{fringe}$ ), RF pulse timing, forward (58 kW) and reflected voltages (9 kW), antenna current (150 Amps RMS) and a probe signal from the Thomson port for a typical high density shot at 180 MHz ( $2\omega_{CH}$ ,  $4\omega_{CD}$ ). Figure 2 shows the antenna current, probe signal and square law detected  $k_{||}$  array probe signals for a medium density shot. Several important conclusions can be drawn from these two Figures. First, we obviously see dozens of eigenmode peaks in as little as 8 msec (in agreement with IV-6.3.). Second, these eigenmodes are very closely packed, overlap, and constitute a virtual continuum. Nevertheless, some resonances are larger than others (because of higher Q or coherent stacking of two modes) and cause dips in the antenna current. The peaks in the probe signal do not necessarily line up with the dips in the antenna current because, with many eigenmodes, fields add differently at different locations around the torus. This effect is further emphasized in Figure 2, where the different probes are only a few centimeters apart along z (III-3.2), and show qualitatively the same behavior, but with some small but

clearly discernable differences, implying the presence of stochastic stacking of relatively short wavelength eigenmodes ( $\approx$  array length).

Figures 3 and 4 show probe signals at very low density ( $\approx 10^{13}/\text{cm}^3$ ) where the eigenmode resonances are well separated, and can be compared to Figures IV-6.7.(3) - (9). From Figure 3 (Min/Max  $\approx 7$ ) and equation IV-6.2(7), we can calculate the damping length as

$$(1) \quad 1/k_i = \pi R \sqrt{\frac{F_R}{F_A}} \approx 4.5 \text{ meters}$$

The eigenmode Q can be crudely estimated from equations IV-6.4.(6) and IV-6.4(21) as

$$(2) \quad Q \approx \frac{k_A}{4k_i} \approx \frac{.2}{4 \times 2 \times 10^{-3}} \approx 25 \text{ at low density} \\ \approx 100 \text{ at high density}$$

$$(3) \quad Q \approx \frac{2n_e}{\Delta n_e} = \frac{2n_e}{\left(\frac{\partial n_e}{\partial t}\right) \Delta t} \approx \frac{2 \times 10^{13}/\text{cm}}{\left(\frac{10^{13}/\text{cm}^3}{10^{-2} \text{ sec}}\right) 10^{-4} \text{ sec}} \approx 200 \text{ at either low or high density}$$

Although these two methods should be redundant, the large discrepancy is easily justified by the crudeness of IV-6.4.(6) ( $k_{ii}$  is not known), and the gross inaccuracy of  $\partial n/\partial t$ .  $F_R/F_A$  is also underestimated, since the antenna current dips ( $F_R$  is underestimated), and several modes exist (skirts overlap and thus  $F_A$  is overestimated).

Figure 3 also shows an  $A \cos \phi$  signal of two  $k_{ii}$  array probe signals. Although the magnitude of probe 76 is almost identical to 77, their phases are considerably different, as expected.

Figures 4 and 5 show probe signals at  $10^{14}$  and  $2 \times 10^{14}/\text{cm}^3$  line average densities and 200 MHz. At  $10^{14}/\text{cm}^3$ , the eigenmodes are still quite discrete, and mode stacking is only partial. At  $2 \times 10^{14}/\text{cm}^3$

and above, the individual modes are more difficult to discern, and radiation resistance spikes disappear, in agreement with IV-6.7, where we concluded that probe signals would always be "noisier" than  $R_R$ . Figure 5 also shows a 23 kHz heterodyned probe signal for phase measurements (III-3.3. and II-2.3).

Figure 6 shows a low density, pure hydrogen, 90 MHz ( $\omega = \omega_{cH}$ ) plasma shot. Very high Q eigenmodes occur early in the discharge. At very low density, late in the discharge, there is an apparent mode cutoff, in agreement with simple theory (this may not be a real cutoff, and Section IV-5.4. showed that no cutoff should occur for the  $m > 1$ ,  $\mu = 1$  modes).

Figures 7 and 8 show evidence of possible mode splitting (IV-3.3) in a very low and a medium density plasma. This effect could also be caused by density fluctuations, or simply by two different perpendicular eigenmodes.

Following the theory of Section IV-6.3, we are now faced with the dilemma of labeling these eigenmode resonances (Figures 1-8). Are they onsets, large  $R_R$  eigenmode resonances (i.e.,  $m = 0 \pm 1$ ,  $\mu > 3$ ), or simply particularly high Q eigenmodes ( $m > 1$ )? Onset has the largest radiation resistance, and probably accounts for the thirty or so larger peaks. For a quality factor as large as 1000, the energy pump time (decay time, IV-6.4.(12))

$$(4) \quad \tau = \frac{1}{\omega_i} = \frac{2Q}{\omega} = 1.6 \text{ } \mu\text{sec}$$

is much less than any experimental resonance width. From Figures IV-5.4.(3-14), we can approximately write

$$(5) \quad \frac{\partial k_{\parallel}}{\partial n} \approx 10^{-14} \text{ cm}^2$$

$$\rightarrow 0 \text{ for } \mu = 1, m > 1$$

$$\rightarrow \infty \text{ for } \mu > 1$$

$$(6) \quad \frac{\partial k_{\parallel}}{\partial t} = \frac{\partial k_{\parallel}}{\partial n} \frac{\partial n}{\partial t} \approx 50 \text{ cm}^{-1} \text{ sec}^{-1} \quad \left( \frac{\partial n}{\partial t} \approx 10^{14} \text{ cm}^{-3} \text{ sec}^{-1} \right)$$

and for

$$(7) \quad \Delta k_{\parallel} = k_i \approx 10^{-3} / \text{cm} \quad (1/k_i = 10 \text{ meters})$$

we have

$$(8) \quad \Delta t \approx \left[ \frac{\partial k_{\parallel}}{\partial t} \right]^{-1} \Delta k_{\parallel} \approx 20 \mu \text{ sec}$$

which is much longer than the pump time.

Until now, we have assumed  $R_R \propto \frac{1}{k_{\parallel}}$  near onset. Taking into account  $k_{\perp i} \approx k_{\parallel i} \approx 10^{-3}$ , and substituting  $k_{\parallel i} 2\pi R \rightarrow k_{\perp i} 4a$  and  $z \rightarrow r$  in IV-6.2.(5), we have instead a coherent, perpendicular stacking, limiting factor of the form

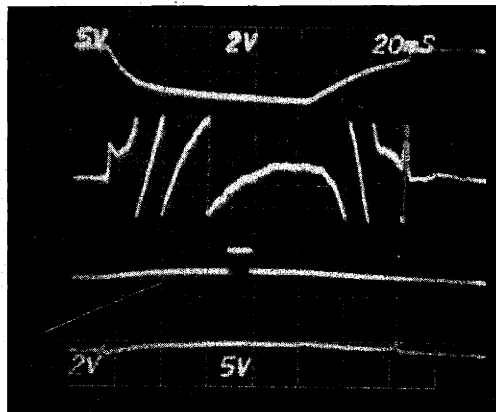
$$(9) \quad R_R \propto \frac{1 + e^{-k_{\perp i} 4a}}{1 - e^{-k_{\perp i} 4a}} \approx 50$$

and  $R_R$  does not go to infinity even for  $k_{\parallel} = 0$ .

Away from onset, toroidal resonance radiation resistance peaks are critically dependent on the evanescent edge and Q. Large perpendicular mode numbers have smaller  $k_{\parallel}$  (small evanescence), but also larger damping

(small  $Q$ ), and the two effects compete. The  $m > 0$  modes have larger  $Q$  (IV-5.3), but cannot be discerned from the  $m < 0$  by probes at the plasma edge. Some peaks may also be beats between different perpendicular eigenmodes (even for  $k_i R > 1$ ).

# Eigenmodes at High Density



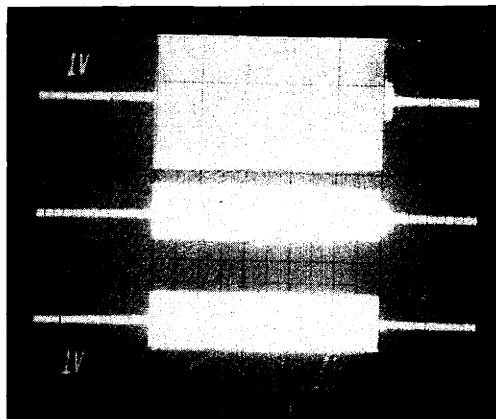
$I_p$

$n_e$

$t$

P13, B4, S50

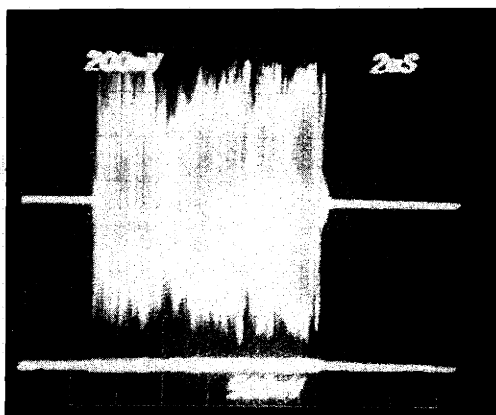
Hydrogen 183 MHz



$V_F$  57.7 kW

$V_R$  8.9 kW

$I_a$  149 amps



$P_{74}$

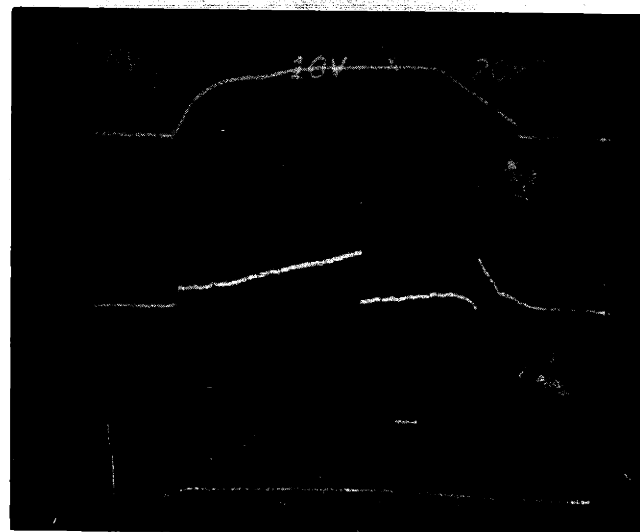
Figure 1



# Radiation Resistance and Probe Signal at Medium Density

S47,B2,P9  
D<sub>2</sub>, 183 MHz

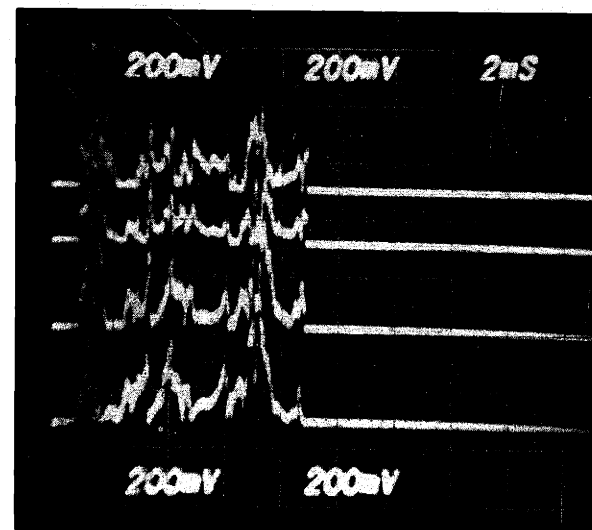
Figure 2



$I_p$

$n_e$

t

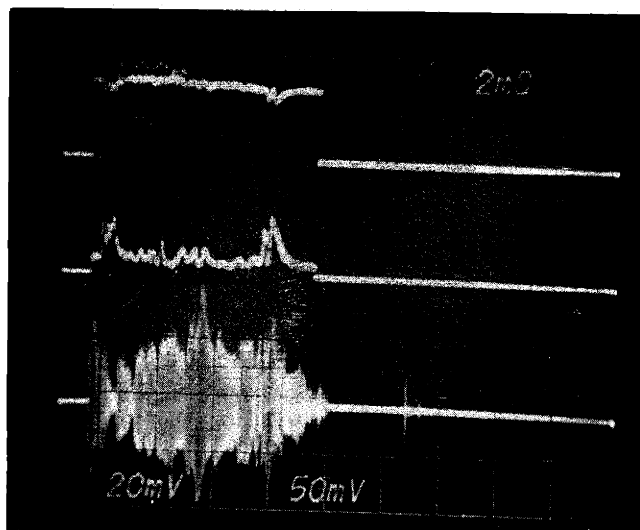


P<sub>70</sub>

P<sub>71</sub>

P<sub>72</sub>

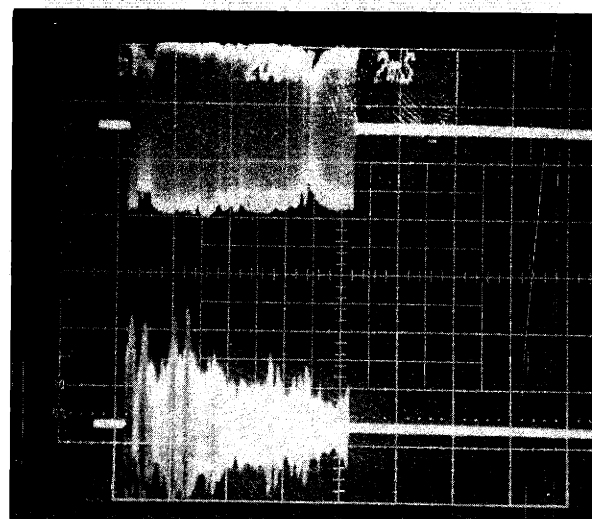
P<sub>73</sub>



$V_F$  (14.2 kW)

$V_R$  (1.1 kW)

P<sub>8</sub>



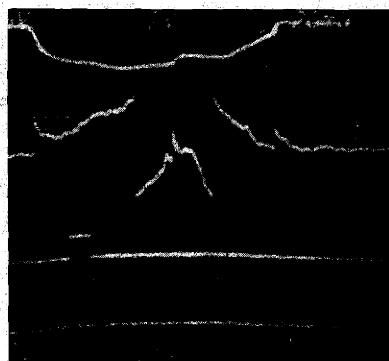
$I_a$   
(96 amps)

$R_R = 1.4 \Omega$

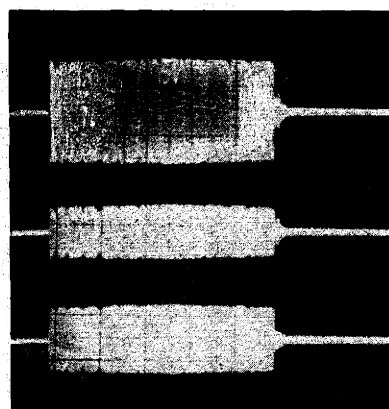
P<sub>79</sub>

# Low Density Phase and Magnitude Eigenmode Measurements

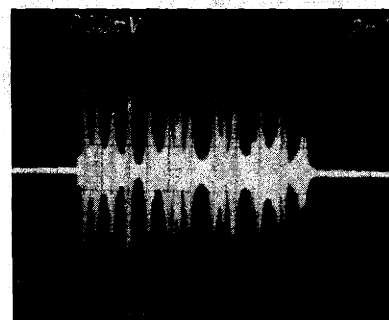
B4, P9, S39  
Hydrogen, 183 MHz



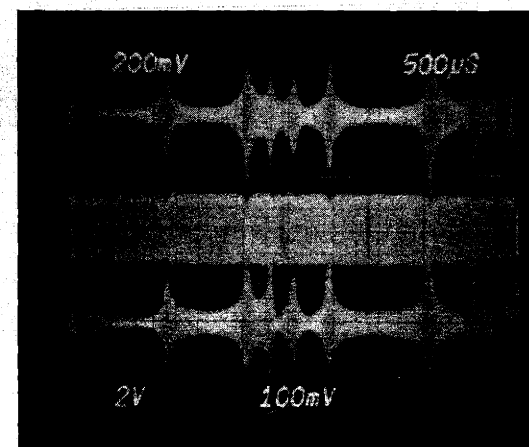
$I_p$   
 $n_e$   
 $t$



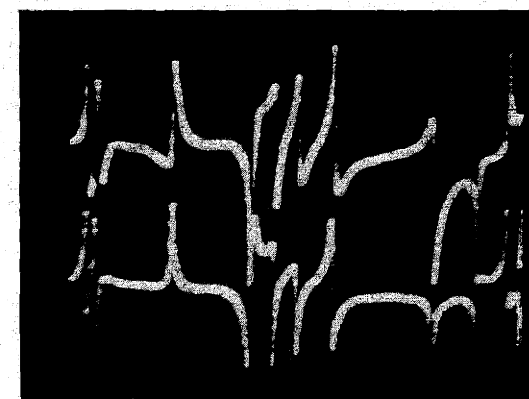
$V_F$  26.8 kW  
 $V_R$  7.6 kW  
 $I_a$  172 amps



$P_{74}$



$P_{76}$   
 $I_a$   
 $P_{77}$

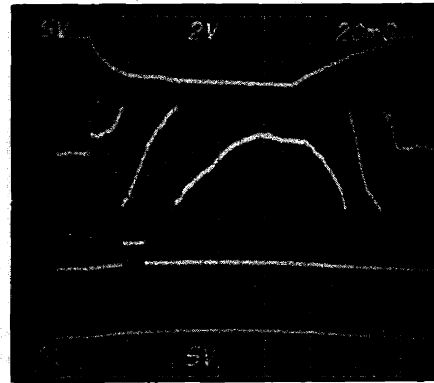


$a_1 \cos \phi$   
 $a_2 \cos \phi$

Figure 3

S32, P6, B4

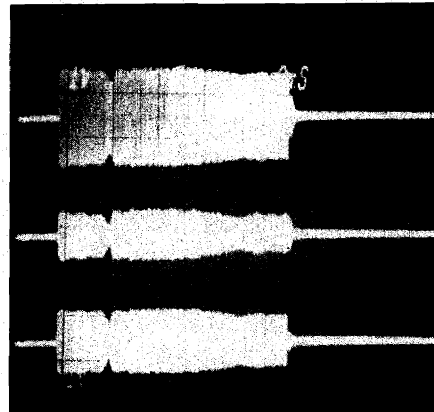
200 MHz at  
Medium Density



$I_p$

$n_e$

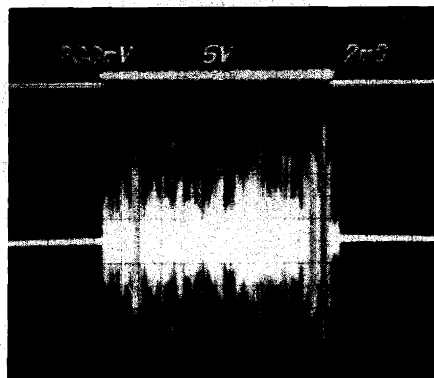
$t$



$V_F$

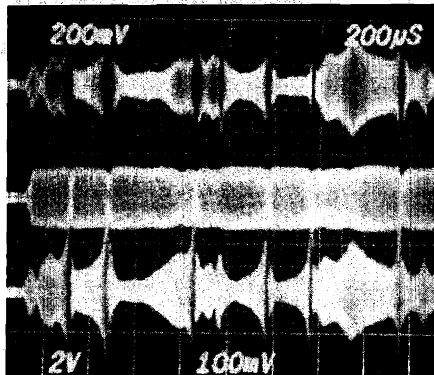
$V_R$

$I_a$



$t$

$P_{74}$



$P_{76}$

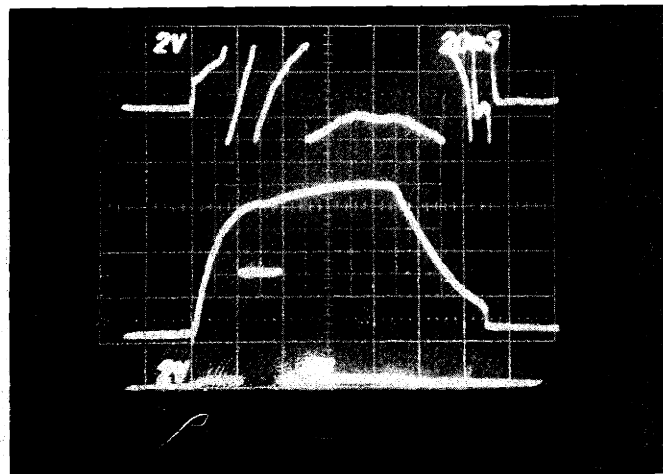
$I_a$

$P_{77}$

Figure 4

Eigenmode Detail  
at Medium Density

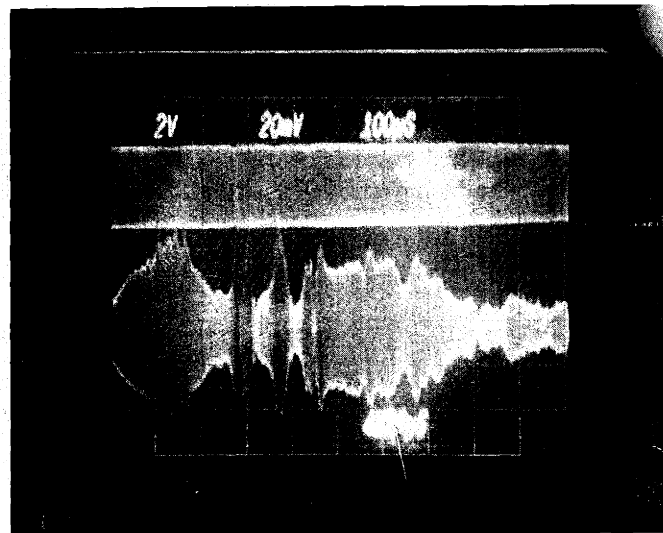
S61, P69, B1  
Deuterium



$n_e$

$I_p$   
 $t$

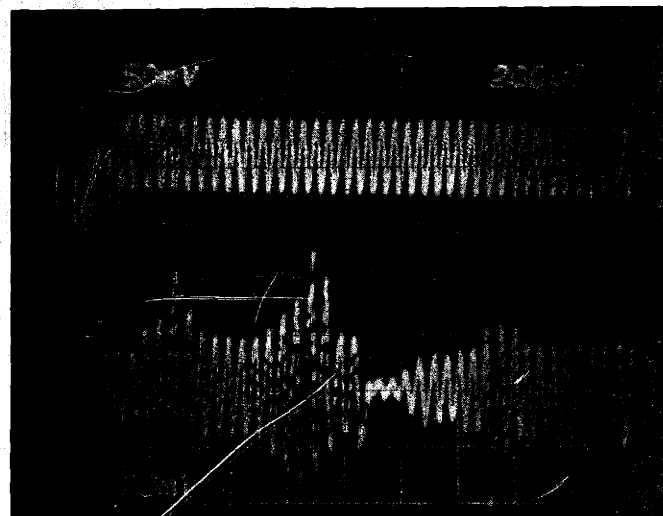
Figure 5



$I_a$

$P_t$

S41, P52, B2  
Heterodyne  
to 23 kHz

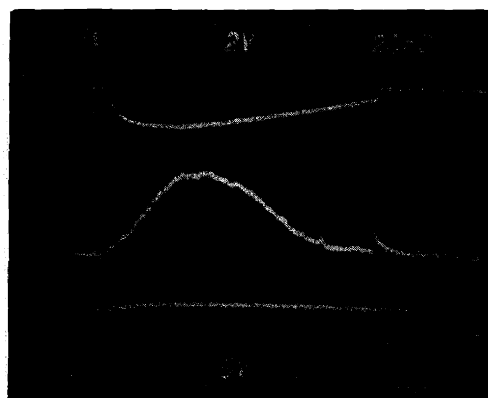


$I_a$

$P_8$

Hydrogen at 90 MHz

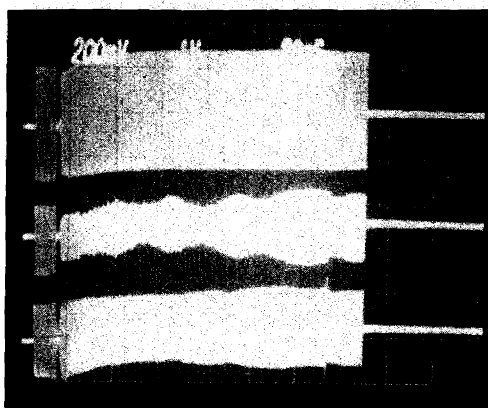
S7, P2, B8



$I_p$

$n_e$

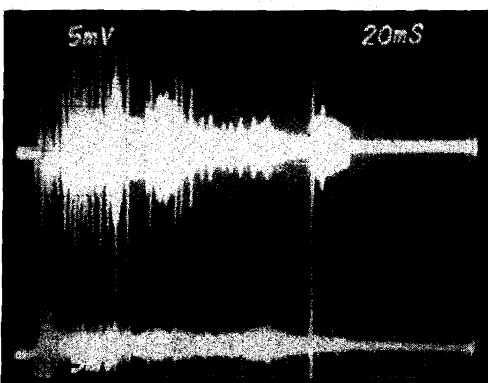
t



$V_F$

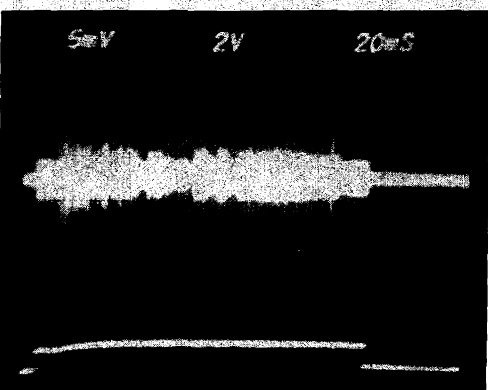
$V_R$

$I_a$



70 HCN

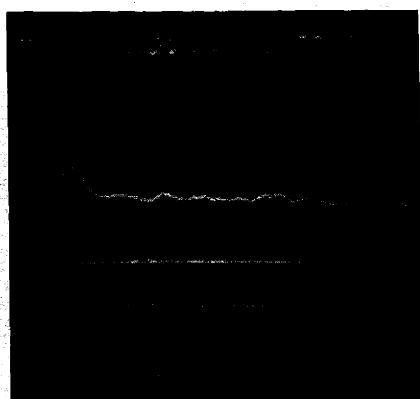
71 HCN



2 Near field

Figure 6

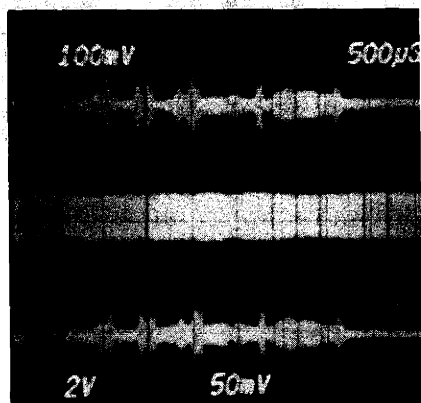
# Evidence of Mode Splitting



$I_p$

$n_e$

t



76

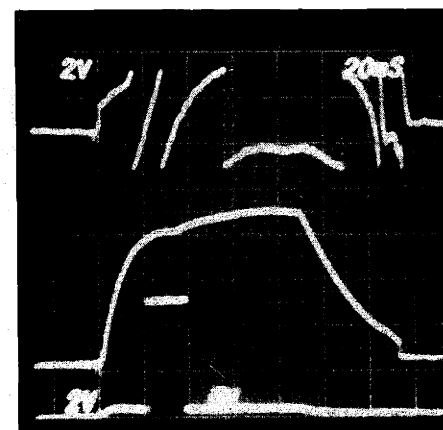
$I_p$

77

S20, P30, B4

H<sub>2</sub>, 200 MHz

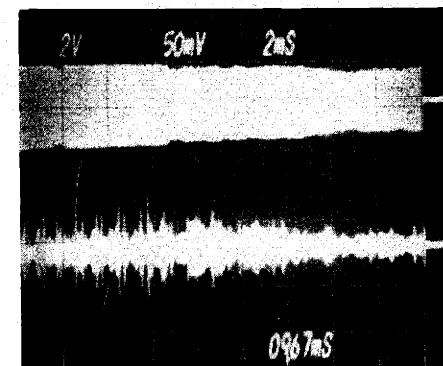
Figure 7



$n_e$

$n_e$

t



$I_p$

P

S64, B1, P70

Figure 8

## II-2.2. Wave amplitude measurements

Neglecting near-field power absorption, we show in IV-6. that, for high Q toroidal resonances and many overlapping eigenmodes, the probe signal average is proportional to  $\langle R_P \rangle / \sqrt{n}$ .

Figure 1 shows a standard high density low power shot with the forward and reflected voltage, antenna current and probe signal at HCN port. Figure 2 shows the radiation resistance  $\left( \frac{P_F - P_R}{I^2} \right)$ , antenna current, antenna electric field  $\left( \frac{IR}{\ell} \right)$  and probe signal for the same shot. Note how the radiation resistance and electric field increase with density, but the probe signal saturates and even decreases with density. Figures 3 and 4 are similar shots, and we notice (Figure 3) the top and bottom antenna currents are loaded in an almost identical manner (good balance in push-push mode). Probe signals around the torus (68 HCN top, 69 HCN bottom and 67 top limiter), on the other hand, have significantly different time histories (Figure 4). Many of these different behaviors are believed to be due to the wandering plasma position. Radiation resistance is usually lower at the end of the shot (for the same density), and is most likely due to the plasma moving inward, since there is usually too much vertical field for the decreasing plasma current. In all cases examined (hundreds of shots in almost all directions of available parameter space) except the fundamental regime (with the cyclotron layer in the center of the plasma), probe signals around the torus ( $\pm 90^\circ$ ,  $180^\circ$  from antenna) were all about the same magnitude (probe position in the complicated port geometry is critical). These measurements were done very carefully, and are strong evidence of long damping length.

Figures 5 and 6 show radial probe scans with a large magnetic probe (Figure III-3.2.(3)) and a standard small unshielded probe

(Figure III-3.2.(2)). With the large probe fully inserted, it becomes a good model of a short section of the  $A_2$  antenna, and we can write (without mode stacking)

$$(1) \quad V_{\text{Probe Theoretical}} = \frac{A_{\text{probe}}}{A_{\text{antenna}}} I_{\text{antenna}} R_R \\ \approx 3 \quad V_{\text{p experimental}}$$

and the discrepancy could be explained by some 9 eigenmodes or, of course, by near-field or single toroidal pass power absorption. Figure 6 shows similar results, but at three different power levels, and also showing a discrepancy factor of three or so. If mode stacking is not invoked, this means that only some 10% of wave power reaches  $90^\circ$  away from the antenna. The measurement is very crude because of the extreme sensitivity of the probe and plasma position (evanescent edge), and the mere fact that power is proportional to voltage squared (50% error can easily be due to the average probe signal value estimation from the oscillograph).

Figures 5 and 6 are indicative of large  $k_{||}$ , since, for small  $k_{||}$ , the edge magnetic field should be roughly independent of distance from the wall, which is clearly not the case here. For large  $k_{||}$  ( $\approx .5/\text{cm}$ ), the edge  $B_z$  vs  $r$  should be either linear or exponential, in agreement with experiment. (Note again the extreme sensitivity of the port geometry.)

The results shown in Figures 5 and 6 can also be used to estimate wall power losses as

$$(2) \quad P_{\text{wall}} = \frac{A_{\text{wall}} J_{\text{wall}}^2}{\sigma \delta} \approx \left( \frac{V_p}{A_p} \right)^2 \left( \frac{1}{f_0 \mu_0} \right)^{1.5} \left( \frac{\pi}{\sigma} \right)^{.5} a R \approx 1.0 \text{ Watt}$$

$$(3) \quad \frac{P_{\text{in}}}{P_{\text{wall}}} \approx \frac{500 \text{ W}}{1 \text{ W}} = 500$$



and wall losses can be totally neglected in any power balance calculation (this is a lower limit due to the bellows convolutions and port geometry).

High and low power experiments showed exactly the same results (except obviously, during multipactor), and all processes (coupling, damping, etc.) were linear over six orders of magnitude (.1 W to 100 kW) (except perhaps some preliminary measurements of low level and frequency parametric activity at the 50 kW input power level).

Figures 7 and 8 show low and high density shots with predominantly  $H_e^+$  plasma composition. The eigenmodes are High Q and very similar to those for  $H_2$  or  $D_2$  plasmas.  $R_p$ , on the other hand, is very small (II-2.4). The lower traces of Figures 7 and 8 show RF pickup by the electrostatic Langmuir probe at the HCN port (Figure III-3.2.(5)). The probe signal was shunted by a 1  $\mu H$  inductor acting as a high pass filter. At low density, the electrostatic and magnetic probe signals show the same behavior (high Q eigenmodes), but differ considerably at high density (activity continues even after RF is shut off).

# Radiation Resistance and Probe Signal at High Density

B2, P61, S58

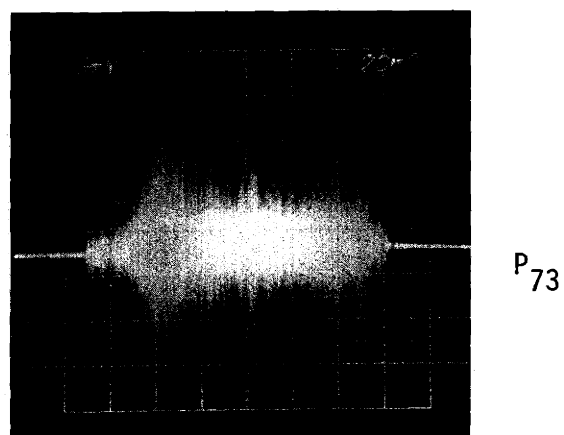
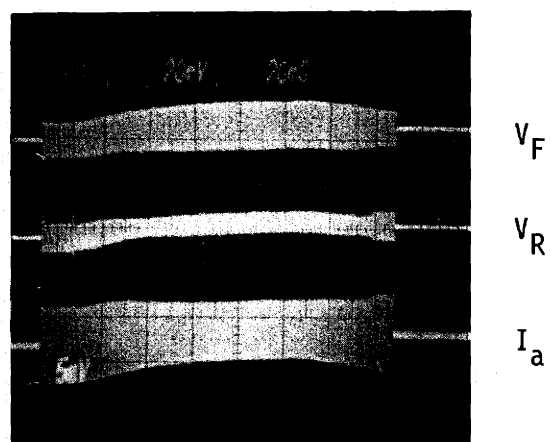
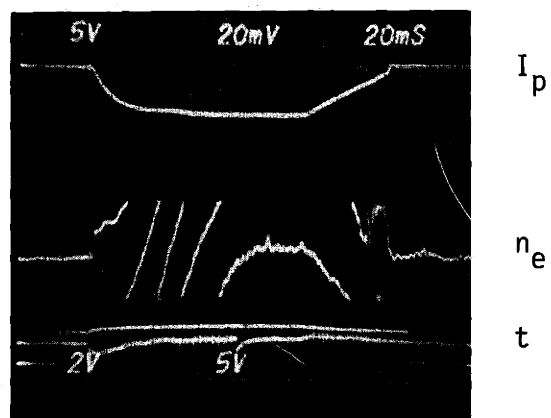


Figure 1

Radiation Resistance, Antenna Electric Field, and  
Probe Voltage as a Function of Density

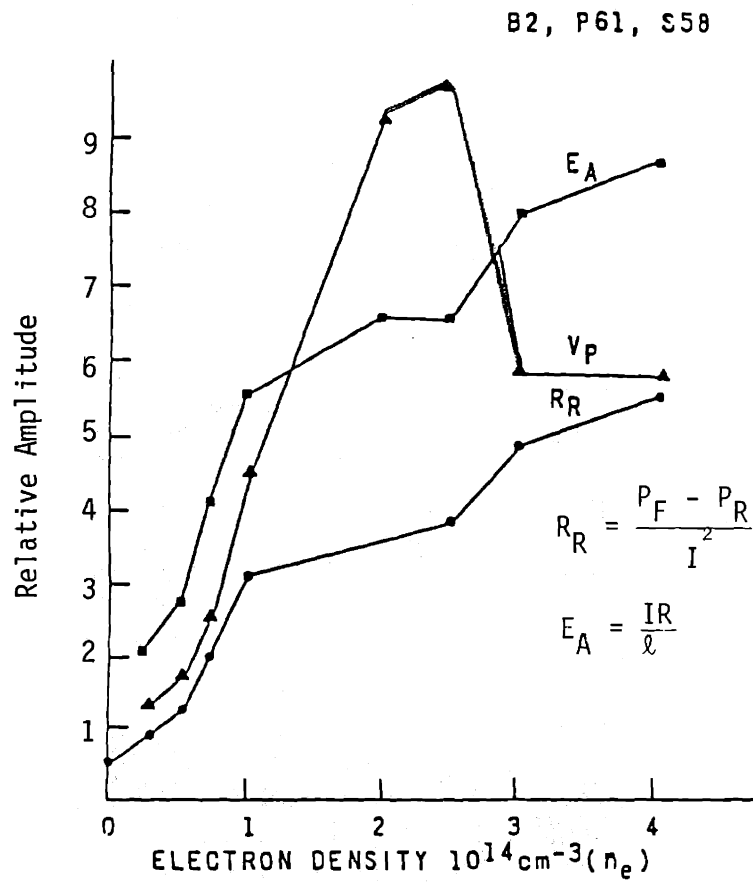
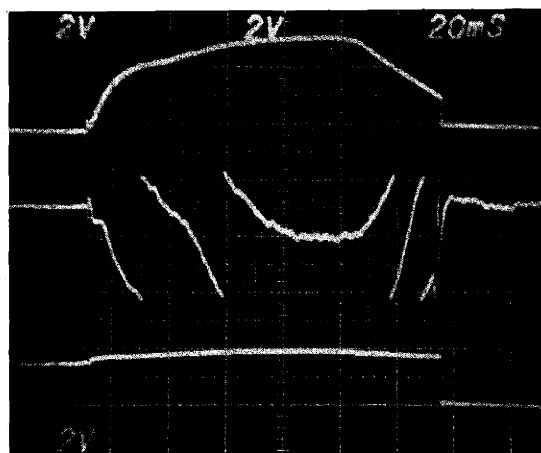
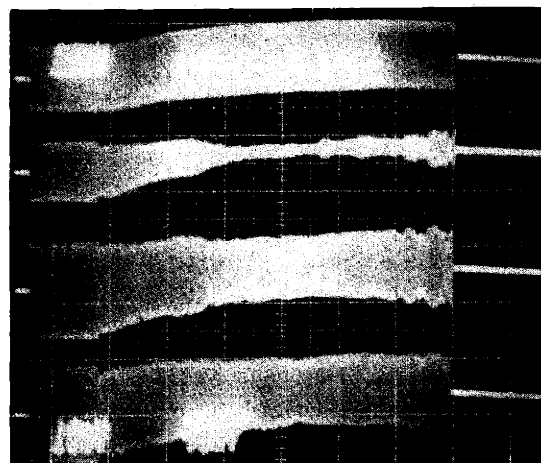
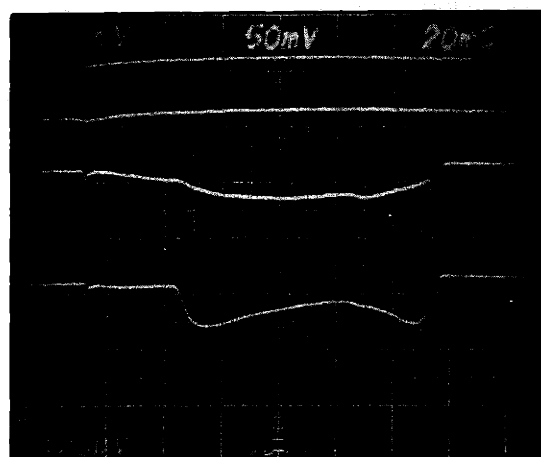
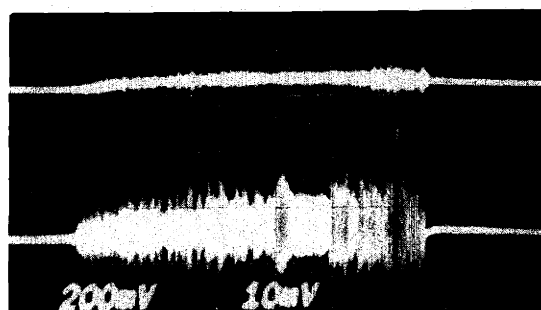


Figure 2

Low Power with  $A_2$  Fed  
in Push-Push Mode

 $I_p$  $n_e$  $t$ 

$H_2$ , 67.5 kA  
200 MHz

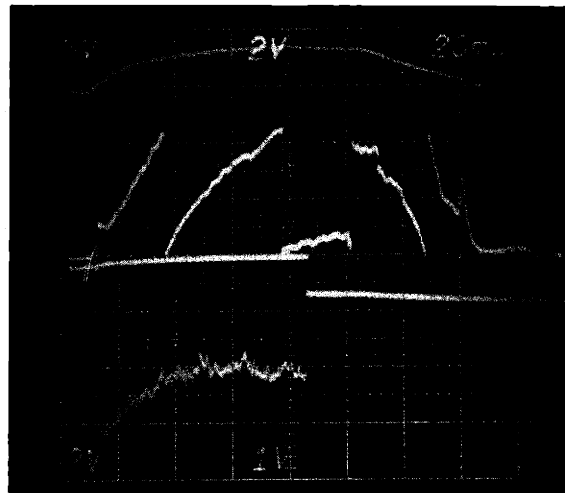
 $V_F$  1.6 W $V_R$  $I_T$  .46 A $I_B$   $R_R = 7.6\Omega$  $L_T$  $L_B$   
pin $H_{\alpha}$ 

68 HCN top

69 HCN Bot.

Figure 3

D<sub>2</sub> and He<sup>+</sup> Mixture  
at 183 MHz

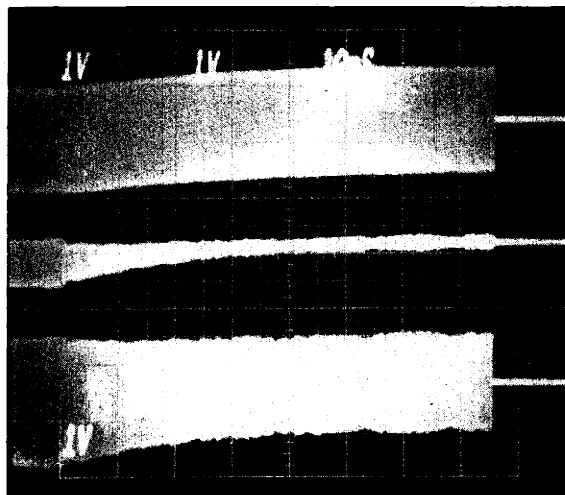


$I_p$

$n_e$

$t$

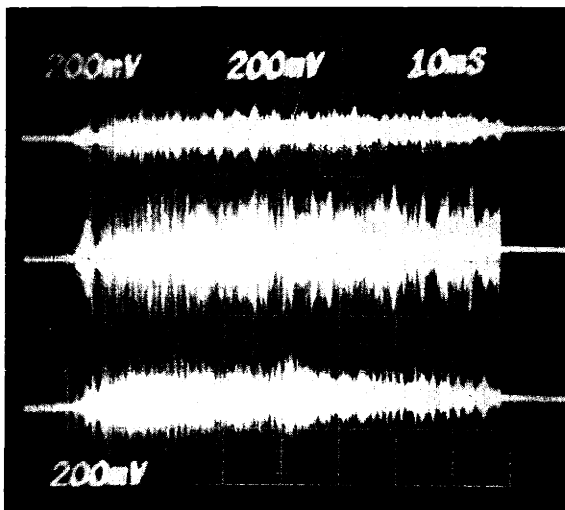
$R_R$



$V_F$

$V_R$

$I_a$



68 HCN top

69 HCN bot

67 Limiter top

Figure 4

# PROBE SIGNALS VS RADIAL POSITION

3/19/80  
B17 P44  
Shot 40 to

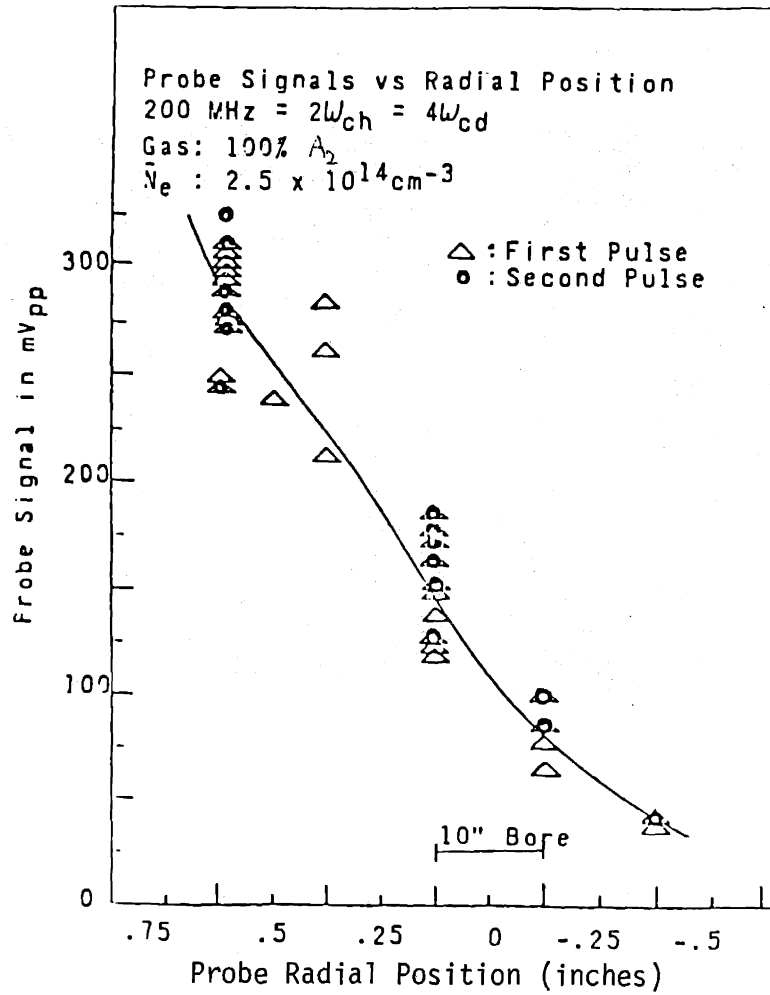


Figure 5

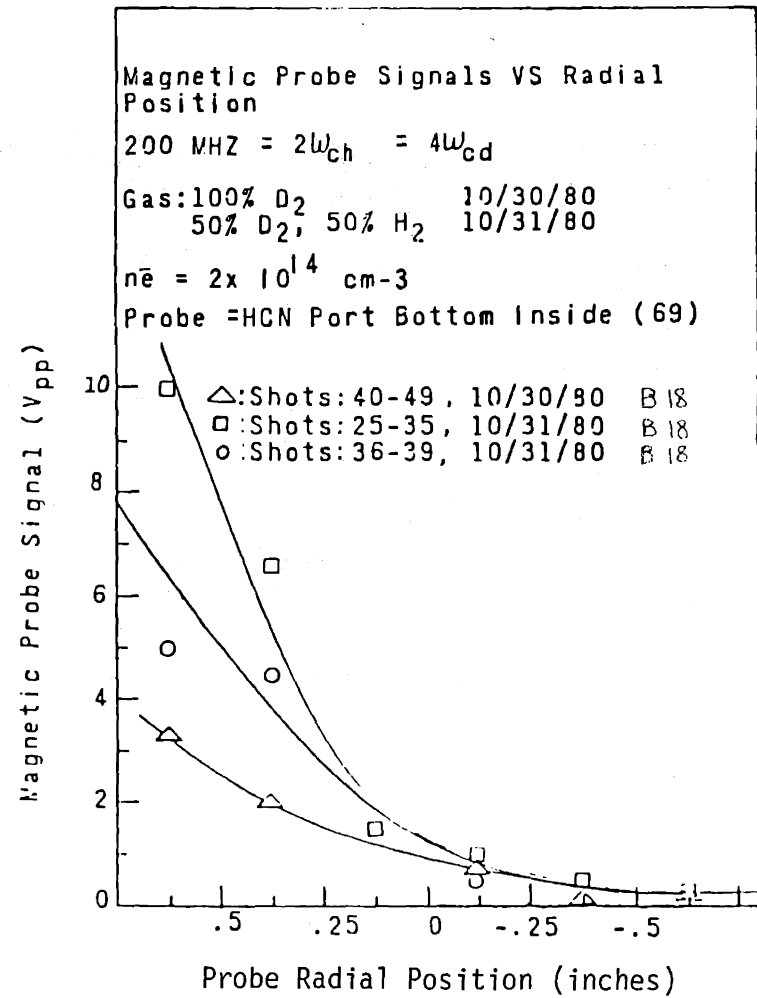
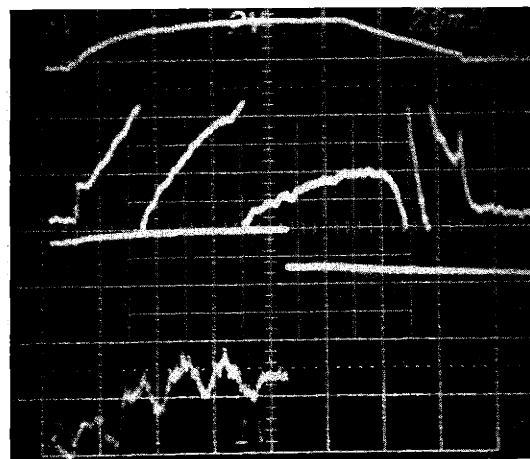
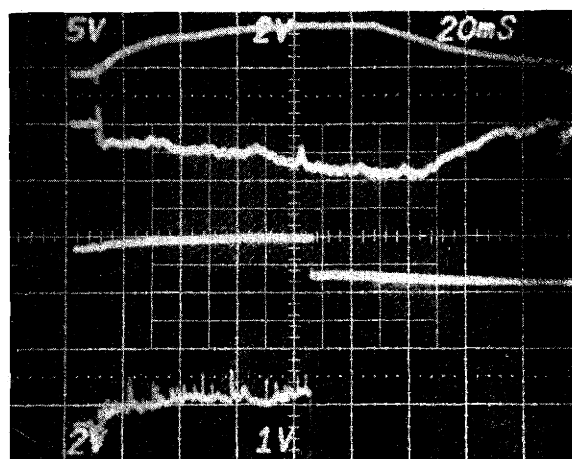


Figure 6

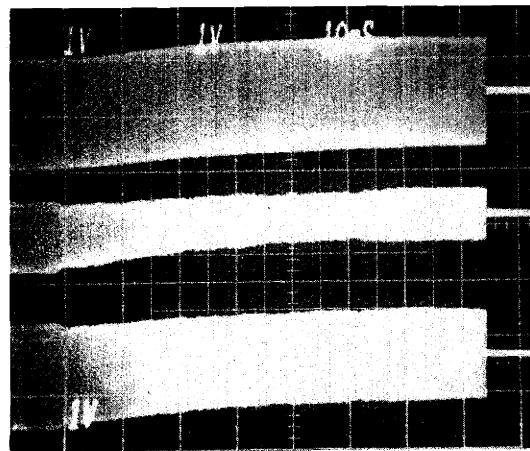
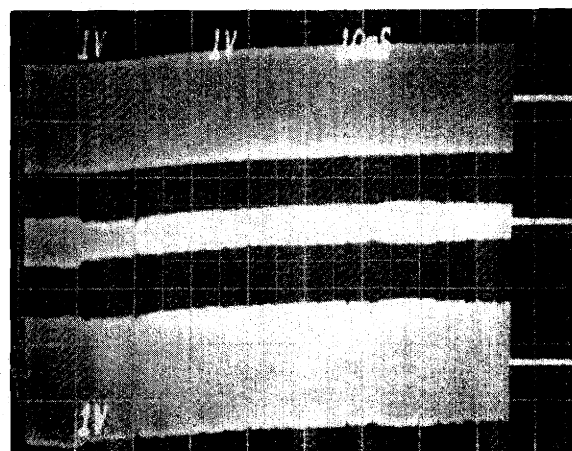


$I_p$

$n_e$

$t$

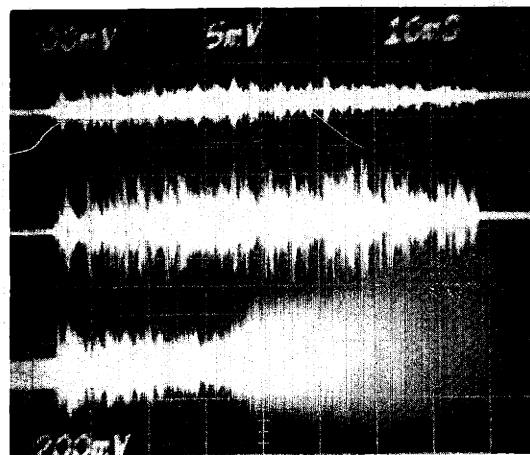
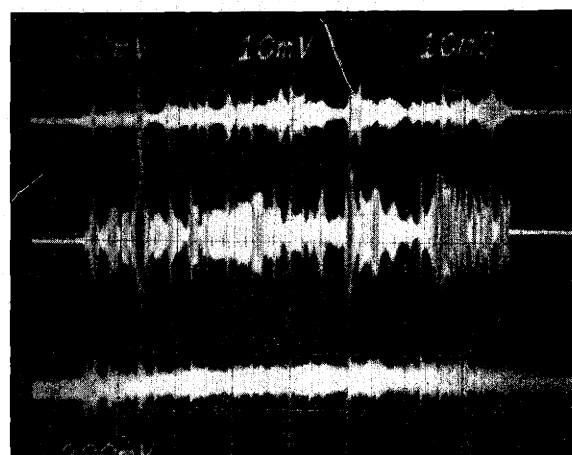
$R_R$



$V_F$

$V_R$

$I_a$



68

69

$L_p$

S45

S43

Figure 7

Figure 8

### II-2.3. Wave phase measurements

As outlined in III-3.3., extensive probe phase measurement schemes were used to estimate parallel and perpendicular wavelengths. Figure 1 shows a typical high density shot with 50% H<sub>2</sub> and 50% D<sub>2</sub>. The lowest trace is the phase (2 $\pi$ /fringe) between the oscillator ( $\approx$  antenna current) and a magnetic probe 90° away from the antenna. For a finite Q system and large dynamic range, the phase is never lost, and the phase difference simply increases as k<sub>||</sub> ( $\propto \sqrt{n_e}$ ). Thus

$$(1) \quad k_{||} = \frac{\phi}{\ell} = \frac{4N}{R} = .61/\text{cm at } 2.2 \times 10^{14}/\text{cm}^3 \quad (N = 8.3)$$

and from IV-2.2. the maximum k<sub>||</sub> (k<sub>⊥</sub> = 0) is

$$(2) \quad k_{||} = \frac{k_A}{\sqrt{1+\Omega}} = \frac{\Omega}{\sqrt{1+\Omega}} \frac{\omega p i}{c} = 4.4 \times 10^{-8} \frac{\Omega}{\sqrt{1+\Omega}} \sqrt{\frac{n}{\mu}} \approx .78/\text{cm}$$

where

$$(3) \quad \begin{aligned} \frac{\Omega}{\sqrt{1+\Omega}} \frac{1}{\sqrt{\mu}} &= 1.15 \quad \text{for } \Omega = 2 \quad \text{H}_2 \\ &= .82 \quad \text{for } \Omega = 2 \quad \text{D}_2 \\ &= 1.26 \quad \text{for } \Omega = 4 \quad \text{D}_2 \\ &\approx 1.2 \quad \text{for } 50\% \text{ H}_2, 50\% \text{ D}_2 \end{aligned}$$

and the eigenmodes could be very near cutoff. Note that including an initial phase shift ( $\phi_o \approx \frac{\pi^2 R}{\lambda_o} \approx \pi$ ) and perhaps one to three initial fringes at breakdown ( $n_e \approx 10^{13}/\text{cm}^3$ )

$$(4) \quad \phi_{10^{13}} = k_{||} .5\pi R \approx 4 \times 10^{-6} \sqrt{n} \approx 2\pi$$

would give k<sub>||</sub> = .76  $\approx$  k<sub>||</sub> (k<sub>⊥</sub> = 0).



At this point, we must caution the reader that, according to Figure IV-2.2.(4), even for a large  $k_{||} = .8 k_{||\text{cutoff}}$ ,  $k_{\perp} = .5 k_A$ , and many radial and poloidal mode numbers are still possible at high density.

Figure 2 shows an expanded view of a pure deuterium shot ( $k_{||} \approx .5/cm$  at  $2 \times 10^{14}/cm^3$ ) with the phase difference between the HCN port and antenna (68/ref), top and bottom of HCN (68/69) and HCN and limiter (68/67). Although more difficult to read (due to the large dynamic range on both inputs to the phase correlator), the phase difference between the HCN and limiter is the same as between the antenna and HCN ports. Of course, the phase difference between the antenna and limiter was twice as large as between the antenna and HCN ports (Figure 3). The shots shown in Figures 2 and 3 were at the 40 kW level, and the antenna broke down after 60 msec.

The phase difference between the top and bottom of the HCN (68/69, Figure 2) does not indicate any average increase in phase, and is very noisy, as expected with a mixture of many randomly phased eigenmodes. Eigenmode identification at high density would clearly be very difficult if not impossible.

Fringe measurements during a single run were also done at 90 and 180 MHz, and as expected from (3)

$$(5) \quad \frac{\phi_{180}}{\phi_{90}} = \frac{1.26}{.82} \approx 1.54$$

and there were about twice as many fringes at 180 MHz as at 90 MHz for about the same experimental conditions. A similar effect was observed by changing the magnetic field from 40 to 80 kG in II-2.4.

Figures 4 and 5 show phase measurements between the reference and a near-field probe (5/2) and reference and HCN (5/68), with a linear

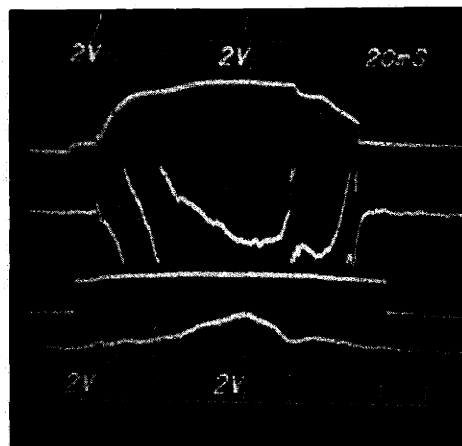
and cosine phase correlator for  $\text{He}^3$  and  $\text{He}^4$  plasmas at 183 MHz. Note how the linear correlator is easier to read. The cosine correlator had a bandwidth of about 10 MHz. As expected, the near-field phase deviation was less than  $\pm\pi$ , and was highly dependent on eigenmode resonances. The radiation resistance for these He shots was much lower than for  $\text{H}_2$  or  $\text{D}_2$  at the same density, in agreement with the accepted belief that helium profiles are much narrower. ( $k_{\text{AHe}} \cong k_{\text{AH}}$ , but evanescent edge is much larger).  $k_{\parallel}$  is also very large ( $\cong .6/\text{cm}$ ), even for medium density ( $n_e \cong 1.6 \times 10^{14}/\text{cm}^3$ ).

Figure 6 shows an expanded view of phase measurement at medium power ( $\cong 50$  kW). Note the slight phase difference between the top and bottom HCN probes, and the usual slight dip in the thermonuclear neutron production rate.

Figure 7 shows a close up of fringes and soft X-rays, where we note the correlation between the "positive going" fringe sawteeth and the central soft X-rays. This correlation could be an argument supporting the hypothesis that these measurements represent bulk  $k_{\parallel}$ .

Phase Measurements at 206 MHz and D<sub>2</sub> 50%, H<sub>2</sub> 50%

B18, P66, S49

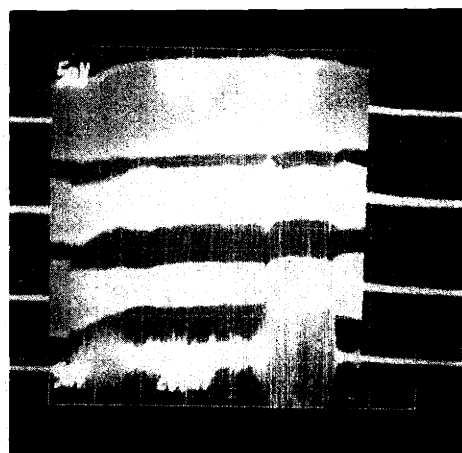


$I_p$

$n_e$

t

Neutrons

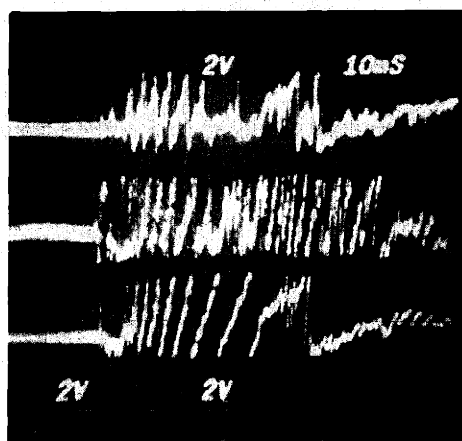


$V_F$

$V_R$

$I_a$

HCN Bottom, 69



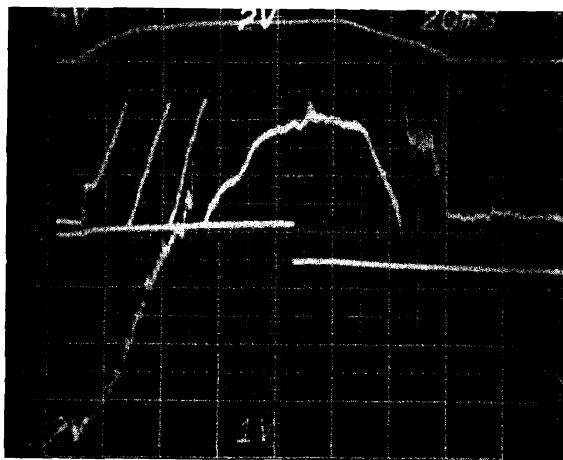
Ref/HCN Top, 68

Ref/HCN Bottom in., 69

Ref/HCN Bottom out., 70

Figure 1

Phase Measurements  
at 183.5 MHz and  
D<sub>2</sub>

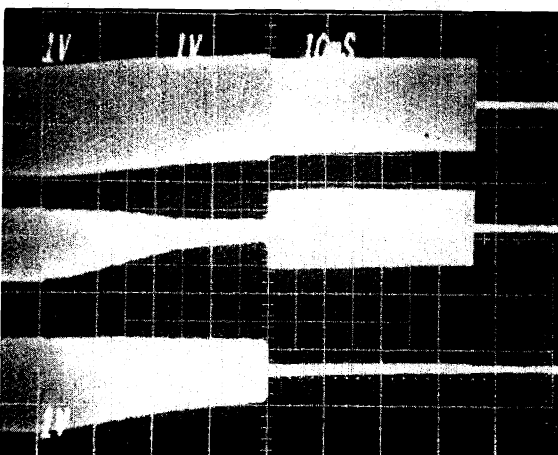


$I_p$

$n_e$

$t$

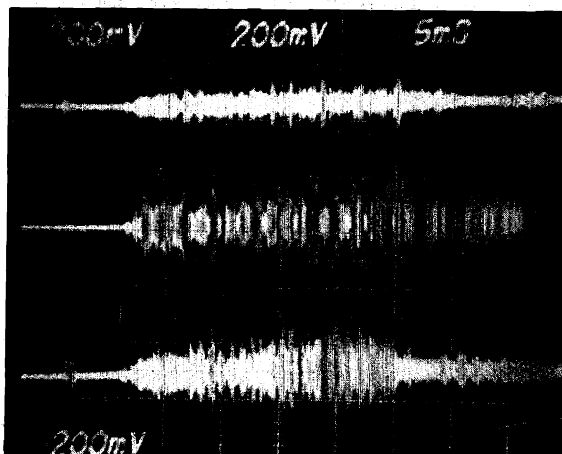
$R_R$



$V_F$

$V_R$

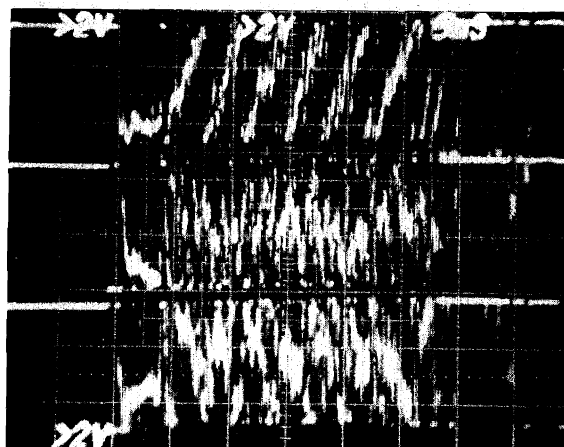
$I_a$



HCN Top 68

HCN Bottom 69

Limiter 67



68/REF

68/69

68/67

Figure 2

B13, P4, S12  
Deuterium  
183.5 MHz

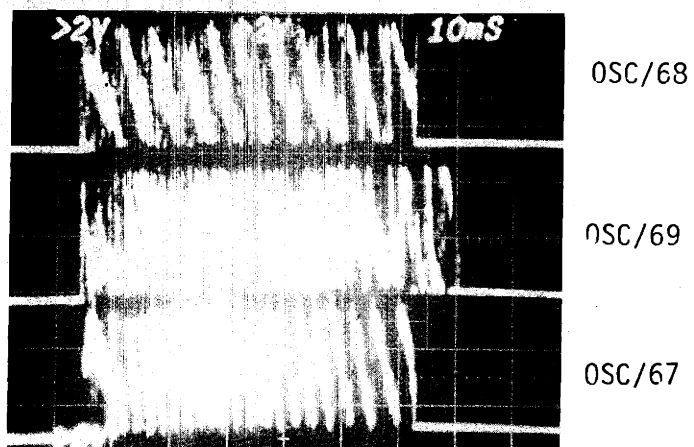
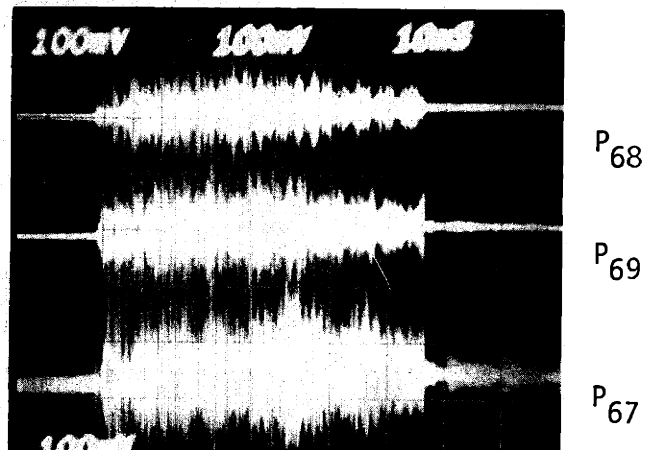
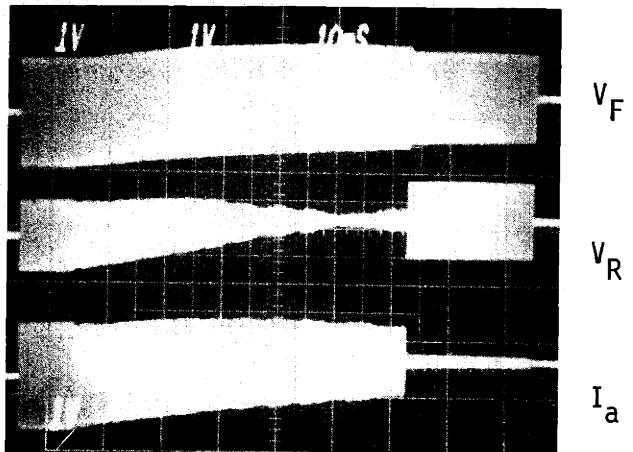
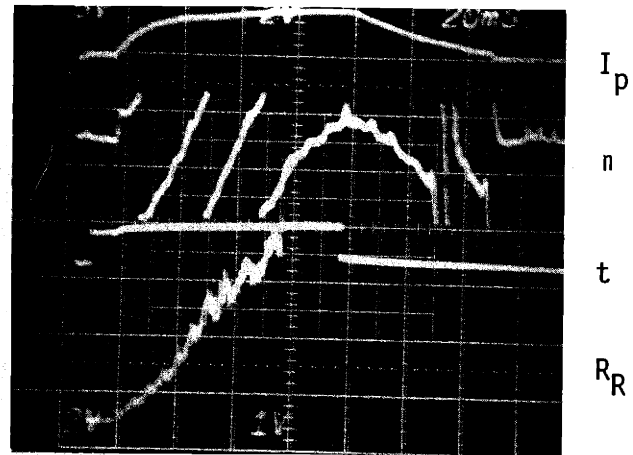
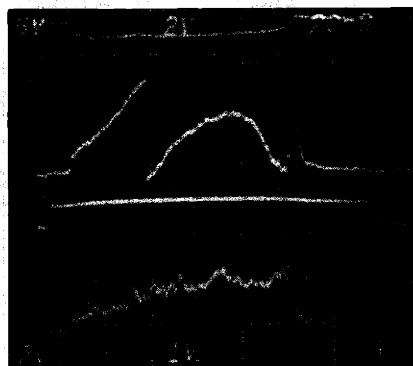


Figure 3

# Phase Measurements at 183 MHz and He<sup>3</sup>

P33, B10, S37-38

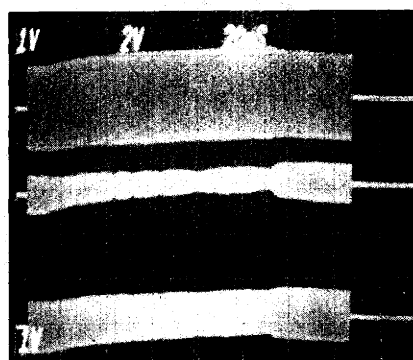


$I_p$

$n_e$

$t$

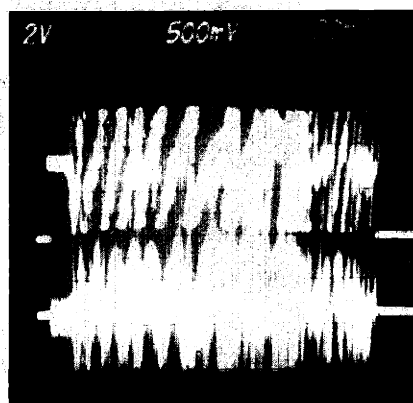
$R_R$



$V_F$

$V_R$

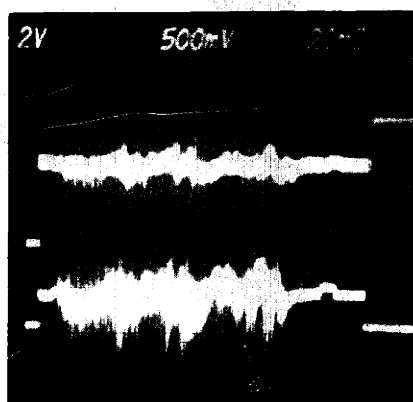
$I_a$



5/68

S37

5/68

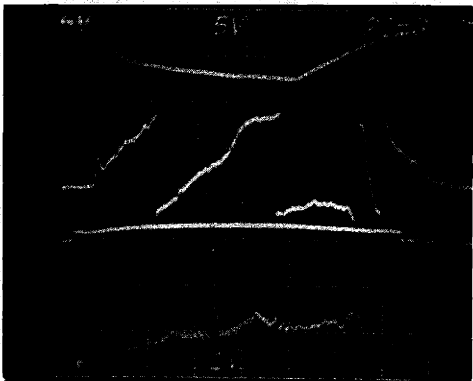


5/2

S38

5/2

Figure 4

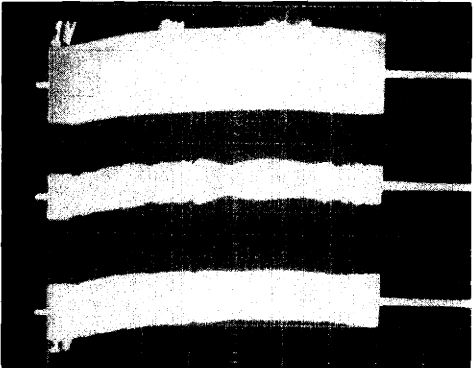


$I_p$

$n_e$

$t$

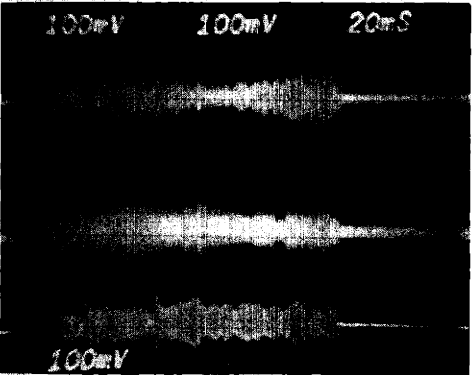
$R_R$



$V_F$

$V_R$

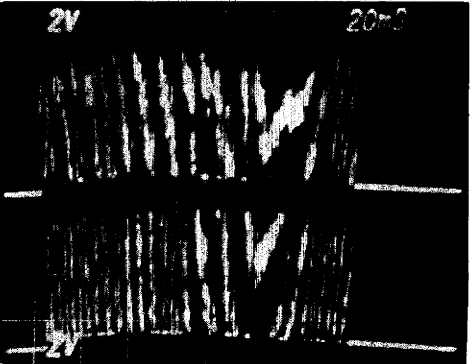
$I_a$



68 HCN

67 Limiter

2 Near-field



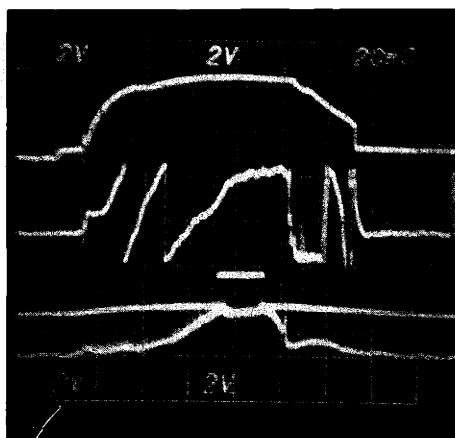
osc/68

osc/67

Figure 5

# Phase Measurements at 206 MHz and D<sub>2</sub>

B18, P51, S53

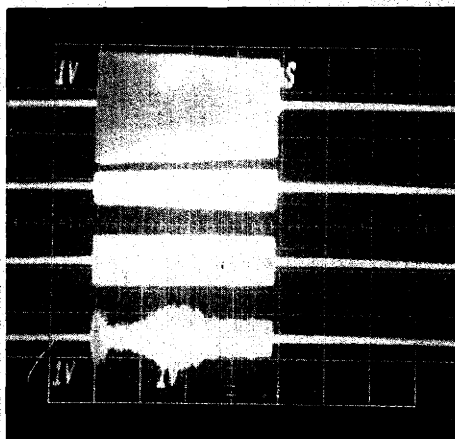


$I_p$

$n_e$

$t$

neutrons

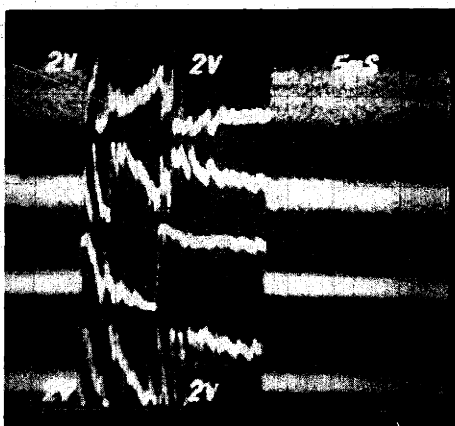


$V_F$

$V_R$

$I_a$

HCN Bottom inside 69



68/69

REF/HCN Top outside 68

REF/HCN Bot. inside 69

REF/HCN Bot. outside 70

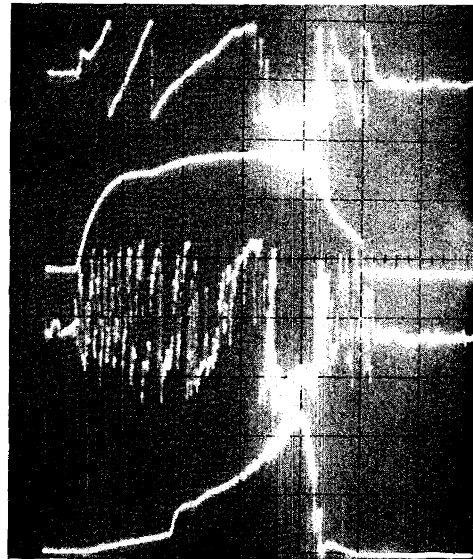
Figure 6



Sawteeth on  
 $k_{||}$  Measurements

205.5 MHz  
100%  $D_2$   
65.3 kA

B18, S45, P64  
11/1/80

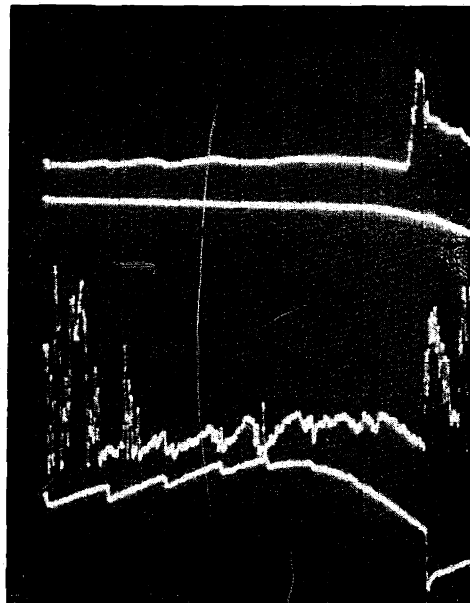


$n_e$

$I_p$

$k_{||}$

center soft X-rays

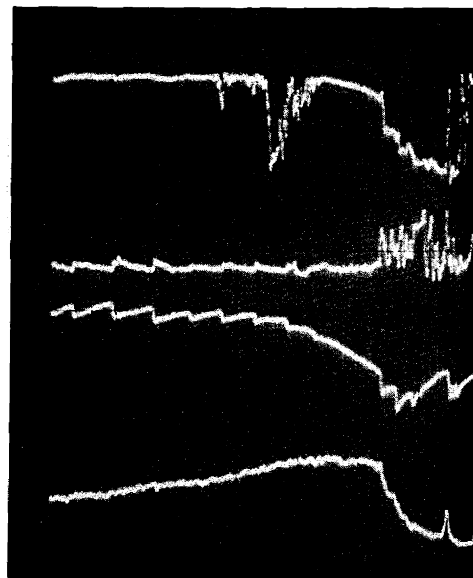


$n_e$

$I_p$

$k_{||}$

center soft X-rays



$n_e$

$I_p$

$k_{||}$

center soft X-rays

Figure 7

## II-2.4. Radiation resistance and magnetic field scans

Radiation resistance is the single most important parameter to characterize ICRF wave coupling. Figure 1 shows how  $R_R$  roughly increases linearly with density, and is slightly higher for deuterium than hydrogen. Although  $R_R$  is usually quite repeatable within a given run, significant changes were noted from run to run with roughly similar experimental conditions.  $R_R$  background seemed essentially independent of plasma currents and magnetic field. Plasma in out position is believed to be responsible for much of the variations, but this was not confirmed, since all the position loops on Alcator were inoperative (Figure 2).

Most of the data in this work was taken with the  $A_2$  unshielded antenna but  $R_R$  behaved essentially the same for the  $A_1$ ,  $A_2$  and  $A_4$  antennas, except of course, for their relative value. At 200 MHz and  $3 \times 10^{14}/\text{cm}^3$ ,  $A_1$  was about 3 ohms while  $A_2$  and  $A_4$  were typically 8 ohms. At 90 MHz,  $R_R$  was lower by a factor of about 4, so that  $A_2$  could only couple 30 kW or less.

Figures 3 and 4 show typical 90 MHz shots with  $D_2$  fill and  $H_2$  minority (recycling), and with the resonant layer either in (65.4 kA) or outside the plasma (79.2 kA). The radiation resistance was calculated with a high speed on-line analog computer (III-3.1.).  $R_R$  background is independent of magnetic field, but the eigenmode spikes on  $R_R$  and probe signal are very sensitive to layer position. Figure 5 is a plot of probe signal versus layer position, and unambiguously exhibits strong magnetic field dependence in agreement with theory. The minority concentration was small (<5%), so the TIIH layer was very near the cyclotron layer. Also note the difference between the field probe signals for the two resonant layer positions. This may

suggest that eigenmode spikes are due to toroidal resonances rather than onsets, because even with short damping lengths (no probe spikes),  $R_R$  and near-field should still exhibit large onsets (which they do not in Figure 3).

Figures 6 to 9 show the results of similar experiments at 180 MHz. Only three fields were used in this case (smaller increments were used in other experiments), but many shots were averaged to gain good statistics. No significant field dependence was found on either the probe signals or radiation resistance (except for the expected  $k_{||} \propto 1/B_0$  dependence). Medium power field scans (II-3.3.) were also done in 50%  $H_2$  and 50%  $D_2$  plasmas, again with about the same result (except for the neutral flux dependence), indicating that second harmonic damping is weak.  $R_R$  background is not necessarily expected to change (both single perpendicular pass and high Q average  $R_R$  are independent of damping), but eigenmode peaks should be inversely proportional to damping if they are representative of the bulk power and second harmonic damping (the most important unknown).

Figure 10 shows a high density  $D_2$  fill and  $He^4$  pulse shot, where we again note (II-2.3.) how  $R_R$  stays small with a predominantly helium plasma. Figures 11 and 12 show  $H_{\alpha}$  and total light in front (MW bottom) and inside (through holding pins) the antenna for two low power, high density, disruptive plasma shots. Note in particular, the close resemblance between the  $R_R$  and the light signals. The increase in  $R_R$  could be due to a broader profile (thinner evanescent layer) or increased near-field losses (also edge density). At medium power, this light increases by 30% (II-3.3) all around the torus. Also note (Figure 12) the close resemblance between the alcohol interferometer fringes ( $\phi \propto n_e$ ) and the ICRF  $k_{||}$  fringes ( $\phi \propto \sqrt{n_e}$ ).

# Radiation Resistance vs. Density

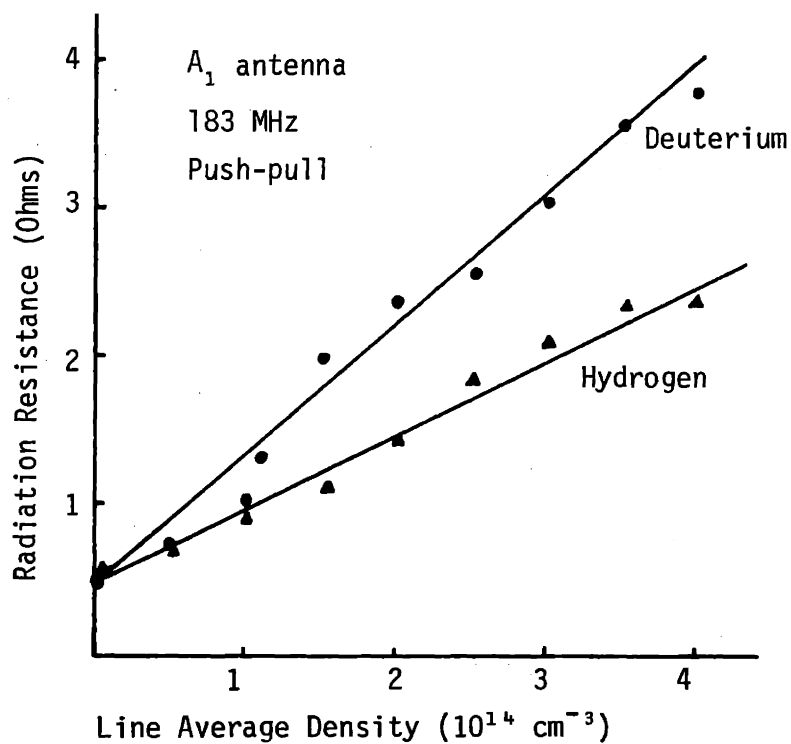


Figure 1

# Radiation Resistance vs. Density

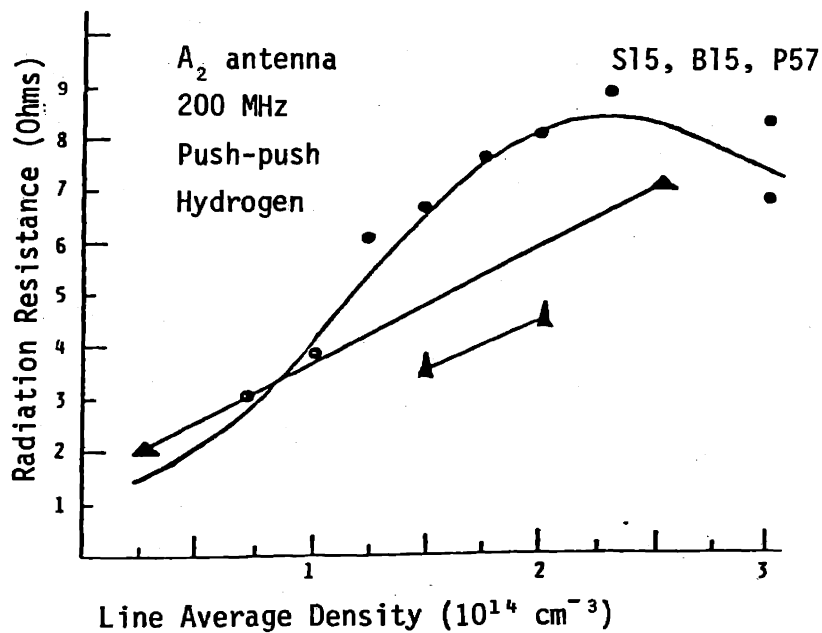


Figure 2

# Radiation Resistance with Resonant Layer at Plasma Center

B9, P7, S21

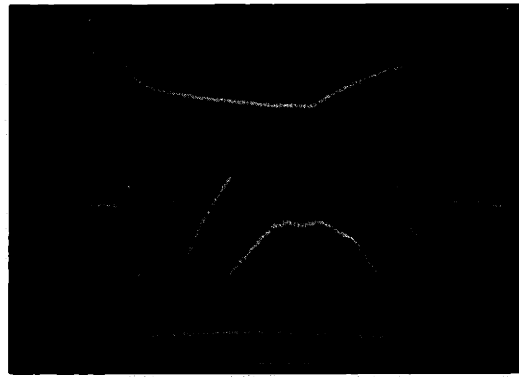
90 MHz

D<sub>2</sub> fill

H<sub>2</sub> minority

65.4 kA

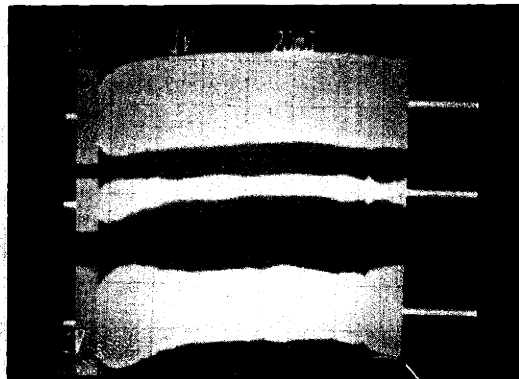
2.3 at 2 fringes



$I_p$

$n_e$

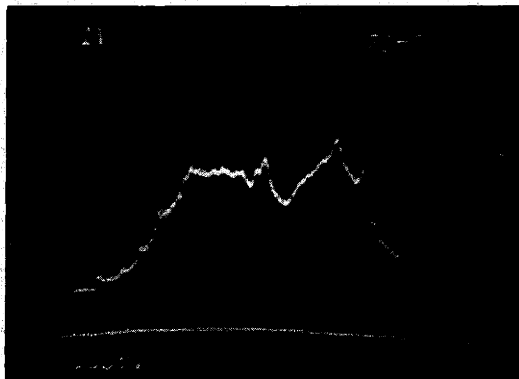
$t$



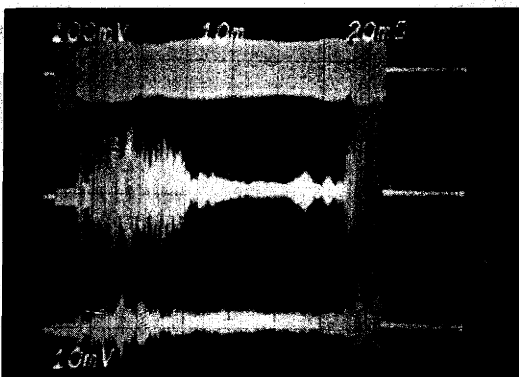
$V_F$

$V_R$

$I_a$



$R_R$



Near-field 2

Limiter 67

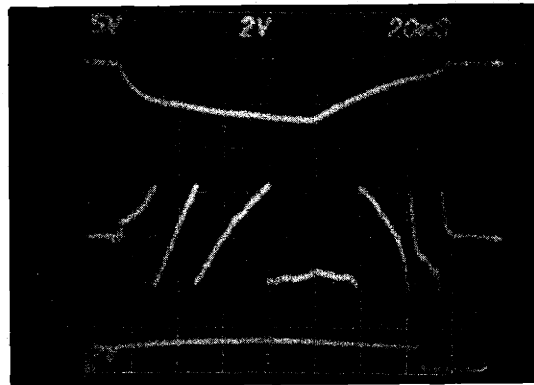
HCN Top 68

Figure 3

# Radiation Resistance with Resonant Layer Outside Plasma

B9, P7, S32

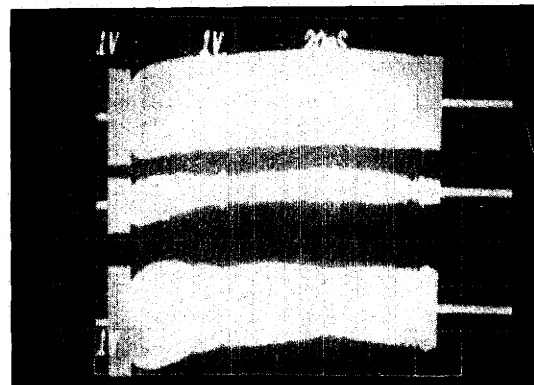
90 MHz  
D<sub>2</sub> fill  
H<sub>2</sub> minority  
79.2 kA  
2.3 at fringes



$I_p$

$n_e$

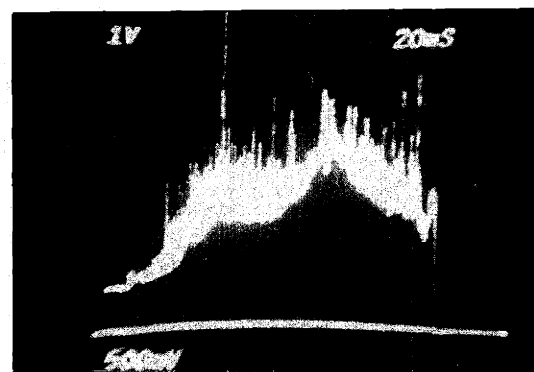
t



$V_F$

$V_R$

$I_a$



$R_R$



Near-field 2

Limiter 67

HCN Top 68

Figure 4

# Probe Signals and Radiation Resistance as a Function of Toroidal Magnetic Field

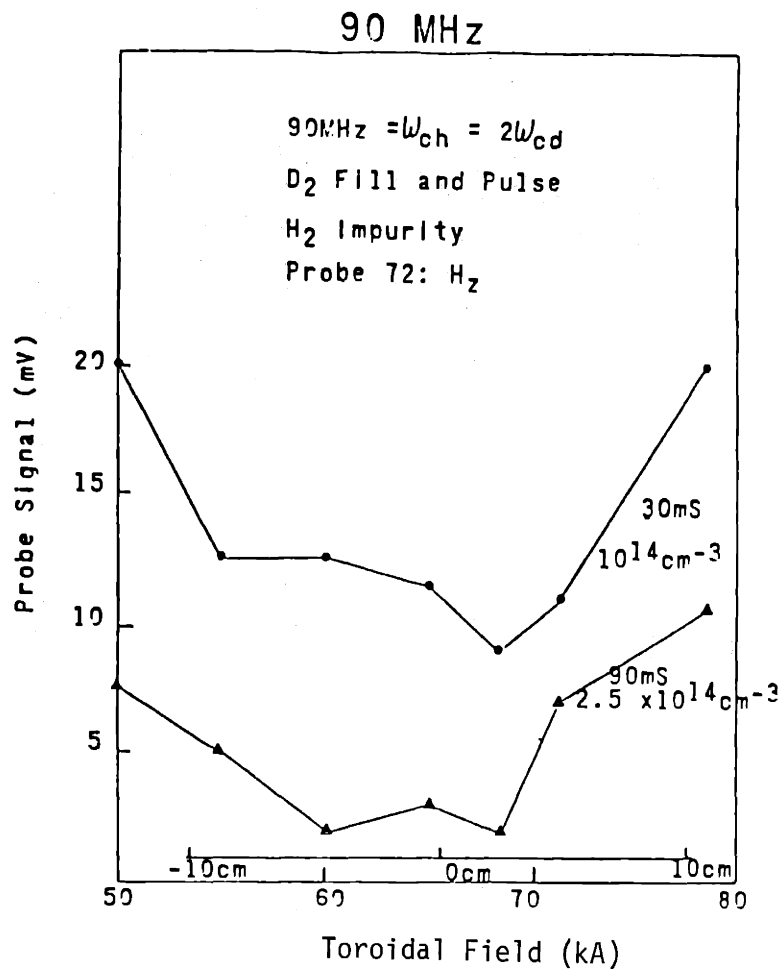


Figure 5

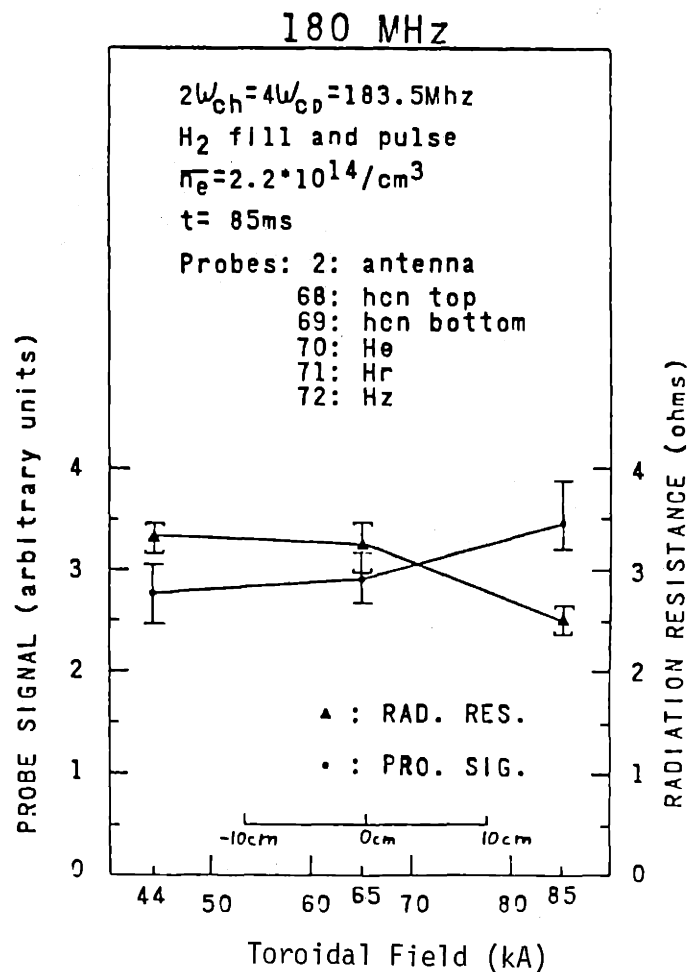


Figure 6

# Resonant Layer on High Field Side

B10, P16, S50

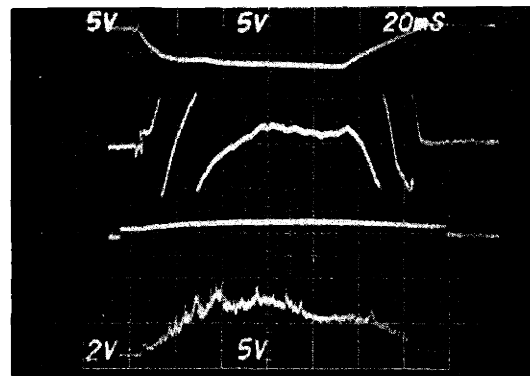
2/13/80

Hydrogen

183.5 MHz

$I_B = 44 \text{ kA}$

$3 \Omega$  at 2 fringes

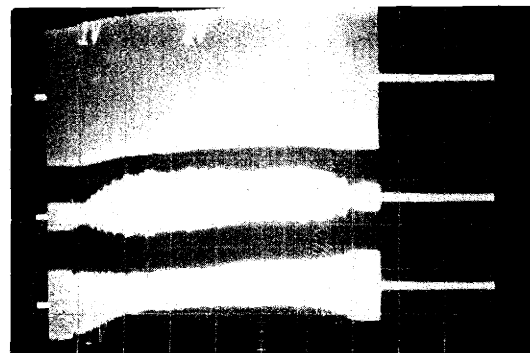


$I_p$

$n_e$

$t$

$R_R$



$V_F$

$V_R$

$I_a$

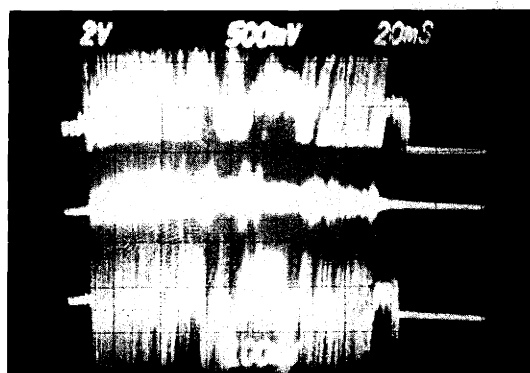


P 2

P 70

P 71

P 72



osc./72

5/72

Figure 7



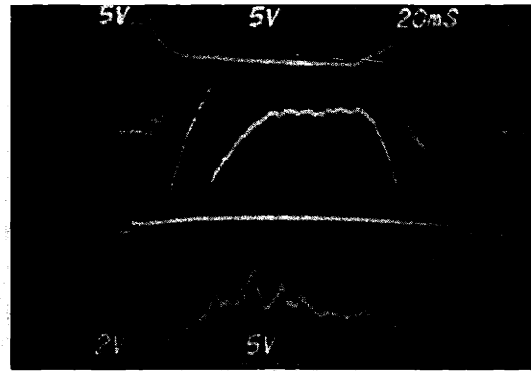
2/13/80

Hydrogen

183.5 MHz

$I_B = 65 \text{ kA}$

$3 \Omega$  at 2 fringes

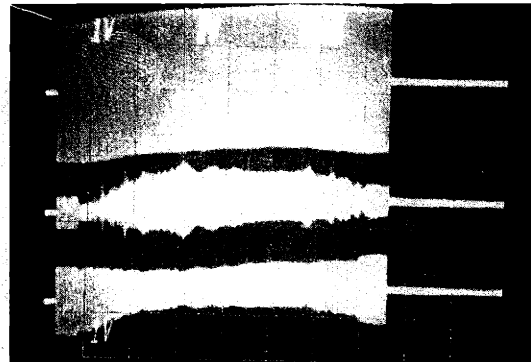


$I_p$

$n_e$

$t$

$R_R$



$V_F$

$V_R$

$I_a$



P 2

P 70

P 71

P 72



osc. /72

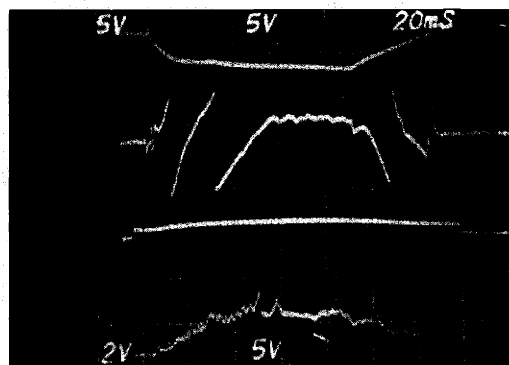
5/72

Figure 8

# Resonant Layer on Low Field Side

B10, P18, S55  
2/13/80

Hydrogen  
183.5 MHz  
 $I_B = 85.8 \text{ KA}$   
 $3 \Omega$  at 2 fringes



$I_p$

$n_e$

$t$

$R_R$



$V_F$

$V_R$

$I_a$

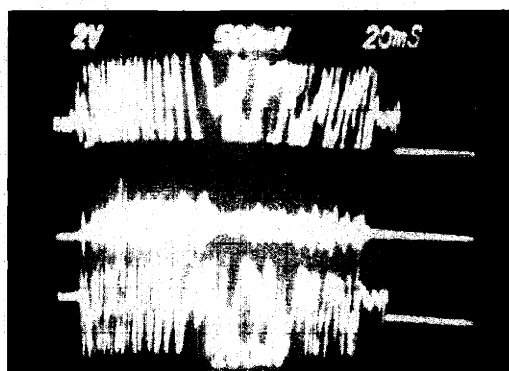


P 2

P 70

P 71

P 72



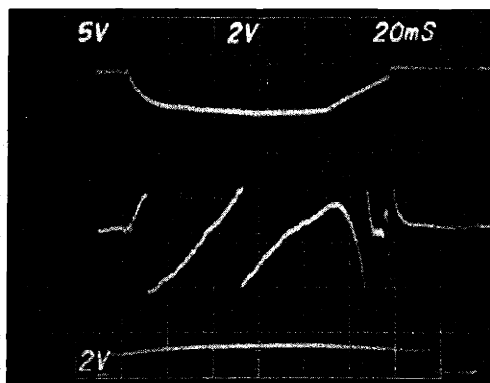
osc./72

5/72

Figure 9

65.2 Ka

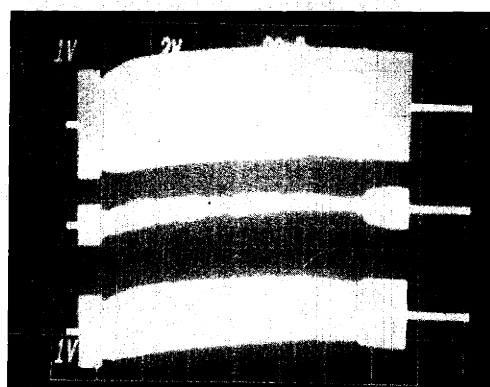
1.6  $\Omega$  at 2 fringes



$I_p$

$n_e$

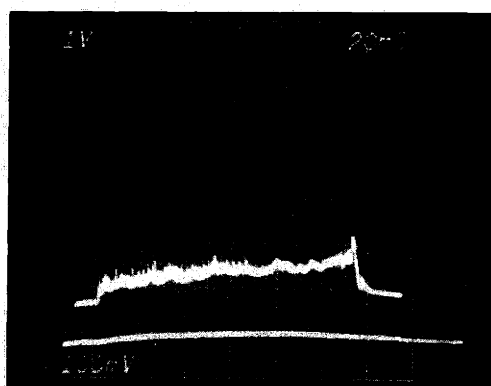
t



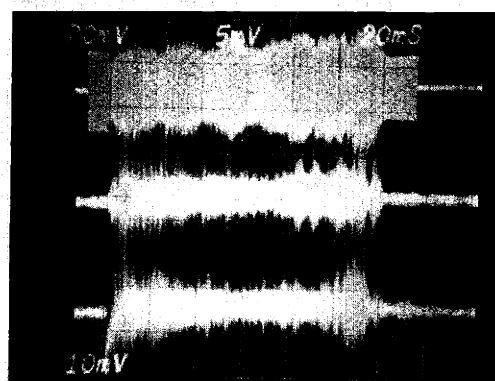
$V_F$

$V_R$

$I_a$



$R_R$

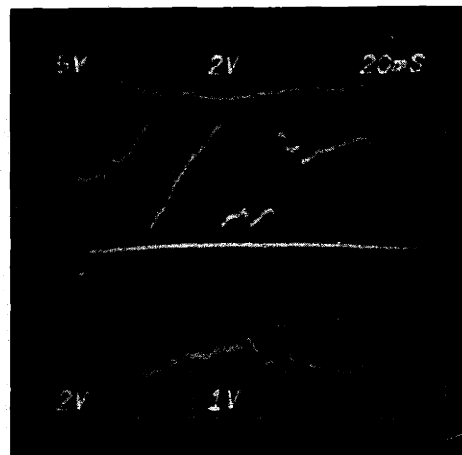


Near-field 2

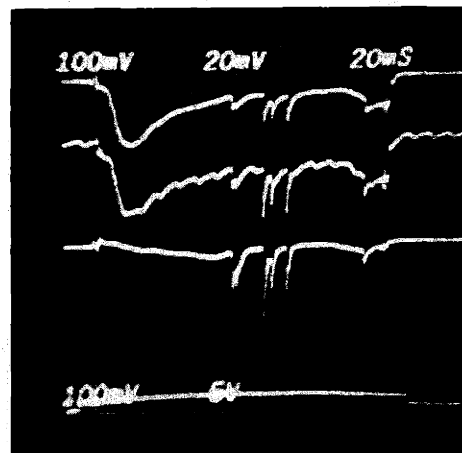
Limiter 67

HCN Top 68

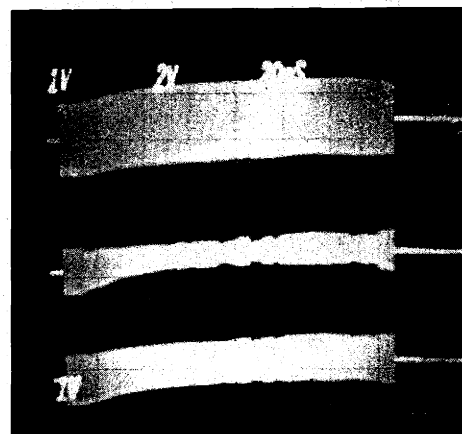
Figure 10



$I_p$   
 $n_e$   
 $t$   
 $R_R$



$H_\alpha$   
 $L_t$   
 $pin$   
 $t$



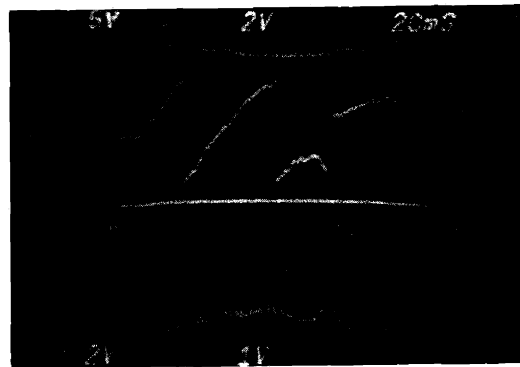
$V_F$   
 $V_R$   
 $I_a$

Figure 11

# Effect of Disruptions on $H_{\alpha}$ , total light and $R_R$

B9, P71, S50

2/10/80 D<sub>2</sub>, 183 MHz

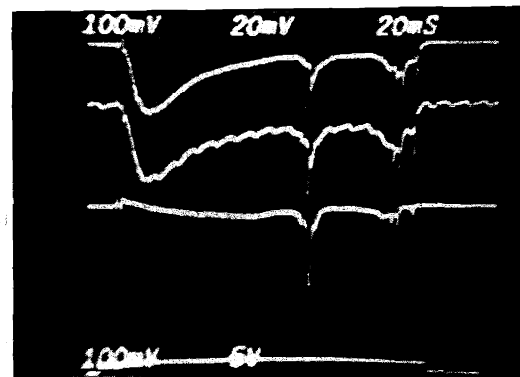


$I_p$

$n_e$

$t$

$R_R$

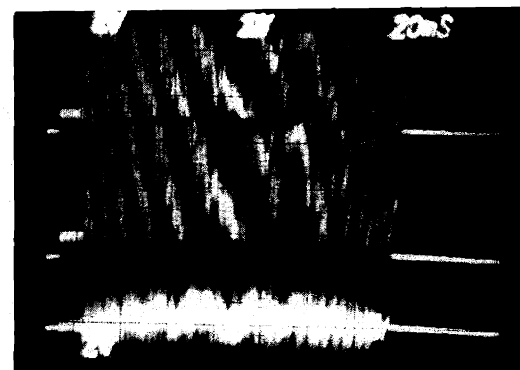


$H_{\alpha}$

Total Light

Arc Det

$t$



OSC./68

OSC./68

2/5

Figure 12

## II-2.5. Wave coupling experiments

Aside from the carefully controlled  $A_1$ ,  $A_2$  and  $A_4$  antenna coupling experiments, many other simple preliminary measurements were also performed. Energizing only half of the antenna produced between half and a quarter of the original  $R_R$ , in rough agreement with IV-1.3, IV-6.5, and IV-7.1.  $R_R$  also seemed independent of phasing, further supporting IV-6.5.

It was most unfortunate that  $R_R \propto d^2$  scaling (center conductor to wall distance) was not proven ( $a + d$  remained constant), but the antenna was kept as close as possible to the plasma to maximize coupling (both the  $A_1$  Faraday shield and the  $A_2$  side limiters showed some minor plasma erosion). The Faraday shield did not affect coupling, as  $A_2$  and  $A_4$  proved to be almost identical in all respects (slightly lower  $R_R$  as expected from bench tests III-2.5.).

Figure 1 shows capacitive probe (III-3.2.) coupling at high density and 180 MHz. The front of the 1.2 inch diameter molybdenum block was about 3/4 inch from the chamber wall. Note how the probe impedance was almost exactly  $50 \Omega$  without any matching network, in plausible agreement with Section III-3.2.

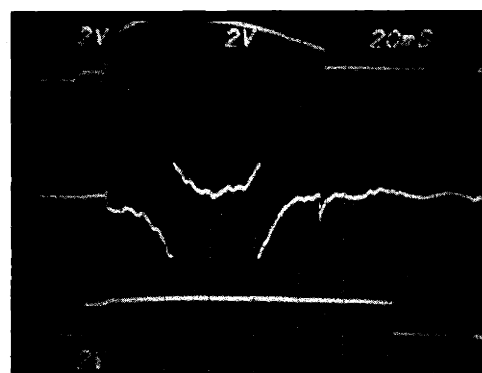
$$(1) \quad Z_p = \frac{V_F + V_R}{V_F - V_R} 50 \Omega \approx R_R$$

From shot to shot during a single run, it was consistently noted that, for a given power ( $\approx 10$  watts), the capacitive probe and the  $A_2$  antenna produced roughly equivalent probe signals around the torus. Although very easily performed, radial probe scans were not done because the probe had to be removed for

the new  $k_{||}$  array (the first array was destroyed by electron runaways). Note the discrete eigenmode structure of the probe signal at extremely low density after the plasma shot. These afterglow effects were never observed with  $A_1$ ,  $A_2$  or  $A_4$  (probe signals are 50 msec/div).

The main tokamak limiter (Figure III-3.2.(13)) was also used as a transmitting antenna and receiving probe. Although the RF connection was very inductive, the limiter provided a reasonably good match. Crudely assuming  $E_r \approx E_\theta$ , and monitoring the received voltage on the limiter, produced roughly consistent results with magnetic probe signals and  $R_p$  (II-2.2).

# Capacitive Probe Measurements at 200 MHz

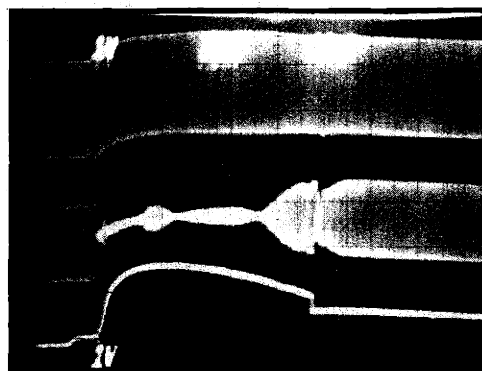


S31, B19, P18  
D<sub>2</sub>, 200 MHz

$I_p$

$n_e$

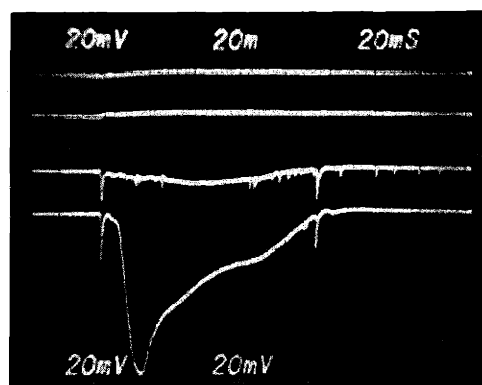
t (not used)



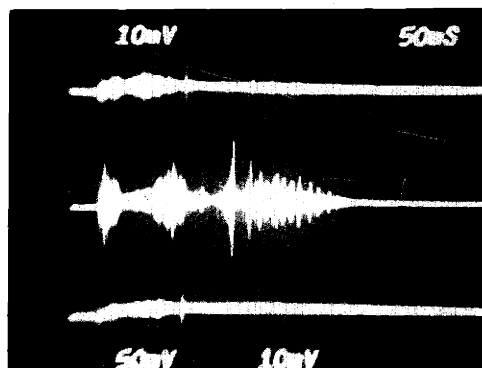
$V_F$

$V_R$

$I_p$



Light



Magnetic  
Probes

Figure 1



### II-3. Medium Power Heating Experiments

#### II-3.1. High density regime

Most of the medium power heating experiments were done in the  $2\omega_{cH}$  and  $4\omega_{cD}$  regimes, for either pure or mixed (50/50%) hydrogen or deuterium plasmas at  $2.5 \times 10^{14} \text{ cm}^{-3}$  and 65 kG.

Experimental conditions (disruptive plasmas and lack of diagnostics and machine time) were not the most favorable, and most of the following results were obtained with the  $A_2$  unshielded antenna. Figures 1 and 2 show typical pure hydrogen shots with 40 kW. Note how the plasma current and density remain unchanged (loop voltage and plasma position also remain constant), while the plasma edge light ( $H_\alpha$  and total light) at the antenna increases by 30%. Figures 3-5 show the fast neutral fluxes for the shots shown in Figures 1 and 2. Figure 5 shows the fast neutral spectra for the third RF pulse of Figures 1 and 3, which has an "effective temperature" of about 300 eV above background (the spectrometer was uncalibrated in these runs). Unfortunately, the flux rise and fall time is less than 100  $\mu\text{sec}$ , and is not indicative of bulk heating. Figures 1 to 5 are for two consecutive shots of the same run, and show the close repeatability of the RF effects on the plasma. From run to run, on the other hand, repeatability was not nearly as good, especially for the usual but inconsistent soft X-ray increase (sometimes only center, then edge also, and ranging from 0 to 50%). Almost identical results were also obtained with pure deuterium, but with a very repeatable slight dip in the neutron rate (Figure II-2.3.(6)). Edge Langmuir probe diagnostics showed no measurable effect on either the plasma edge temperature or density (II-3.3).

Figures 6 to 17 show the basic results of a carefully executed field scan between 50 and 80 kG, for a 50% H<sub>2</sub> and 50% D<sub>2</sub> plasma at 200 MHz, 55 kW and with the A<sub>2</sub> antenna. Figures 6 and 7 show the high (5 keV) and low (500 eV) energy components of the deuterium fast neutrals versus magnetic field (resonant layer position). Note how the low energy component doubles independently of layer position, while the high energy component increases by an order of magnitude when the layer is at the plasma center. Figures 8 and 9 show similar results for the hydrogen spectra, except for a very large increase in both the low and high energy components when the resonant layer is at the low field side of the plasma in the antenna near-field. Figures 10 to 13 show the unreduced neutral fluxes, as well as the neutral spectras for the resonant layer at the plasma center.

Figures 14 to 17 show the radiation resistance and probe signals at the HCN bottom inside (high field side), bottom outside and top outside (Figure III.3.(1)). Radiation resistance is again (II-2.4) independent of layer position (within experimental repeatability), and probe signals are only weakly dependent (slight changes in plasma position could produce even greater differences). One could possibly argue that a slight dip in probe signal exists when the resonant layer is at the center and high field side of the plasma (at the particular probe position that is monitored).

To lowest order and for bulk heating, we can typically calculate

$$(1) \quad P_{OH} = IV \approx 150 \text{ kA} \times 2.0 \text{ V} \approx 300 \text{ kW} \approx 10 P_{RF}$$

$$(2) \quad t_{RF} \gg \tau_E \approx 10 \text{ msec}$$

$$(3) \quad T_{RF} \approx T_{OH} \left[ 1 + \frac{P_{RF}}{P_{OH}} \right] \approx 770 \text{ eV}$$

and the high energy (5 keV) neutral flux is

$$(4) \quad J \propto e^{-E/T}$$

$$(5) \quad \frac{J_{RF}}{J_{OH}} = e^{E \left( \frac{1}{T_{OH}} - \frac{1}{T_{RF}} \right)} \approx 1.9$$

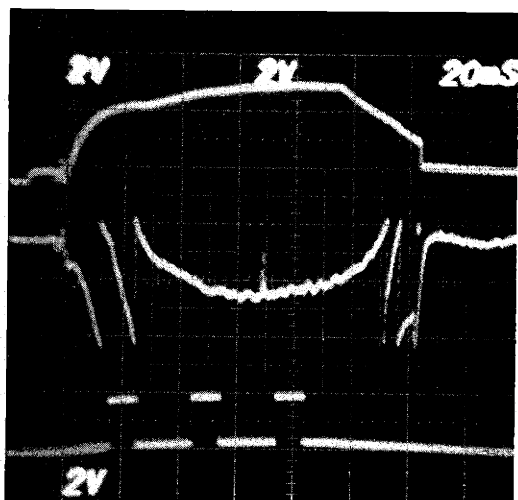
and we can unequivocally conclude that no such bulk temperature increase was observed. (Of course, one might argue that it is not clear that the spectra is itself representative of the plasma bulk temperature).

In the minority heating regime, we can rewrite (3) (again to zeroth order) for the minority species (5% H<sub>2</sub>)

$$(6) \quad T_{RF} \approx T_{OH} \left[ 1 + \frac{n_D}{n_H} \frac{P_{RF}}{P_{OH}} \right] \approx 3 T_{OH}$$

$$(7) \quad \frac{J_{RF}}{J_{OH}} \approx 117$$

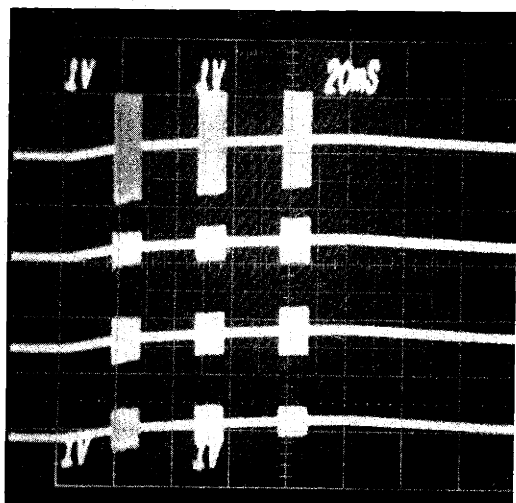
Although medium power 90 MHz heating experiments were only preliminary ( $R_p$  is 4 times smaller, and charge exchange was only available late in the experiment), no such increases in the minority tail were observed in any form (even 1 kW could produce  $\Delta J \approx J$ ). From available evidence ( $n_e$ ,  $I_p$ , CX, etc.), 90 MHz unfortunately behaved much like 200 MHz at high density.



$I_p$

$n_e$

t

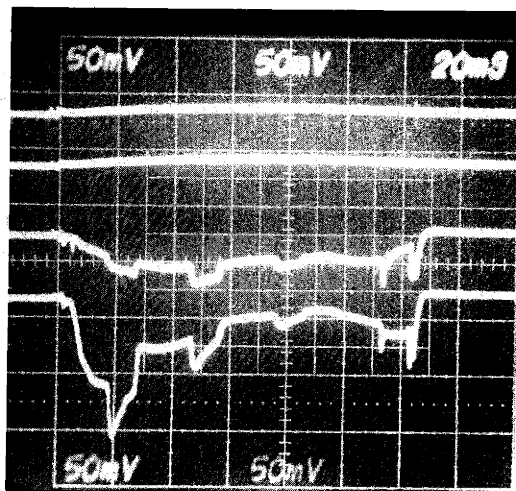


$V_F$

$V_R$

$I_T$

$I_B$

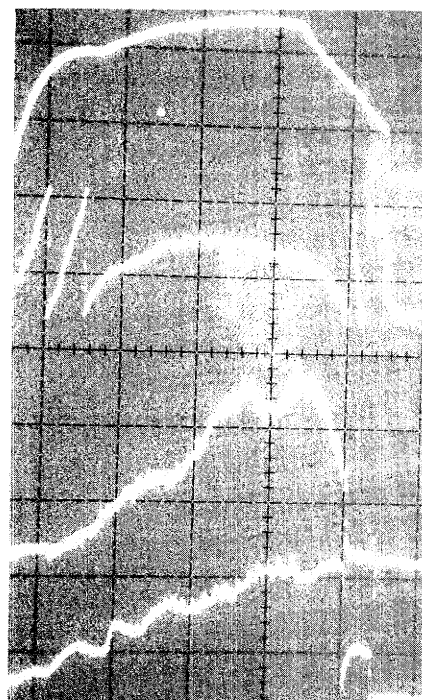


$L_T$

$L_B$

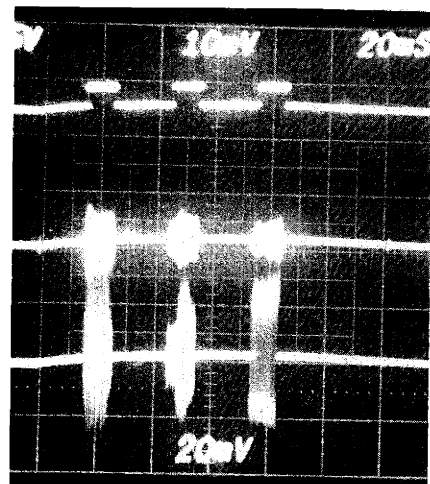
$L_a$

$H_\alpha$



$I_p$

$n_e$



t

69 HCN  
top

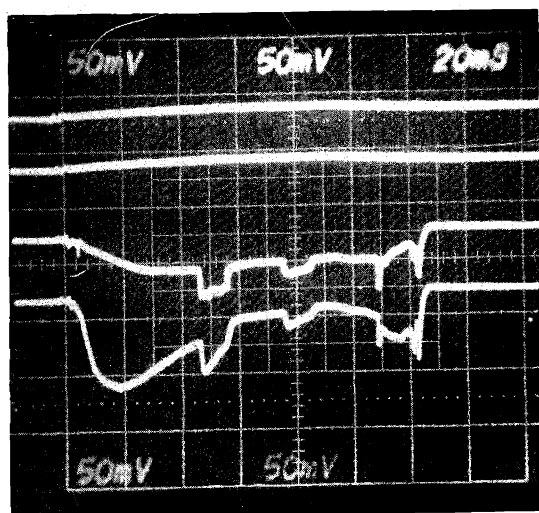
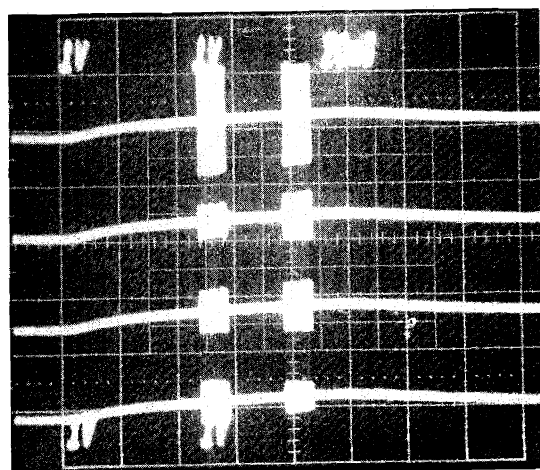
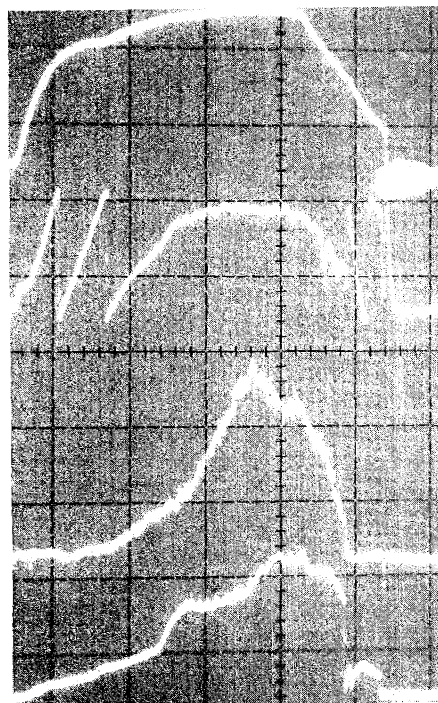
68 HCN  
Bot

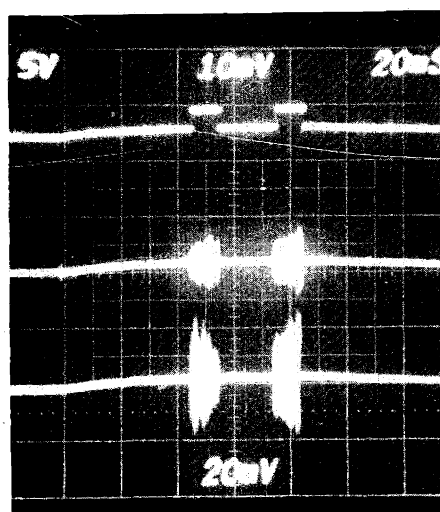
Medium Power Shot 42

S42, P28, B15

$H_2$ , 67 kA, 200 MHz

Figure 1


 $I_p$ 
 $n_e$ 
 $t$ 
 $V_F$ 
 $V_R$ 
 $I_T$ 
 $I_B$ 
 $L_T$ 
 $L_B$ 
 $L_a$ 
 $H_\alpha$ 

 $I_p$ 
 $n_e$ 
 $z$ 

 soft  
X-rays

 $t$ 

 69 HCN  
top

 68 HCN  
Bot

Medium Power Shot 43

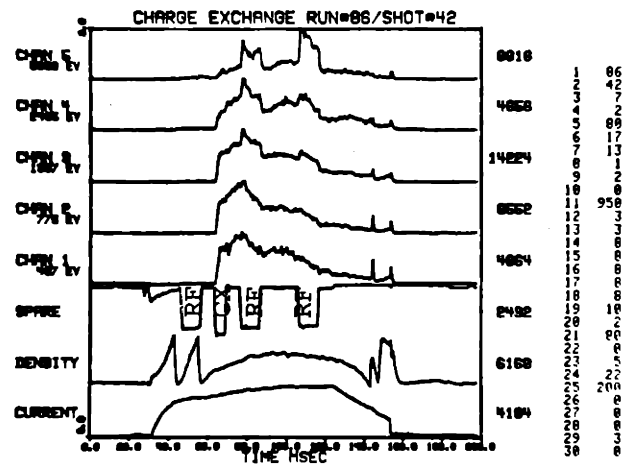
S43, P29, B15

 $H_2$ , 67 kA, 200 MHz

Figure 2

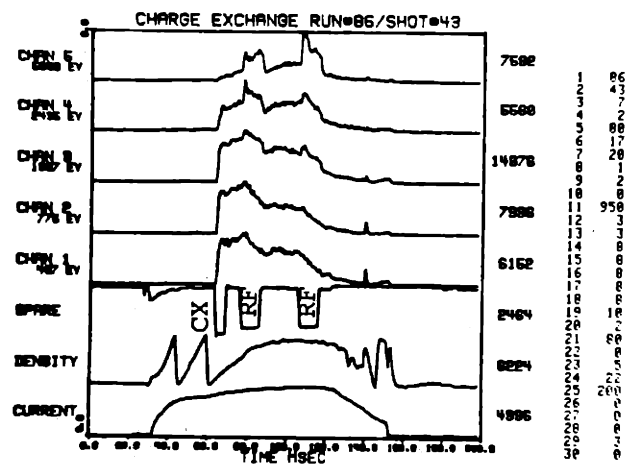
Hydrogen

Figure 3



Hydrogen

Figure 4



Neutral spectra at third RF pulse for shot 42

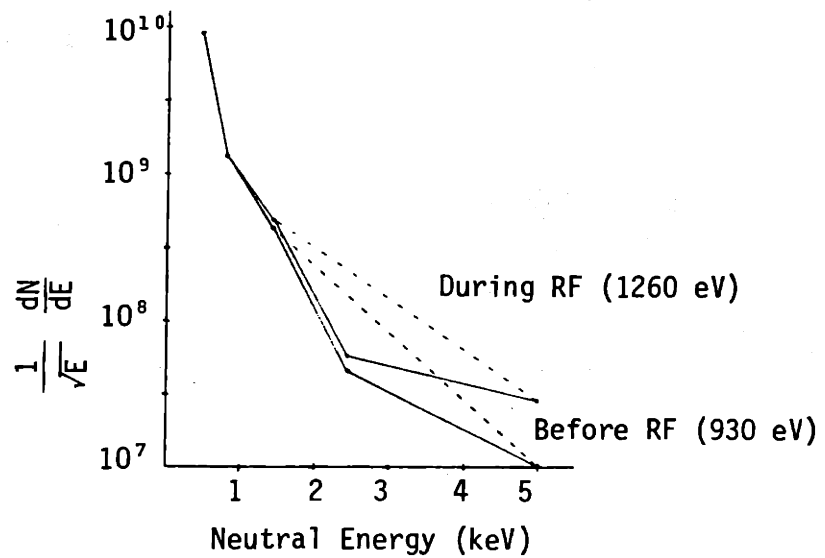


Figure 5

# DEUTERIUM NEUTRALS VS TOROIDAL FIELD

5Kev

10/31/80  
B18, pp46-51  
S43-53

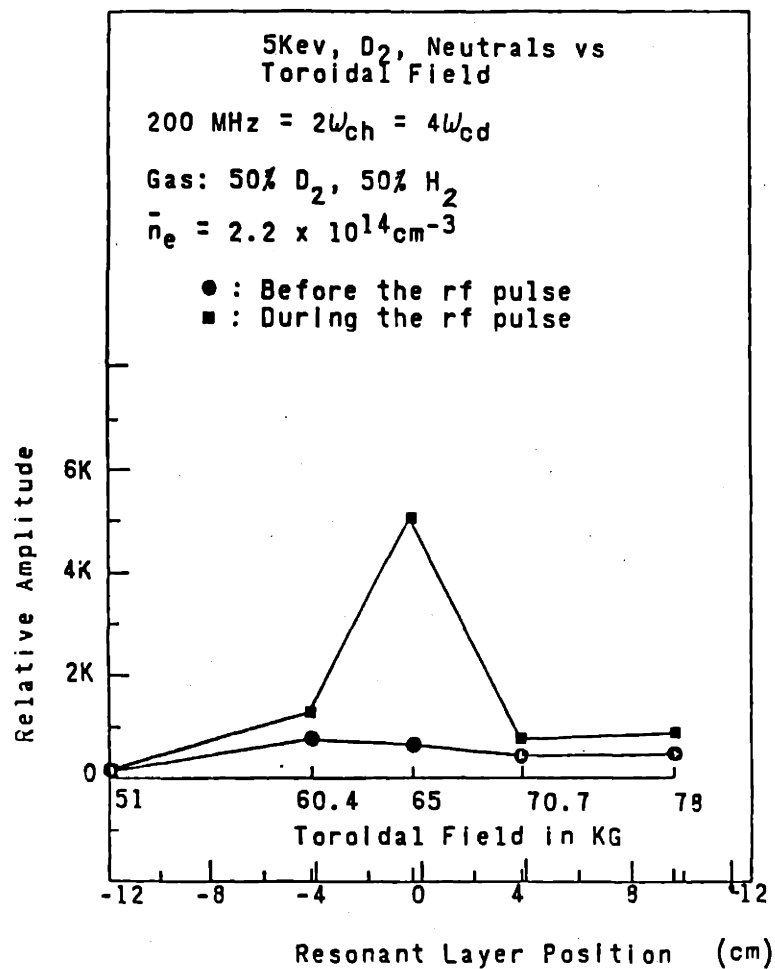


Figure 6

487ev

10/31/80  
B18 pp 46-5  
S43-53

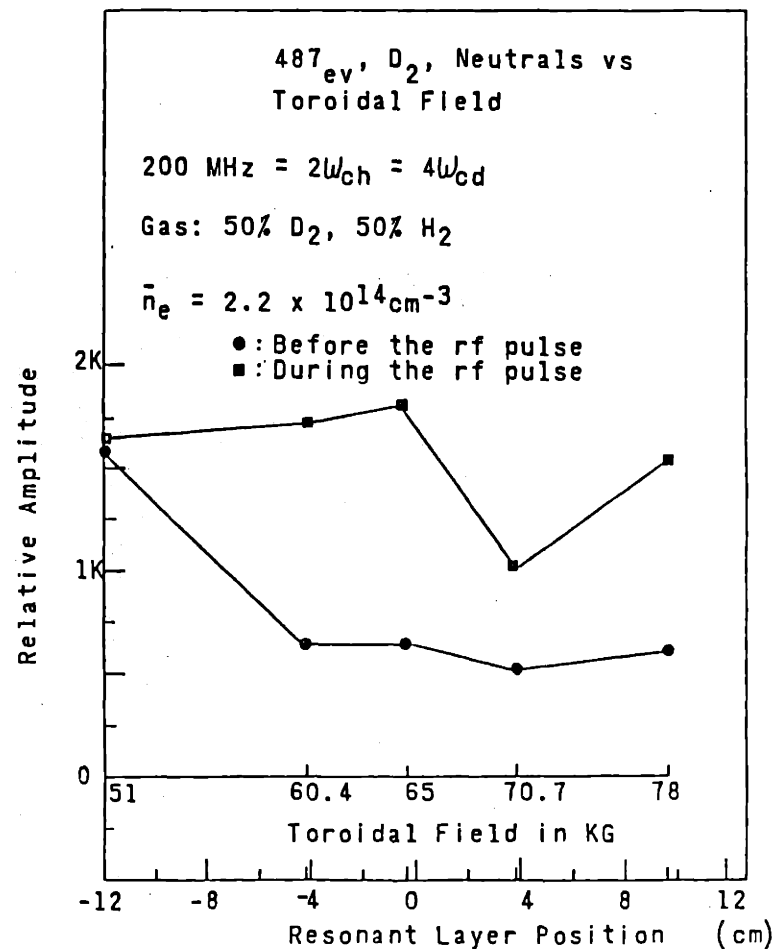


Figure 7

# HYDROGEN NEUTRALS VS TOROIDAL FIELD

5Kev

10/31/80  
B18, pp46-51,  
S43-53

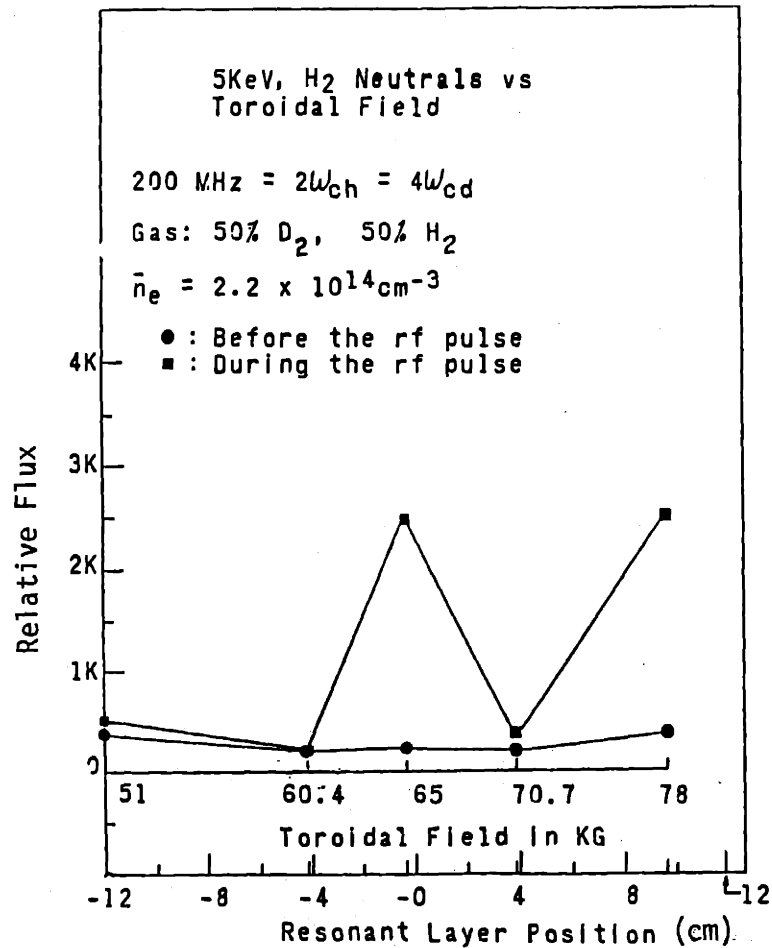


Figure 8

487ev

10/31/80  
B18, pp46-51,  
S43-53

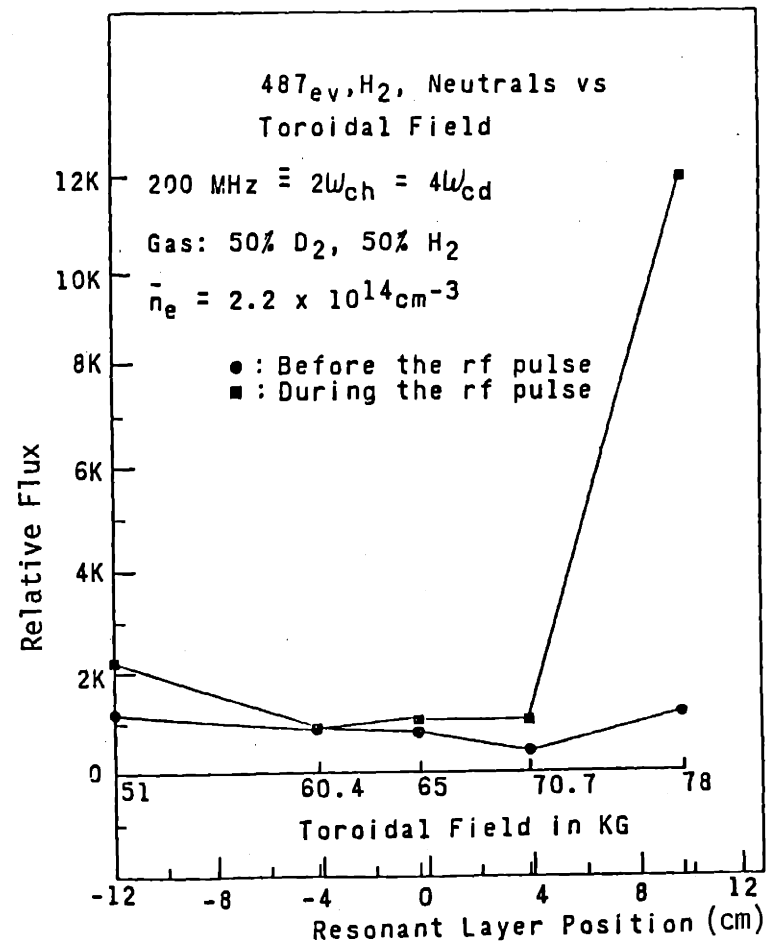


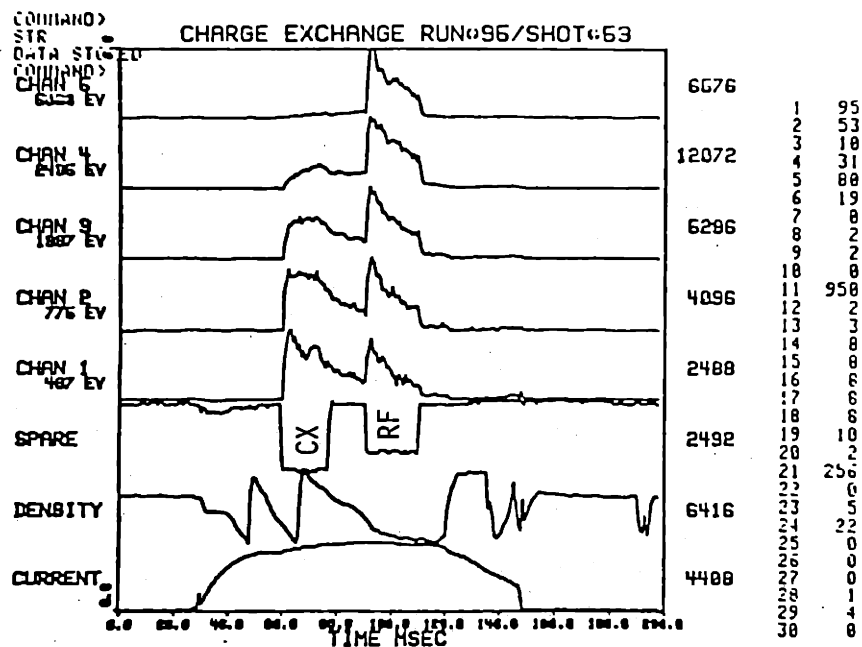
Figure 9



# FAST NEUTRALS SPECTRA

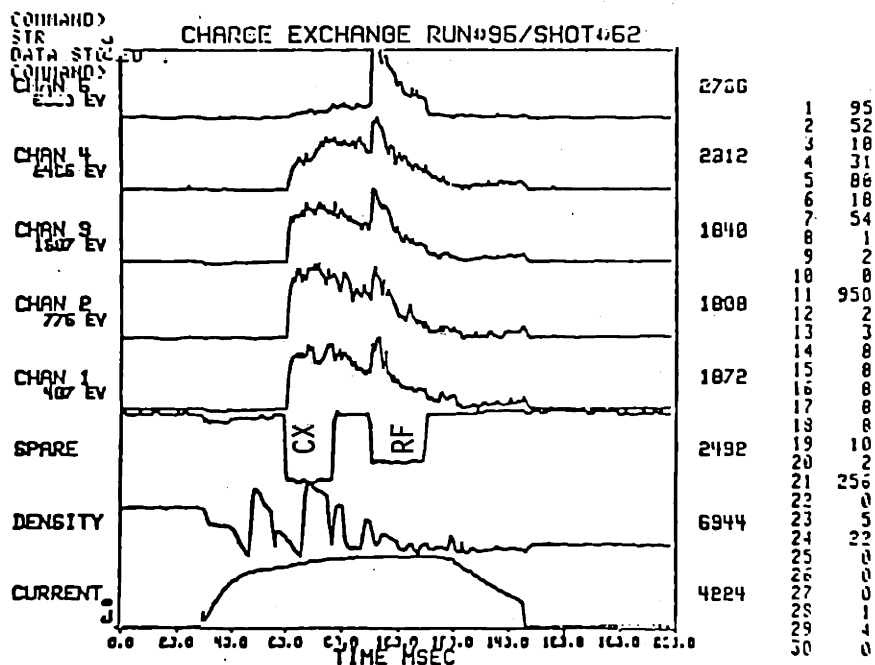
DEUTERIUM

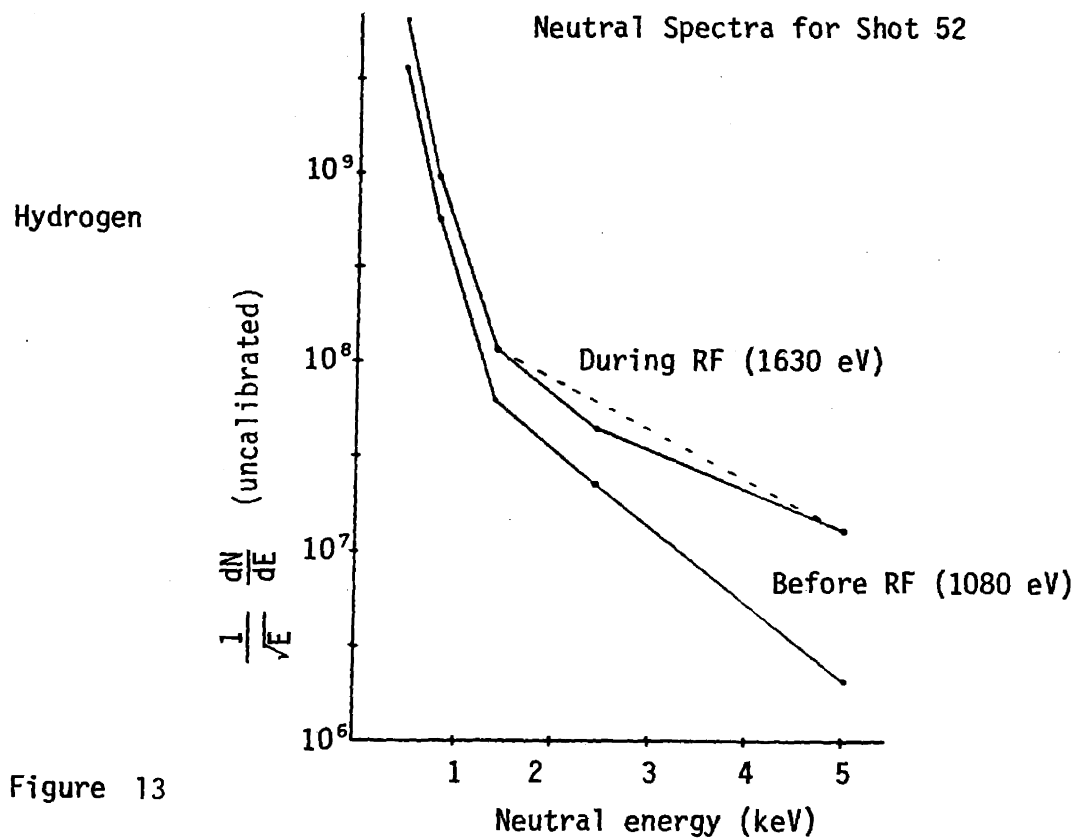
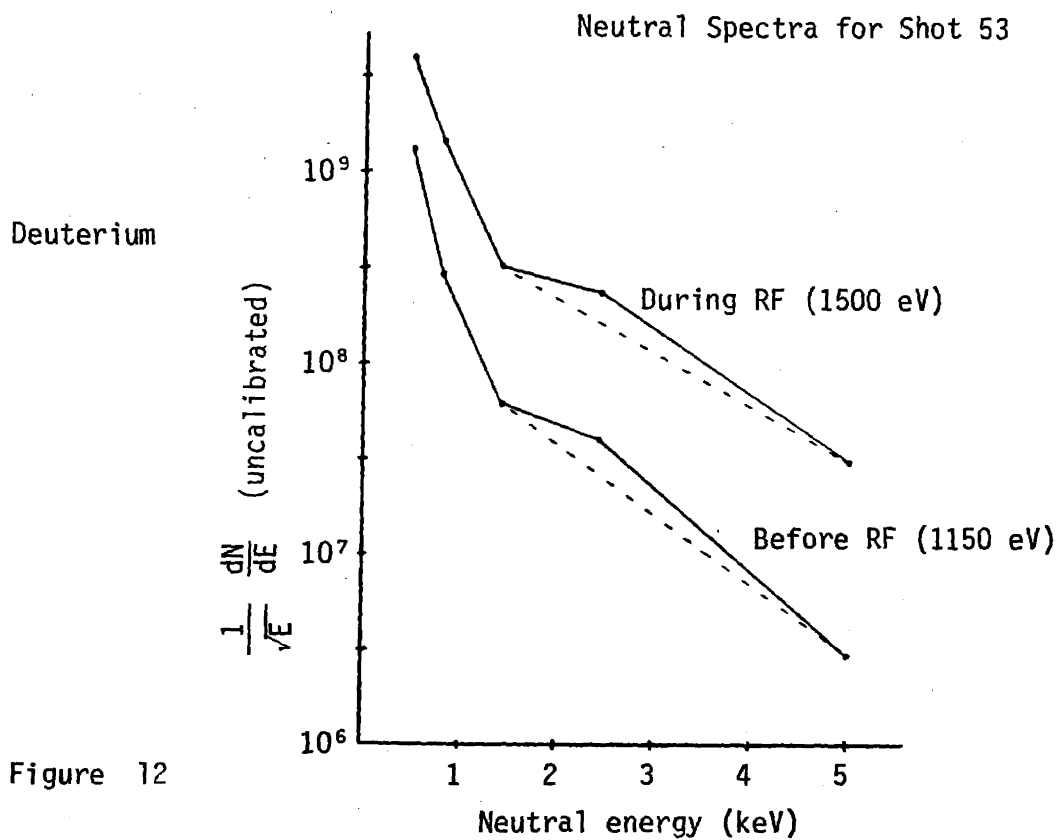
Figure 10



HYDROGEN

Figure 11





# RADIATION RESISTANCE & PROBE SIGNALS VS TOROIDAL FIELD

B18, pp46-51  
S43-53

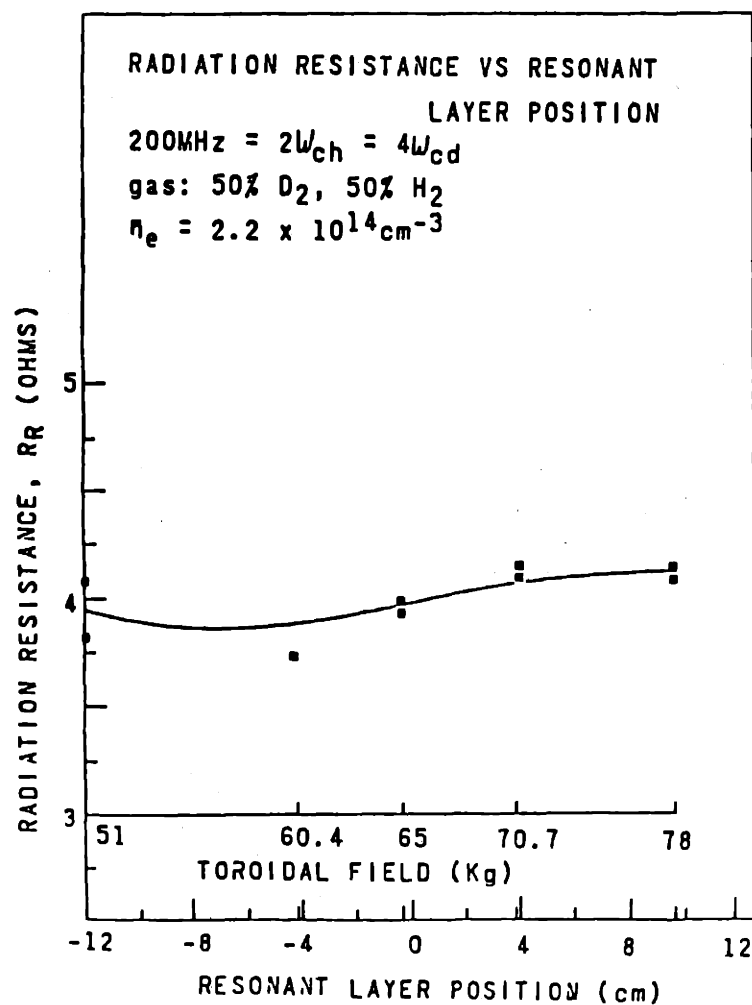


Figure 14

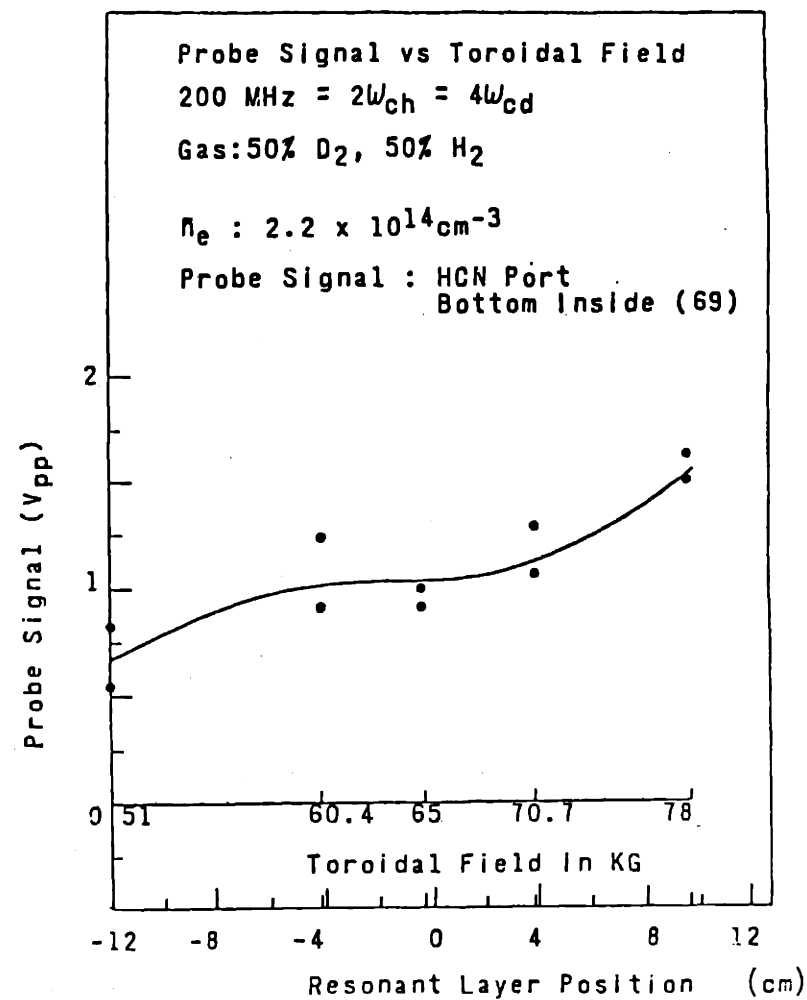


Figure 15

# PROBE SIGNALS VS TOROIDAL FIELD

top outside 10/31/80  
B18, pp46-51  
S43-53

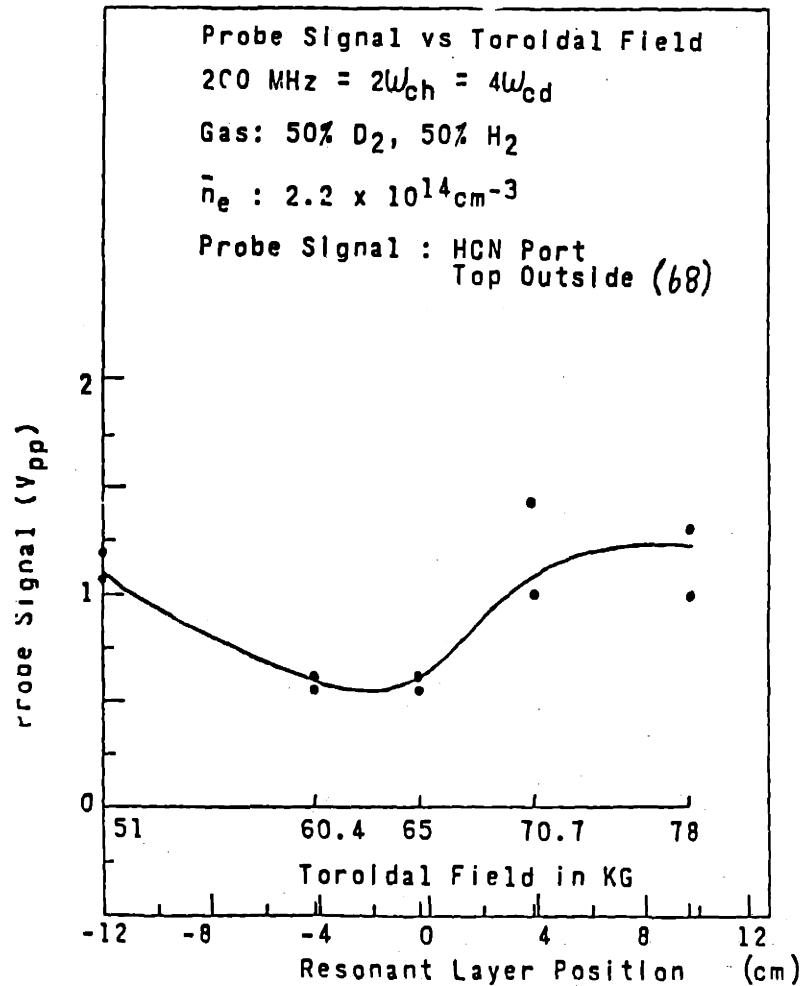


Figure 16

bottom outside 10/31/80  
B18, pp46-51  
S43-53

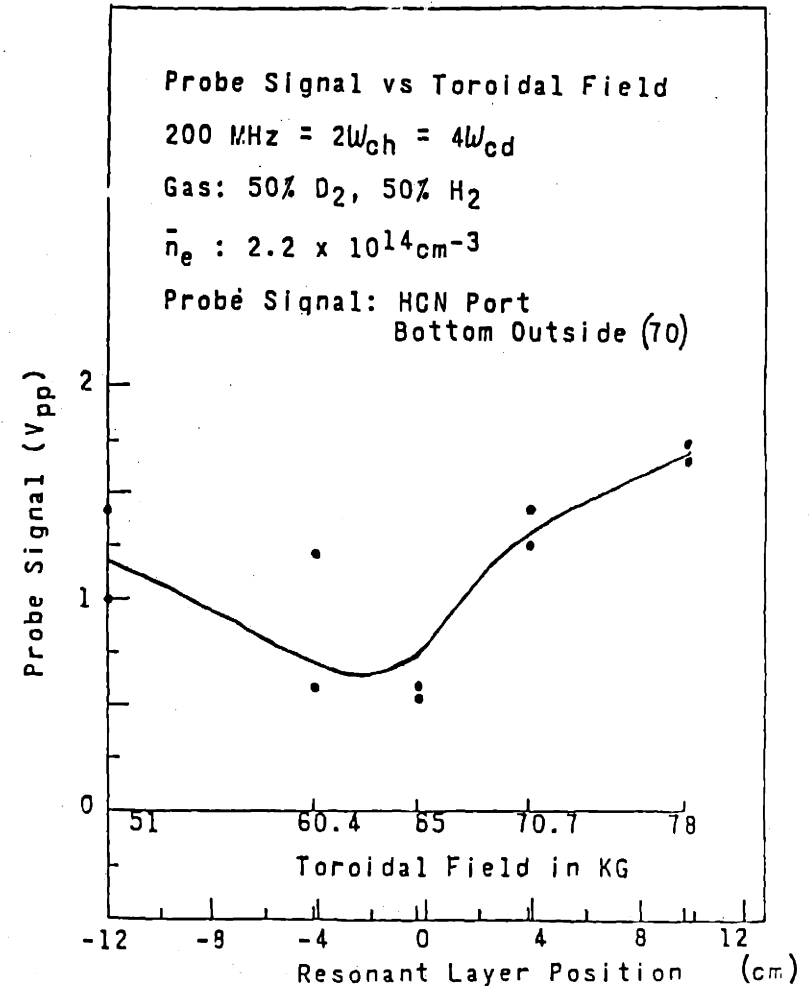


Figure 17

### II-3.2. Low density regime

After the difficulty in heating high density plasmas was clearly observed, some very preliminary heating experiments were performed at low density ( $5 \times 10^{13}/\text{cm}^3$  is still considered high density for other tokamaks). Note that  $R_R \propto n_e \omega^2$  scaling is very unfavorable for low density coupling, especially at 90 MHz where  $R_R < R_{\text{losses}}$ . Fortunately, fewer particles need to be heated, but on the other hand,  $P_{\text{OH}}$  remains approximately the same.

Figures (1) and (2) show the dramatic neutral flux increase for a medium power  $5 \times 10^{13}/\text{cm}^3$ , pure hydrogen and 200 MHz shot with the A<sub>2</sub> antenna. Neutral flux up to 10 keV with tail temperature 250 eV above background, were observed in this regime. Early in the discharge, the apparent flux decay time seemed to be due to the density increase since pulsing 5 msec later gave only a very small increase. However, similar pulses were also done at the density maximum of low density shots ( $< 5 \times 10^{13}/\text{cm}^3$ ) with similar energetic neutral tails but without the decay time.

# LOW DENSITY REGIME

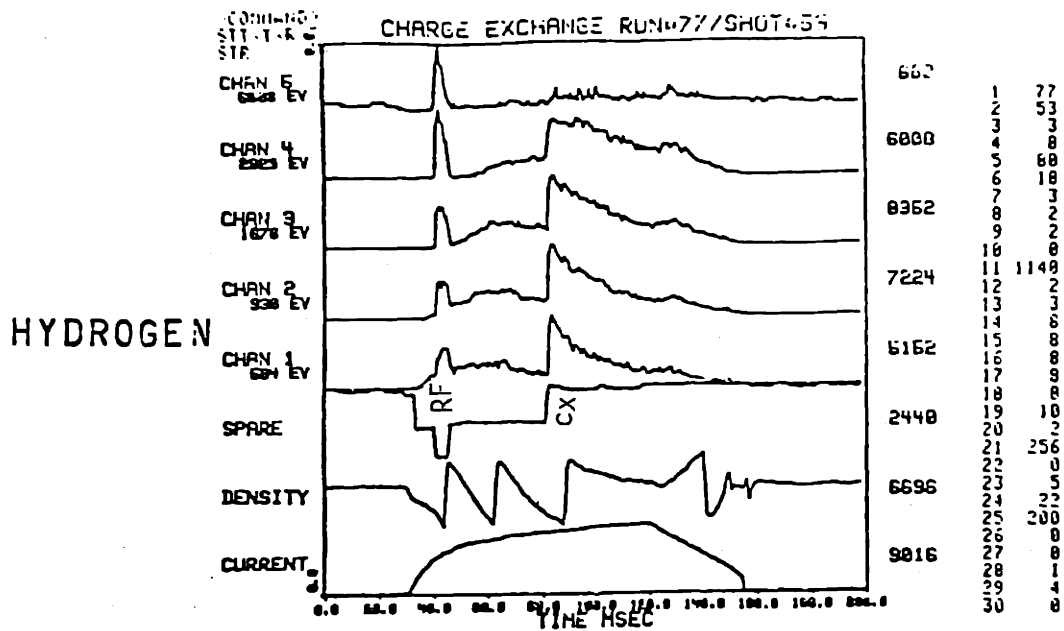


Figure 1

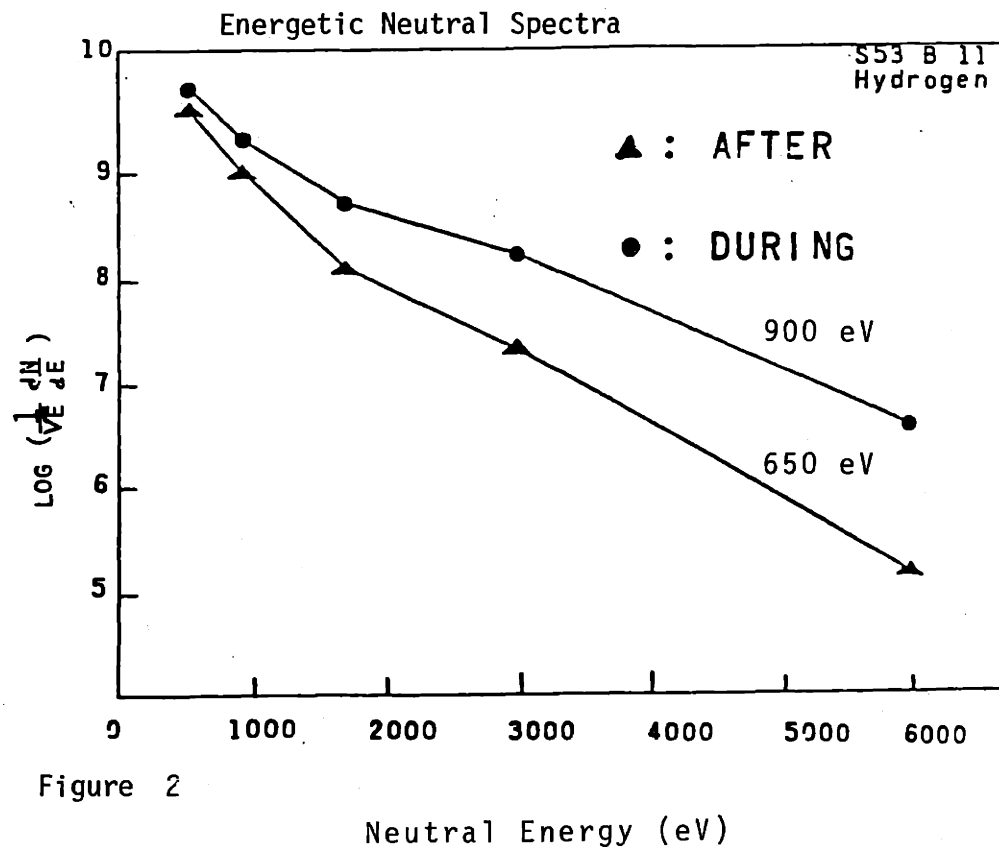


Figure 2

### II-3.3. Impurities and radiation measurements

Figure 1 shows a typical disruptive high density plasma shot with mixed hydrogen and deuterium. Note how the basic parameters ( $I_p$ ,  $n_e$ ,  $I_a$ ,  $V_{probe}$ ,  $H_\alpha$ , pin light) are well behaved and consistent over all three 40 kW pulses. The soft X-rays and thermonuclear neutrons, on the other hand, show a clear increase and flattening, respectively, over all three pulses (this effect was reproduced over several shots).

Figure 2 shows a similar plasma shot, where we note that the top electrical break broke down ( $L_T = 250$  mV,  $I_T = 0$ ) without affecting the plasma current and density, but, nevertheless, causing the usual increase in  $H_\alpha$ , antenna light and fast neutral flux.

Figure 3 shows a similar shot where the current from an edge Langmuir probe (HCN top) is also displayed. No significant change in the edge temperature or density could be measured. Figures 4 and 5 show a reduced edge density and temperature radial scan for a typical high density shot. Considerable variation was observed from shot to shot, and even more from run to run, as expected from the steep density and temperature profile and the uncontrolled plasma position.

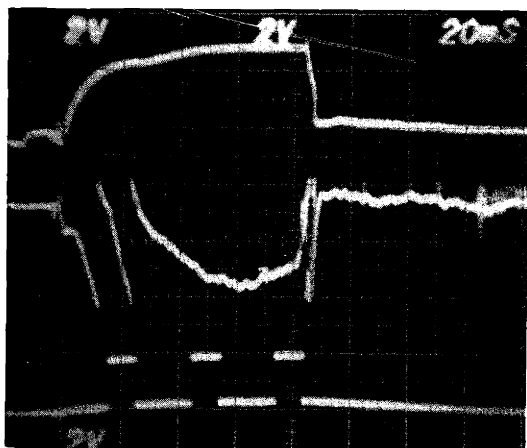
Figure 6 shows the extreme ultra violet spectrum (550 to 1600 Å) before and during a particularly bad RF pulse (typically there is no discernable increase in radiation). This small increase in radiation is most likely produced by light impurities (O, N, C) at the plasma edge ( $r > 8$  cm), where  $T_e < 40$  eV and  $n_e < 10^{14}/\text{cm}^3$ , and also indicates no significant edge heating. The increase in central (and not edge) soft X - rays may be indicative of electron heating, but was not accompanied by any decrease in loop voltage or increase in electron cyclotron

harmonic emission. Hard X-rays were never affected by the RF.

All previous measurements were done with molybdenum limiters. Surprisingly, completely different plasma behavior with RF was observed with a carbon limiter (although  $R_R$  and probe signal behaved the same). Returning to a different molybdenum limiter immediately reproduced the original results (so conditioning and exact limiter radius and geometry factors can be ruled out as the cause).

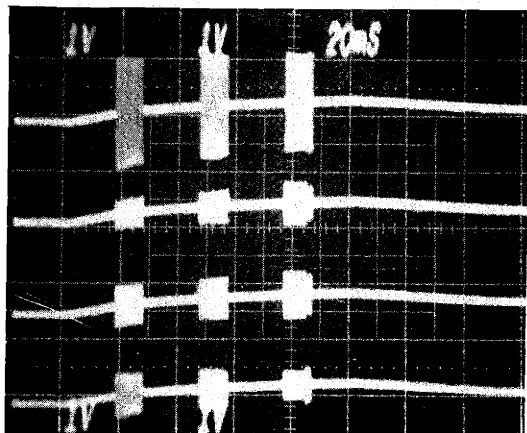
Figures 7 and 8 show the results of a dramatically affected (by RF) plasma shot, with a carbon limiter (although not very repeatable, no such behavior was ever encountered with a molybdenum limiter). Note how the electron density, central soft X-rays and total radiation (pyrometer) unambiguously increase with a long time constant during the medium power RF pulse. The Langmuir probe electron current was also greatly reduced, possibly indicating an increased plasma potential (ion saturation remained unchanged, III-3.4). Significant increases in the neutral tail were never observed either in this mode, as shown in Figure 7. Visual inspection of the antenna showed carbon deposits at high electric field points in the antenna, but not anywhere else near the antenna, which is far away from the limiter.





$I_p$   
 $n_e$

t

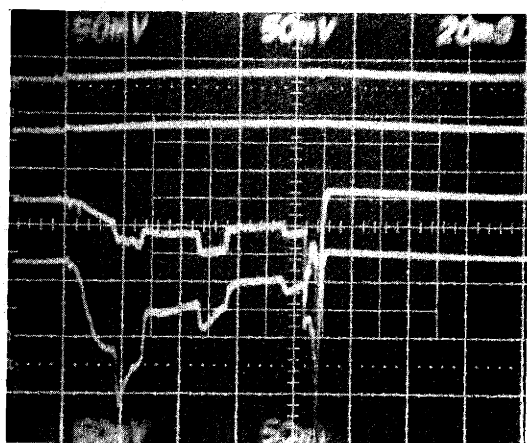


$V_F$

$V_R$

$I_T$

$I_B$

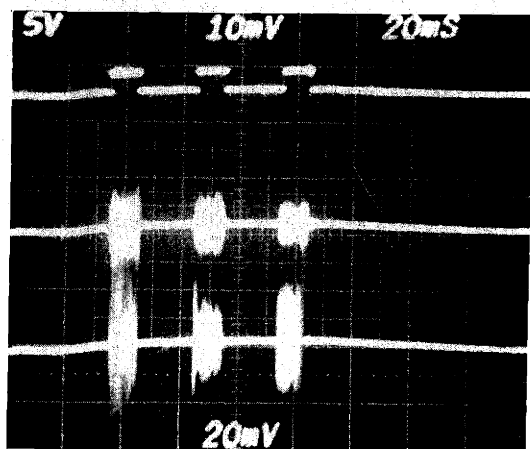


$L_T$

$L_B$

$L_a$

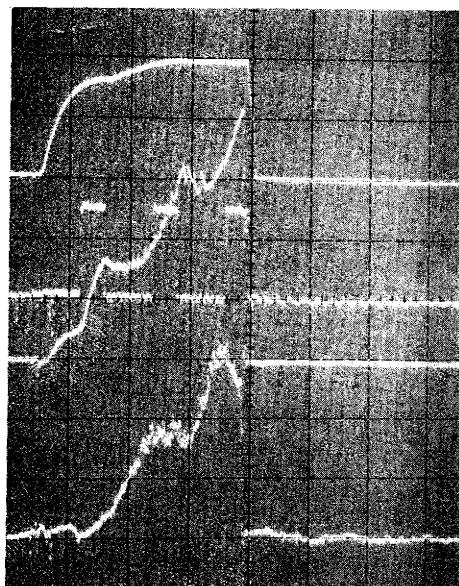
$H_\alpha$



t

69 HCN  
Bot

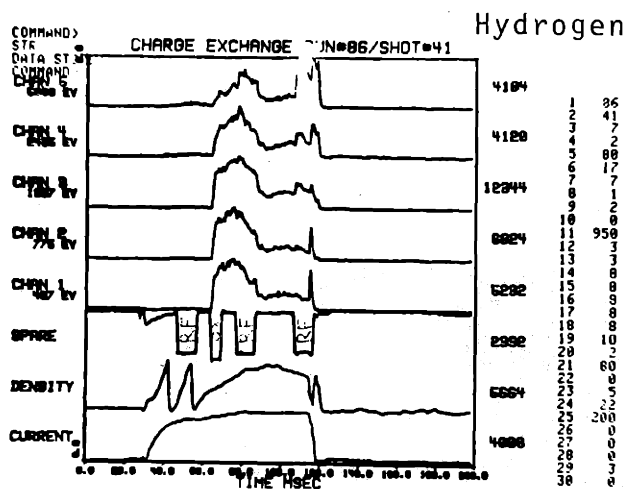
68 HCN  
Top



$I_p$

X-rays

neutrons



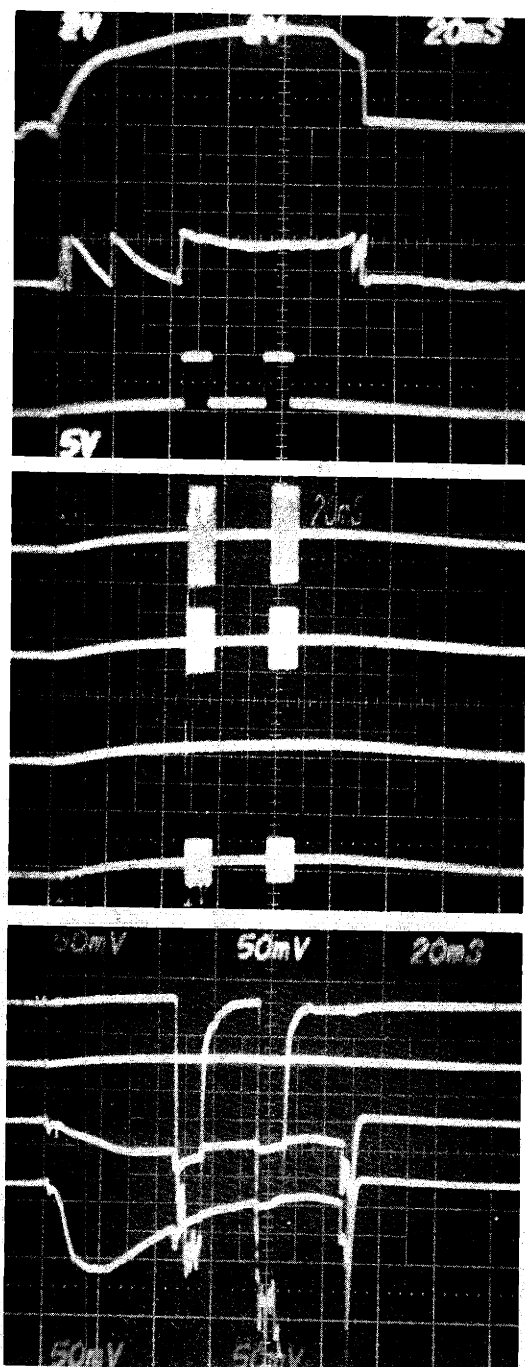
Hydrogen

Disruptive Plasma

S41, P28, B15

$D_2/H_2$ , 67 kA, 200 MHz

Figure 1



$I_p$

$n_e$

$t$

$V_F$

$V_R$

$I_T$

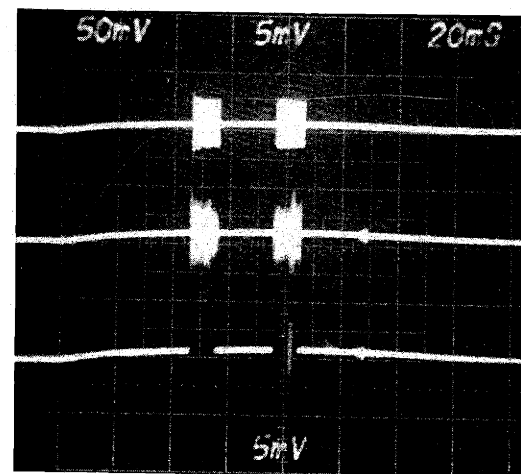
$I_B$

$L_T$

$L_B$

$L_a$

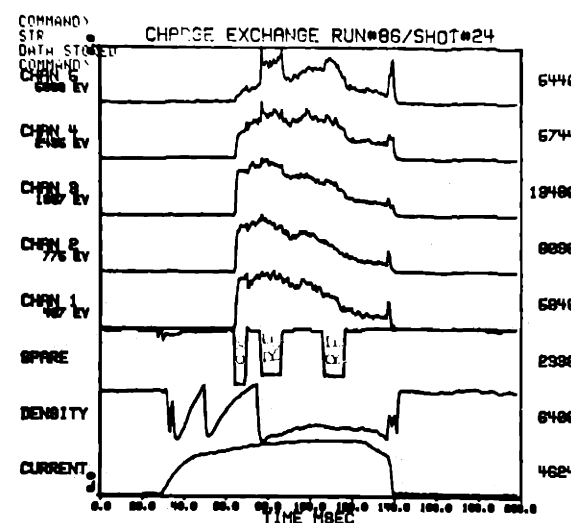
$H_\alpha$



6 Res Top

69 HCN Bot

68 HCN Top

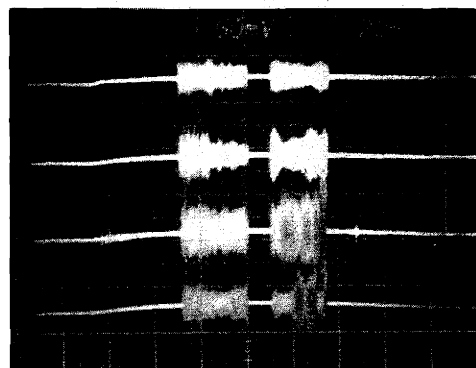
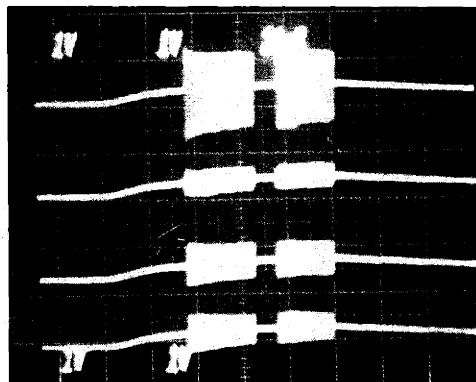
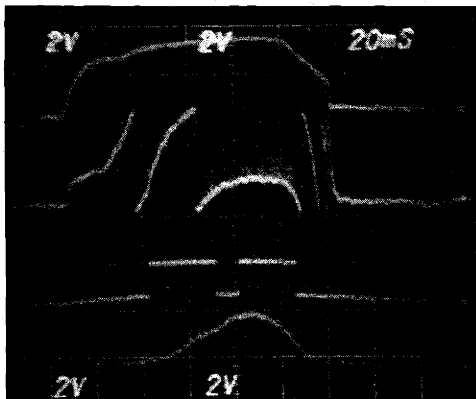


Breakdown in Top Coax

S24, P23, B15

H<sub>2</sub>, 67 kA, 200 MHz

Figure 2



$I_p$

$n_e$

$t$

$n$

$V_F$

$V_R$

$I_T$

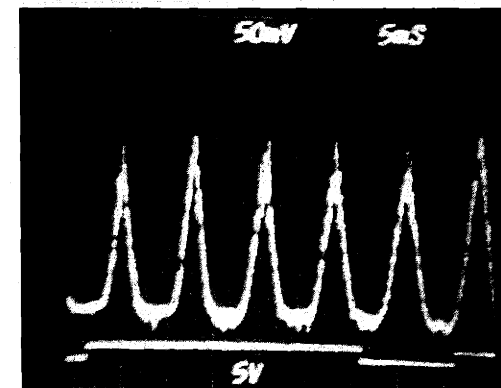
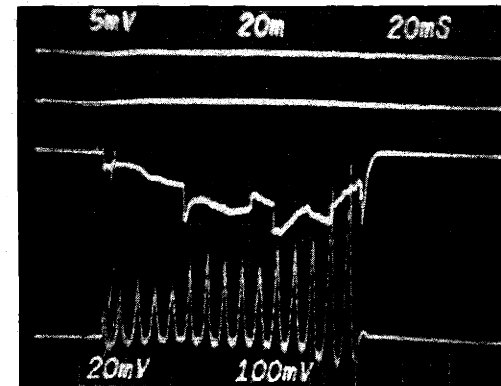
$I_B$

HCN

&

Limiter

Probes



$L_T$

$L_B$

$L_{pin}$

$I_L$

$I_L$

$t$

Typical Langmuir Probe Raw Data

Figure 3

50%  $H_2$ , 50%  $D_2$ , 200 MHz, 65.5 kA

S60, P15, B18

Edge Temperature and Density Profiles (Langmuir Probe)

B18, PF2-29, s26-57

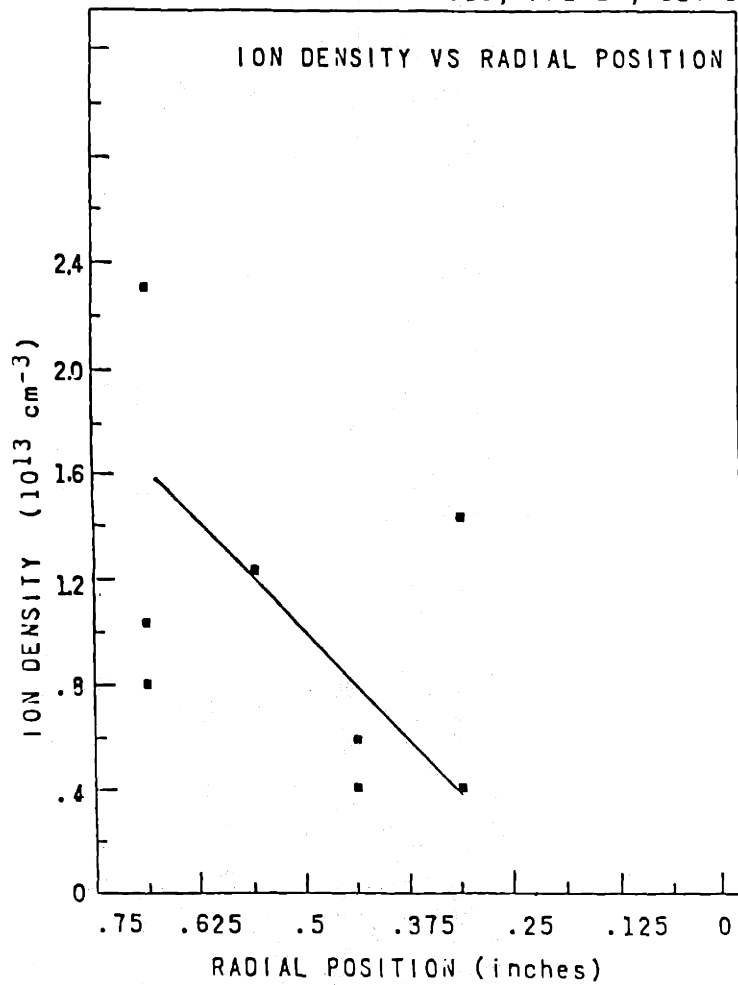


Figure 4

B18, PF2-29, S26-57

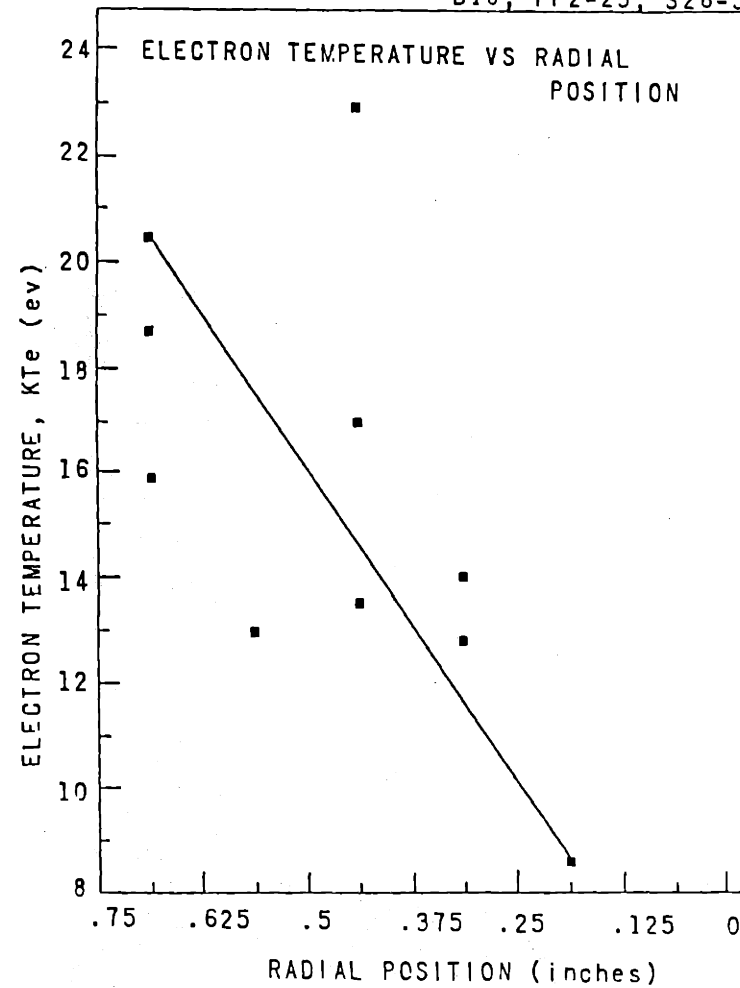
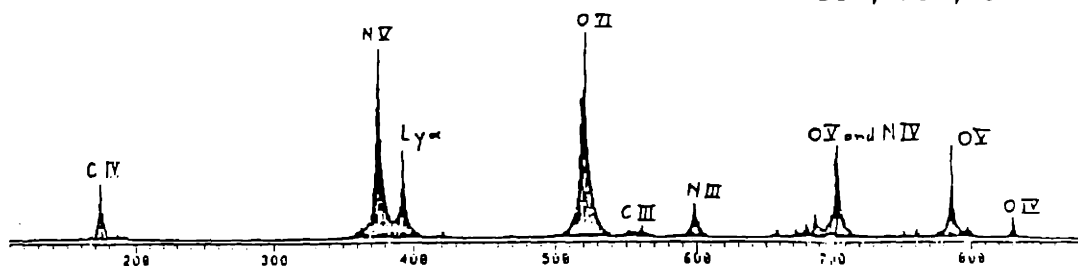


Figure 5

# EXTREME ULTRA VIOLET SPECTRUM

Before Rf pulse 75.5 - 79 ms

B16, P19, S48



During Rf pulse 83 - 87 ms

$r > 8\text{ cm}$

$T_e < 40\text{ eV}$

$\bar{n}_e < 10^{14}/\text{cm}^3$

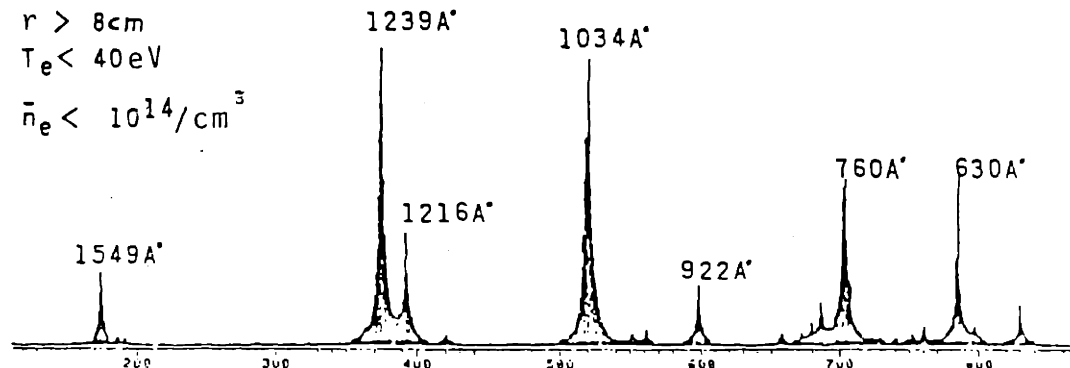


Figure 6

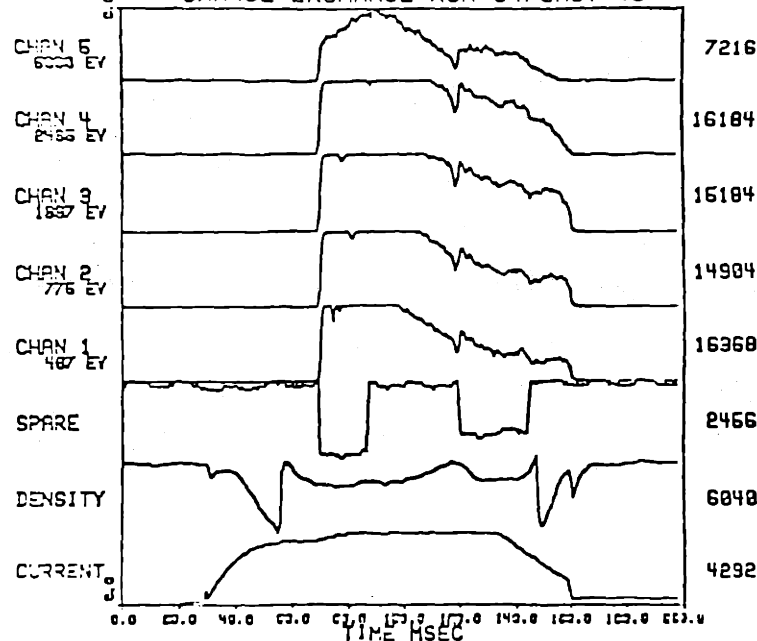
## ENERGETIC NEUTRALS WITH CARBON LIMITER

318, P25, S46

COMMAND

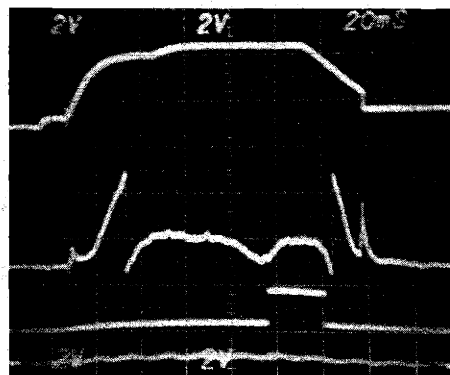
CHARGE EXCHANGE RUN094/SHOT046

H<sub>2</sub>



1	94
2	46
3	10
4	2
5	80
6	21
7	17
8	1
9	2
10	0
11	950
12	2
13	3
14	8
15	8
16	8
17	9
18	9
19	10
20	2
21	256
22	0
23	5
24	22
25	0
26	0
27	0
28	1
29	4
30	0

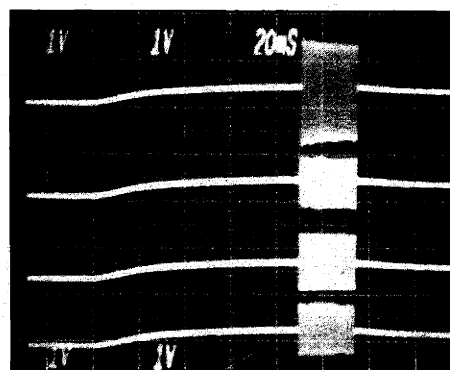
Figure 7



$I_p$

$n_e$

t

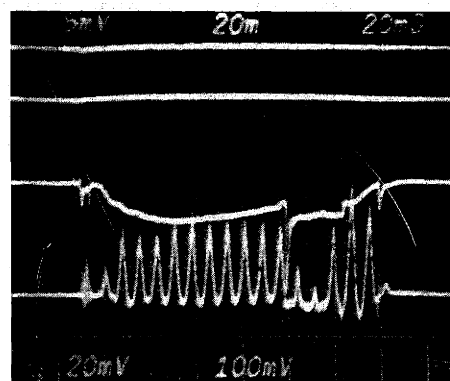


$V_F$

$V_R$

$I_a$  Top

$I_a$  Bottom

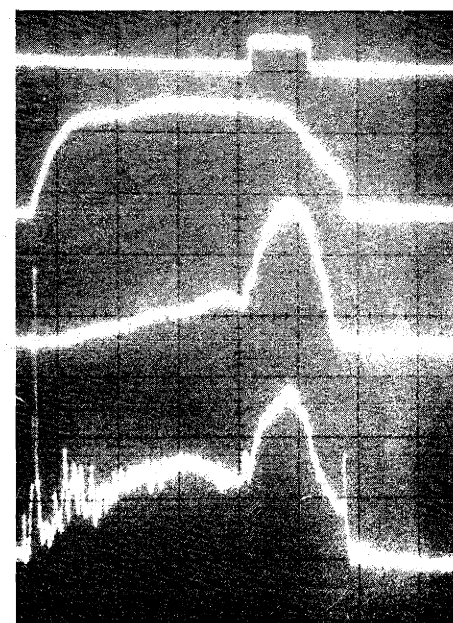


T Up

T Down

Pin

$I_L$

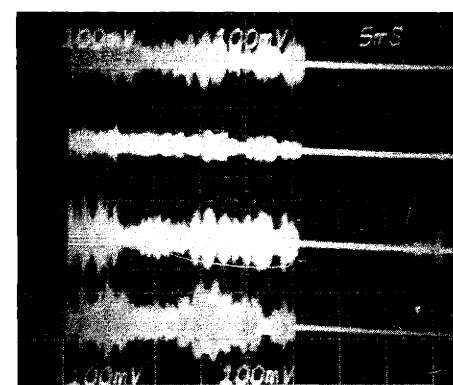


t

$I_p$

center  
soft X-rays

Pyro



P67 Limiter

P68 HCN-top

P69 HCN-Bot-Out

P70 HCN Bot-In

Medium Power Experiments with Carbon Limiter.

B18, P25, S46

200 MHz,  $H_2$ , 62.2 kA

Figure 8

### III - EXPERIMENTAL APPARATUS

#### III-1. Transmitter Chain and Engineering Support

##### III-1.1. Transmitter chain

Usually, the largest investment in any high power RF heating experiment is the transmitter chain and its engineering support.<sup>66</sup> In this case, we were fortunate enough to acquire most of the high power components as a gift from the Air Force, making the present installation the largest of its kind in the world (Figure 1,2). The system was built in view of a multimegawatt experiment on Alcator C, making the apparatus a gross over-kill for Alcator A.

Four "A<sup>2</sup>" high power amplifiers<sup>67</sup> and "B<sup>2</sup>" driver amplifiers were installed, yielding a maximum power of 6 MW, at 180 MHz. One of the B<sup>2</sup> was also operated at 90 MHz with major modifications. Figure 2 shows the initial basic transmitter chain from the oscillator, then excitors, B<sup>2</sup>, A<sup>2</sup>, RF switch gear, dummy loads, resonator, and finally, antenna. An RF feedback control system was built to stabilize either the antenna current or forward power. An auto-tune processor and double-stub tuner with stacked 9" transmit-receive switches as shorting elements was also planned to dynamically match the antenna during a plasma shot. Fortunately, since<sup>134</sup>

$$(1) \quad .33 < Z/Z_o < 3$$

$$(2) \quad \frac{P_{\text{reflected}}}{P_{\text{forward}}} = \left| \frac{Z - Z_o}{Z + Z_o} \right|^2 < .30$$

and this system became unnecessary since, for practical purposes,  $R_p$  never varied over much more than a factor of 3 during a given plasma shot. Figure 3 shows the simplified version of Figure 2 that was actually

used during the Alcator A experiment. A broad-band 1 kW distributed transmission line amplifier was used instead of the exciters and the narrow band 5894, and the 4cx250 stages of the B<sup>2</sup> were bypassed (Appendix 12). Any power level from a fraction of a Watt to 500 kW could be injected into either Alcator or a 2MW dummy load through a network of motorized type-N, 3" and 9", single pole double throw coaxial switches.

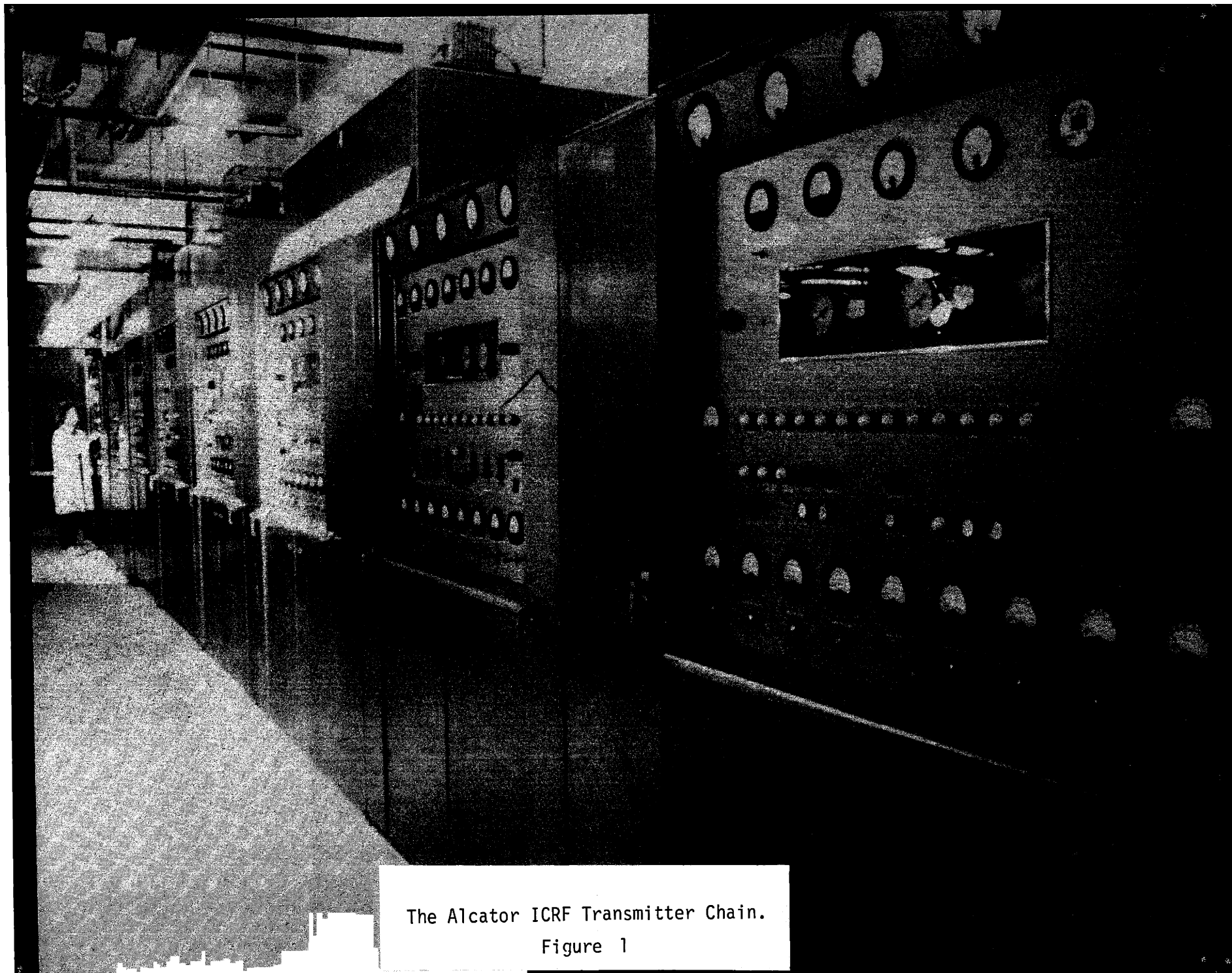
Figures 5 and 6 are very simplified diagrams of the A<sup>2</sup> and the B<sup>2</sup>. The B<sup>2</sup> uses an RCA 2041 high power tetrode mounted on a  $\lambda/2$  plate coaxial cavity with an output coupling tap angle of about 30°. The input resonator is a simple  $3/4 \lambda$  delay line with variable tap angle adjustment, and can be operated at either 90 or 200 MHz without modifications. For 90 MHz operation, the plate cavity is extended some 2', and less tuning flexibility is possible, due to the smaller relative size of the output tuning slug. The B<sup>2</sup> is keyed on by a small screen modulator very similar to the A<sup>2</sup> grid modulator.

The A<sup>2</sup> uses an RCA 6950 coaxial super tetrode. The A<sup>2</sup> grid is much more complicated than the B<sup>2</sup>'s, because the 200 lb tube is  $\lambda/4$  long, and the voltage maximum must be tuned to the center of the tube grid. The  $\lambda/2$  plate cavity is 30 $\Omega$  and 4 feet in diameter. Four magnetic output coupling loops at the top of the cavity then combine to a single output 9" coax. A fifth output coupling loop is used to provide positive feedback, that is combined with the B<sup>2</sup> output, through a variable delay line and a 4 port hybrid ring. This feedback system was bypassed for more stable operation in varying loads.

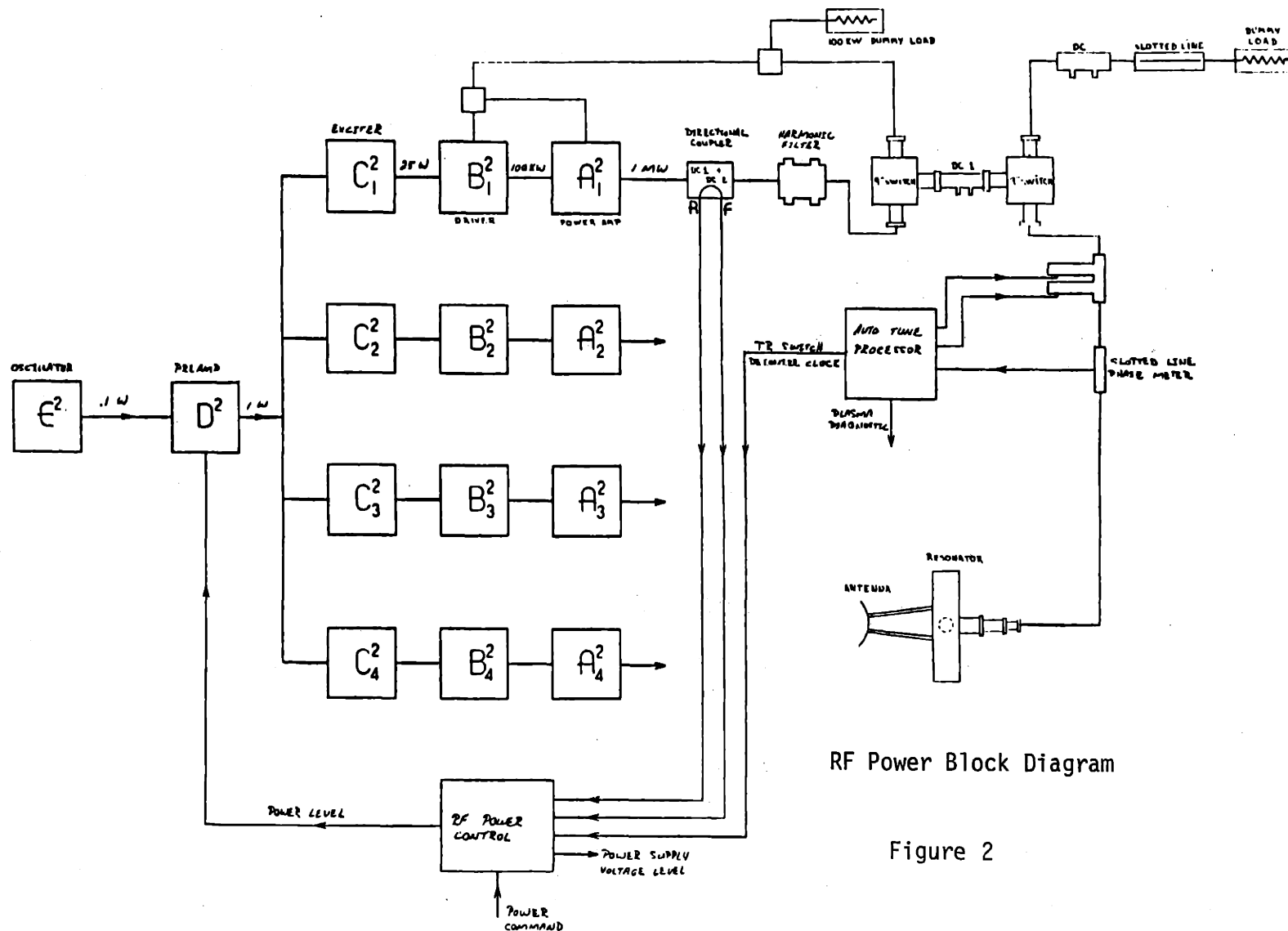
At conservative power levels, the B<sup>2</sup> generates 100 kW of RF with 12 kVdc at 15 amps, and the A<sup>2</sup> generates 1.5 MW with 20 kVdc at 100 amps. One 6950 alone requires 8 kW of filament power and 150 gallons per minute of plate



cooling water. All the tuning elements of the A<sup>2</sup> and B<sup>2</sup> are motor operated, and the transmitter cabinets are thoroughly RF leak tight. The A<sup>2</sup> and B<sup>2</sup> DC plate supply leads are brought through two high power LC RF feedthroughs.



The Alcator ICRF Transmitter Chain.  
Figure 1



RF Power Block Diagram

Figure 2

# Simplified Alcator A ICRF RF Power System

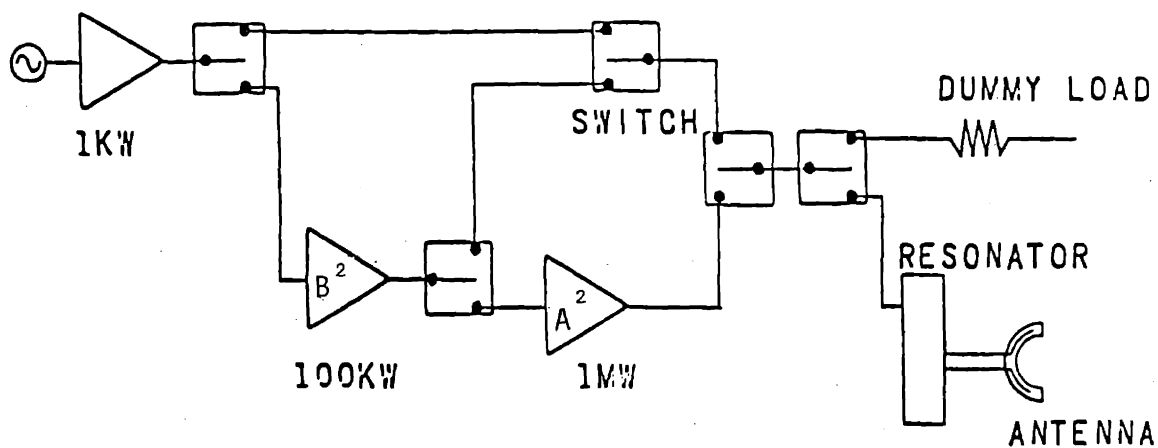
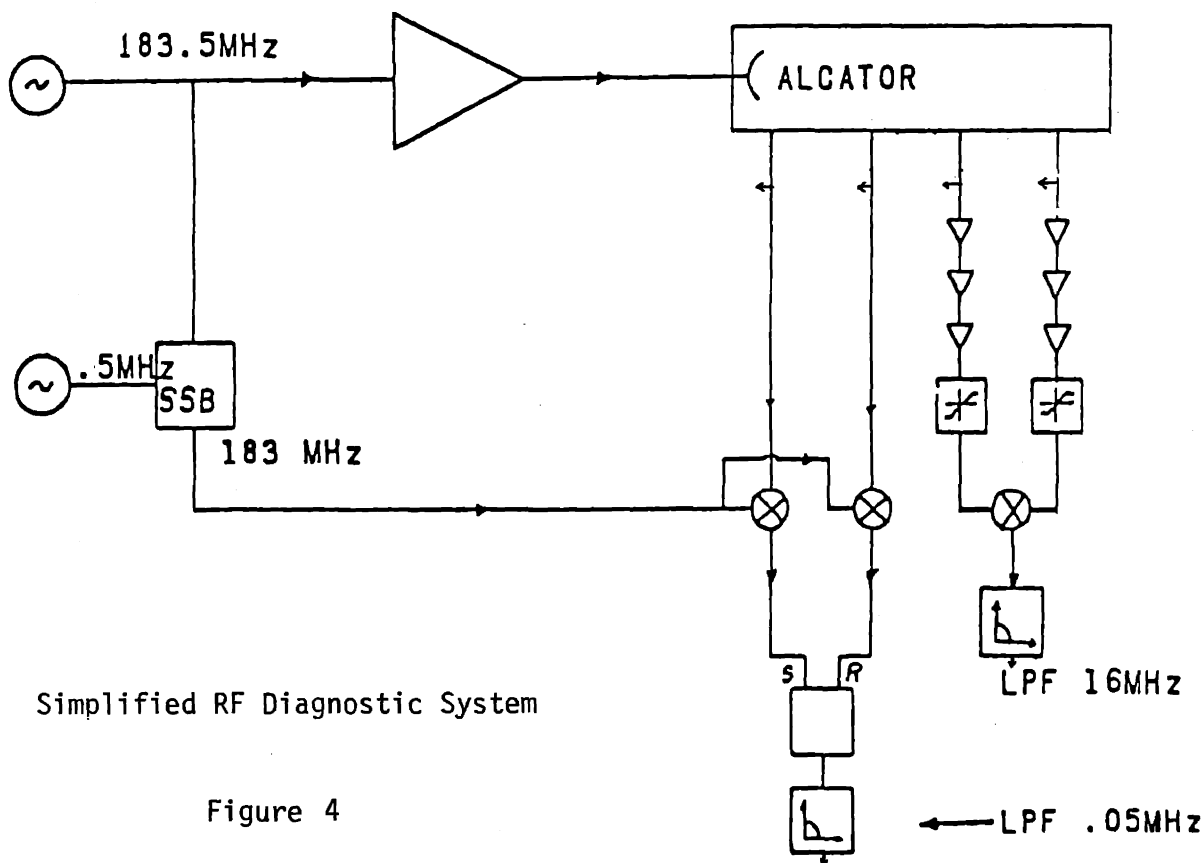
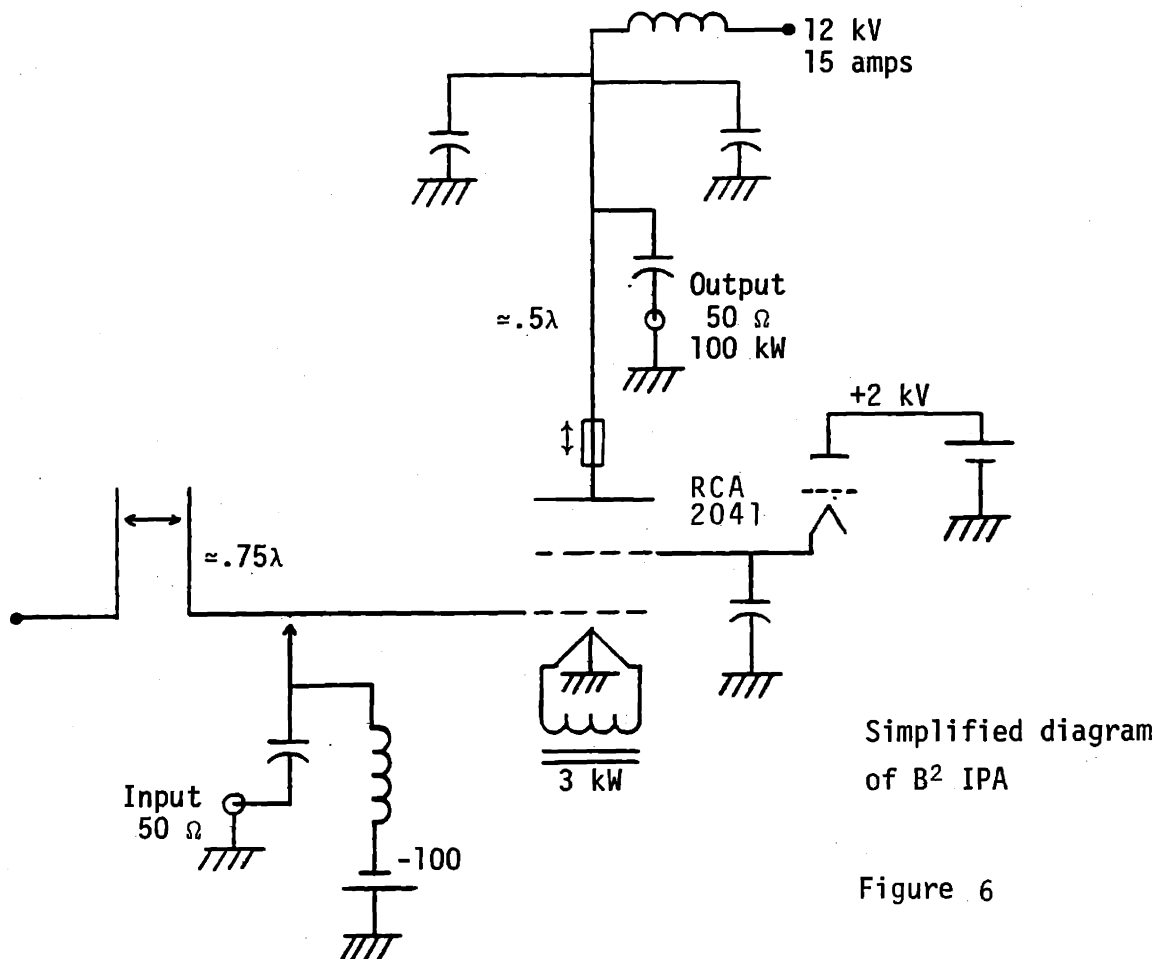
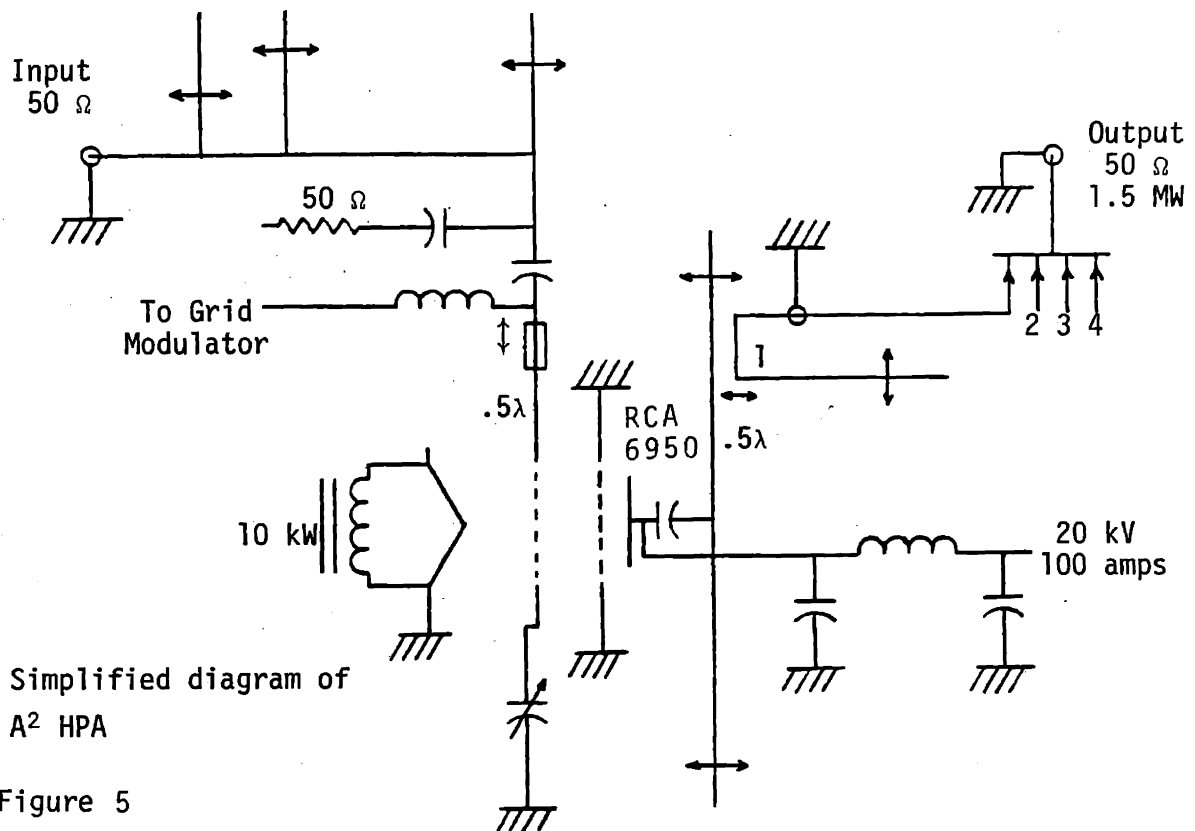


Figure 3



Simplified RF Diagnostic System

Figure 4



### III-1.2. High power DC and control systems

Two multi-megawatt DC power schemes could be used to feed the B<sup>2</sup> plate with 12 kV, and the A<sup>2</sup> plate with 20 kV, as can be seen from the simplified schematic of figure 1 . All four power supplies are located underneath the transmitter room in an explosion and fire proof vault (Appendix 12). In the first and simplest scheme, two 3 ampere DC power supplies are used to charge the 600  $\mu$ F and 400  $\mu$ F capacitor banks to produce a 40 ms, 500 kW RF pulse.

$$(1) \quad W = \frac{4 \times 10^{-4} \text{ F}}{2} (20^2 - 15^2 \text{ kV}^2) = 35 \text{ kJ}$$

$$(2) \quad P = \frac{W}{T} = 875 \text{ kWdc}$$

Both capacitor banks are split in half, isolated with critically damped current limiting RL networks (not shown in Figure 1), and equipped with mechanical and ignitron crowbars. 1000  $\mu$ F could also be installed in the vault permitting up 1.25 MW RF for 40 ms.

The second scheme uses 2 custom made long pulse 300 and 100 ampere ignitron power supplies powered from the 4160 volt ac 1200 ampere Magnet Laboratory bus system. Figure 2 is a simplified block diagram of this system. The main sequencer triggers the "domino block generator", which in turn, sequentially closes the 4160 Limitamp breakers feeding each of the A<sup>2</sup> and B<sup>2</sup> high voltage transformers. The sequencer also triggers the rectifier phase controller, the grid modulator and the fast crowbar system. The ignitron phase control system is very simple, since this is basically a predictable capacitive-resistive load and no power inversion is necessary. The phase control can be run either open or closed loop. The rectifier is a full wave, three phase array of 12 GL-5630 ignitrons (Appendix 12). These high voltage ignitrons are rated 30 coulomb, 30 kVdc, with gradient and screen grids,

and one holding and two triggering anodes.

The 500 gallons per minute at 80 psi of highly demineralized cooling water is provided by a custom made heat exchanger unit using Charles River water as primary coolant.<sup>69</sup>

A multitude of control and safety systems was built to manage the complex high power apparatus. The safety system can be broken down into four basic blocks operating at different speed levels (Figure 3). First, a number of kirk key and panic button interlocks are located throughout the system and, in particular, at the main control panel, vault, RF switch yard, Limitamps, Alcator A sequencer and cell, physics station and matching system. These interlocks, combined with literally hundreds of relay contacts, monitor the status of everything from slow, high voltage overcurrents to a low water flow in an A<sup>2</sup> grid. In the event of a fault, the main high voltage interlock relay K<sub>3</sub> is opened, thus disabling all power systems operating above one kilovolt. Smaller interlock loops will shut down only subsets of the system to protect a particular part of the equipment.

The next two faster levels are solid state, monitor fast current, voltage or reflected power transients, and disable the main sequencer, which in turn triggers the crowbars and cuts off the RF drive. At the fastest speed level, some twelve short dipoles located near the transmitters along the transmission line and in the Alcator cells monitor RF leakage, and, for power levels exceeding OSHA recommendations, the RF drive to the exciters is quickly cut off.

The most important control element is the central high power control unit (Appendix 12), which can be divided into three subsets, K<sub>1</sub>, K<sub>2</sub>, K<sub>3</sub>. For all practical purposes, K<sub>1</sub> enables all non-solid state systems (as long as the safety circuits are satisfied). K<sub>2</sub> enables the 4160 Alcator starters, and K<sub>3</sub> enables all high voltage DC systems above 1 kV.

The main sequencer,<sup>68</sup> fast crowbar system, grid modulator and RF leakage detector system<sup>70</sup> were custom built in house. Many channels, remote control and the recyclable features were necessary for the sequencer, so that during a run, different RF scenarios could be quickly implemented between shots, (i.e., low power long pulse for tuning, three short interspaced pulses for high power heating, or simply a short medium power pulse every second for antenna conditioning).

Even when measured with a half wavelength 50 ohm matched dipole, RF leakage signals of, at most, one volt were measured under the most adverse conditions (i.e., a panel left off the resonator, and  $P_f \approx 50$  kW). The Poynting flux,

$$(3) \quad S = \frac{P}{A} \approx .07 \text{ W/m}^2 = 7 \text{ } \mu\text{W/cm}^2$$

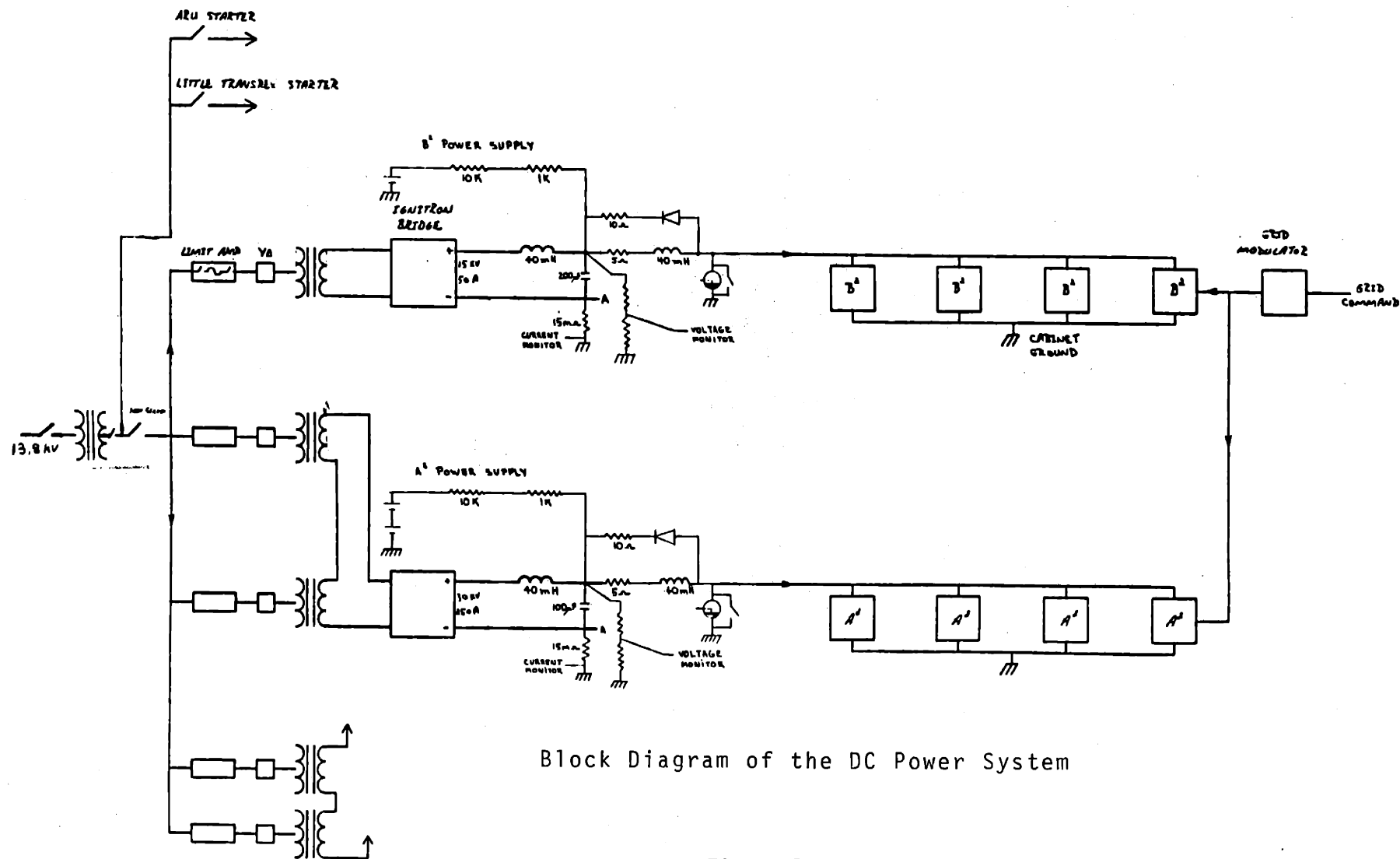
where

$$(4) \quad P = \frac{V_{\text{rms}}^2}{50\Omega}$$

$$(5) \quad A = \frac{1.65 \lambda^2}{4\pi}$$

was much less than the OSHA recommended  $10 \text{ mW/cm}^2$ , and the system was not needed. Normal readings in the cell were at the nanowatt/cm<sup>2</sup> level. Nevertheless, the system should be used with high power A<sup>2</sup> experiments.





Block Diagram of the DC Power System

Figure 1

Ignitron A<sup>2</sup> and B<sup>2</sup> Phase Control Block Diagram

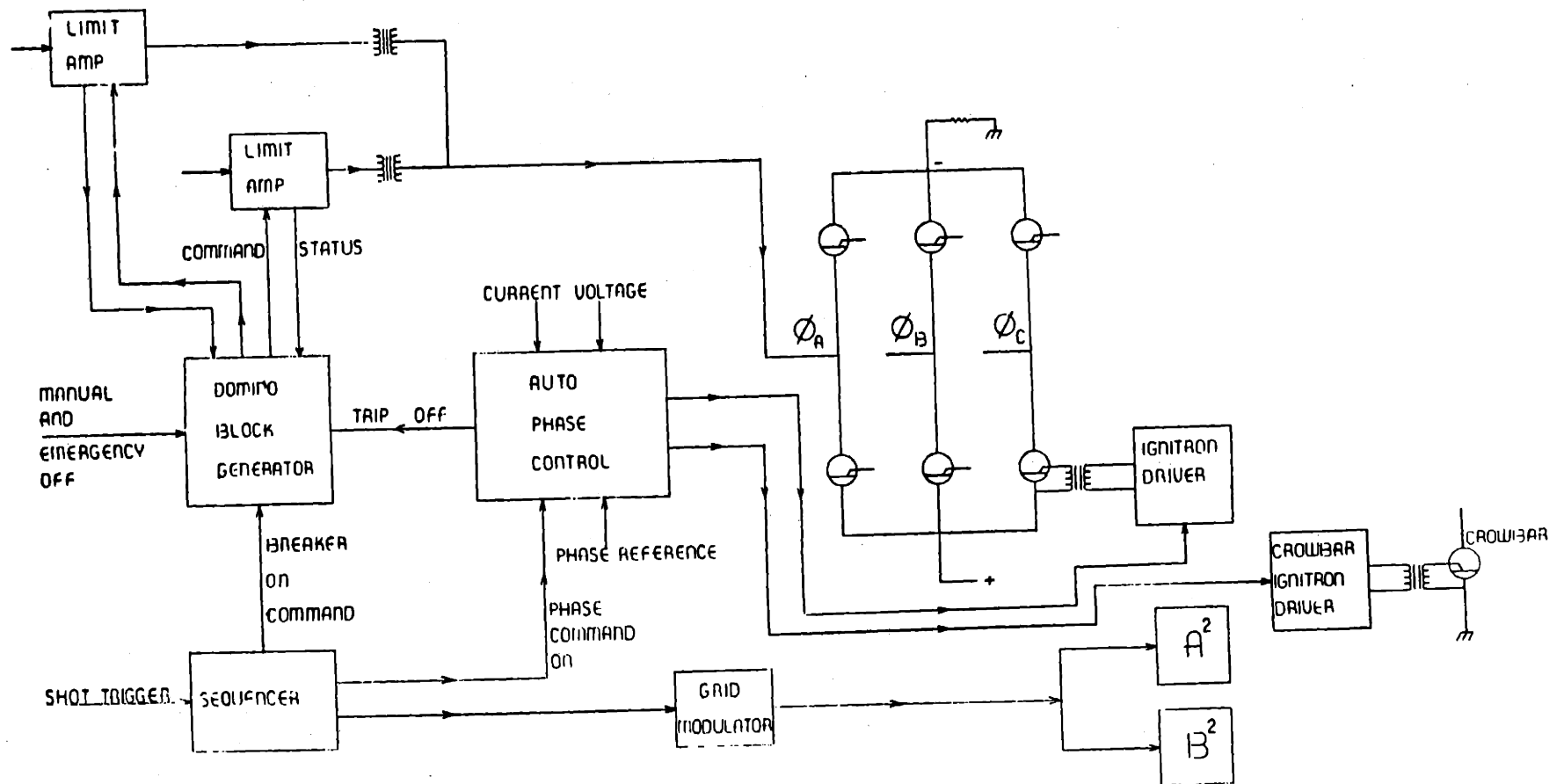


Figure 2

# Simplified Flow Diagram of Safety System

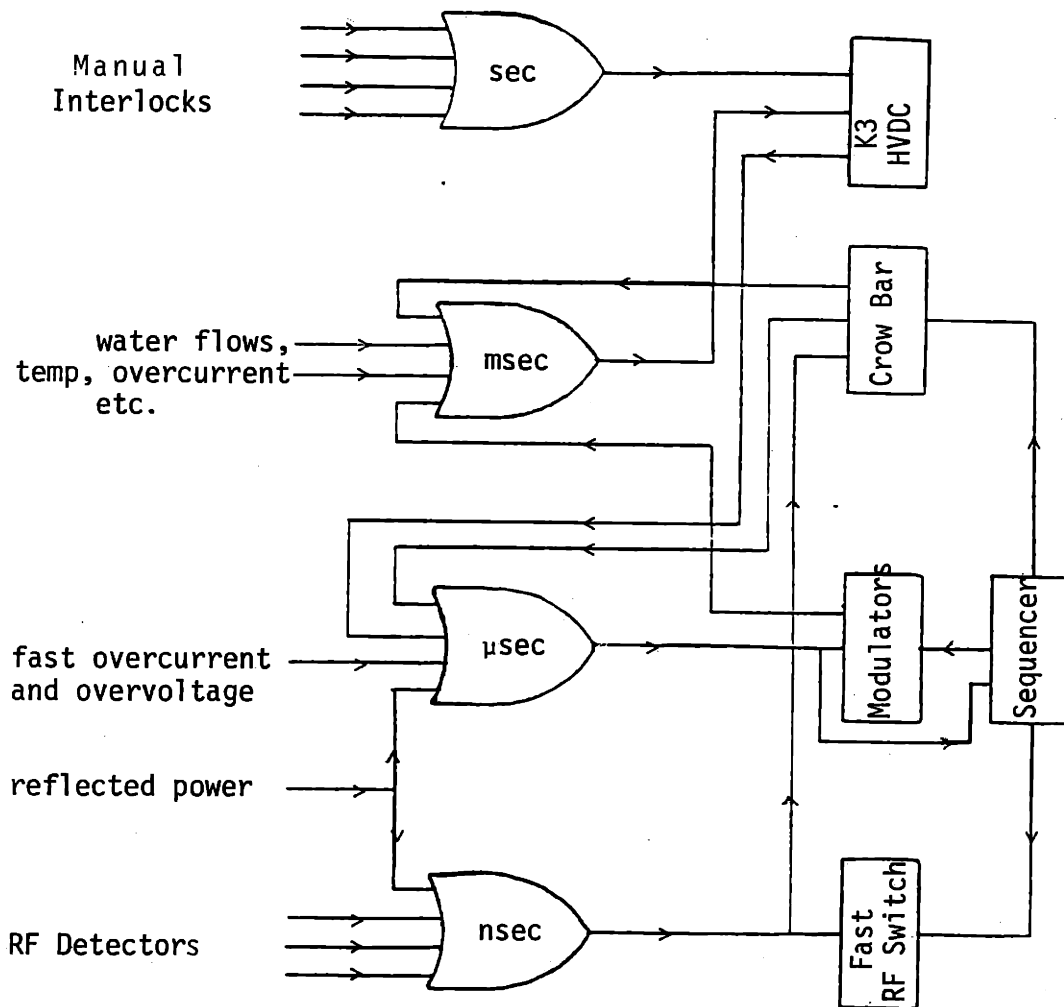


Figure 3

### III-2. Matching and Launching Structures

#### III-2.1. Basic matching and launching system

As we mentioned earlier, a series of three antennas,  $A_1$ ,  $A_2$  and  $A_4$  were designed and tested (Figure 1). The first,  $A_1$  is an all metal ultra compact shielded antenna. Significant attention was given to RF properties, cooling, and especially to the stringent mechanical constraints of the Alcator tokamak <sup>71</sup> (Section III-2.2.). The second antenna,  $A_2$  is unshielded, also all metal, and has 2.5 times more loop area (Section III-2.4.). The  $A_2$  antenna structure with side virtual limiters is more than an order of magnitude larger than the tiny 1.25 x 3.4 inch access port, and extensive use of "Boat in the Bottle" technique was necessary to install these unfolding structures inside the Alcator vacuum chamber (Figure 2).

$A_4$  is a combination of  $A_1$  and  $A_2$  with an increased loop area, more finely slotted Faraday shield and side limiters, and a Langmuir probe mounted in the front of the shield. Figure 3 is a simplified diagram of the basic antenna structure.

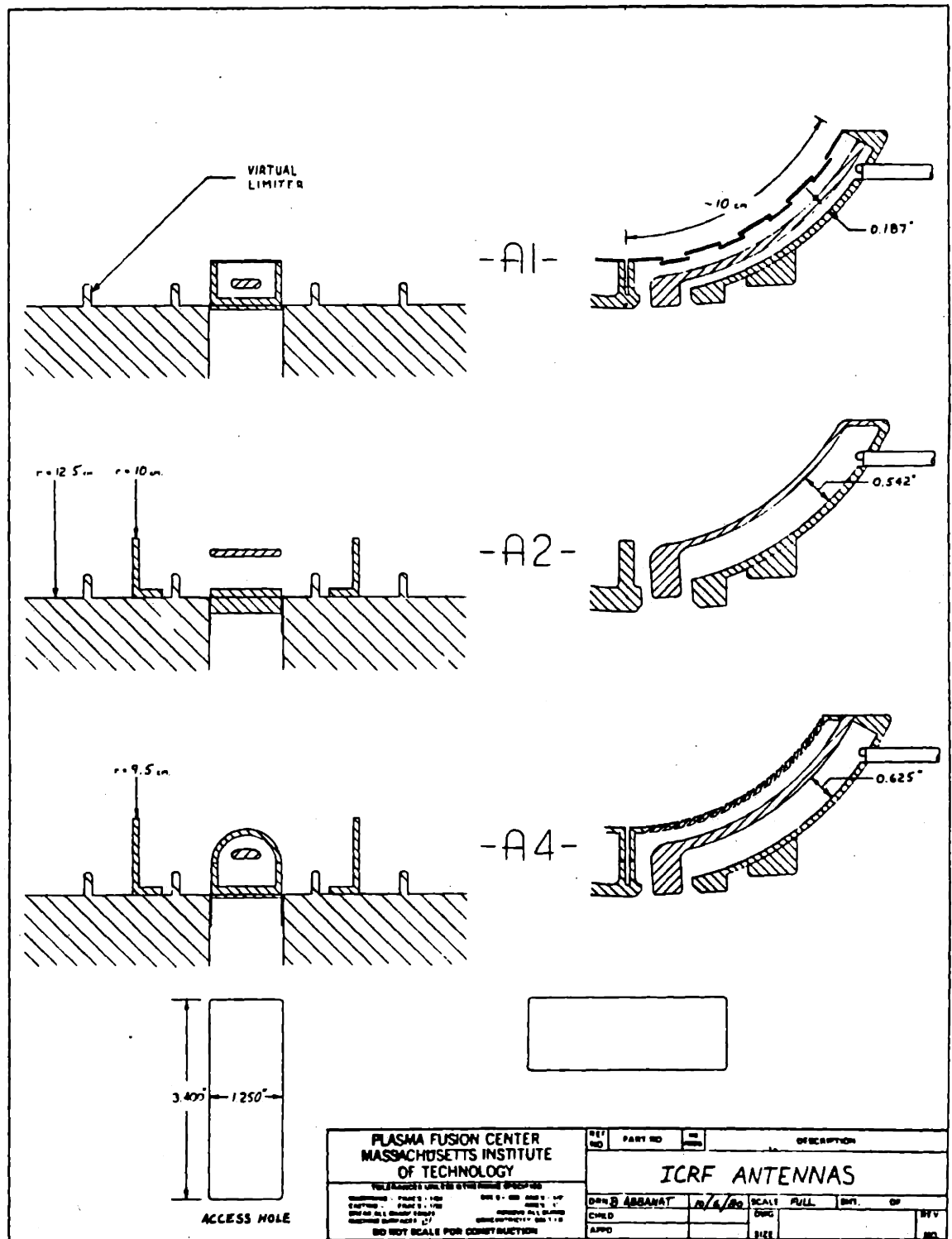
All three antennas were located on the low field side of the plasma, with their back plane electrically connected to the vacuum vessel ( $r(\text{wall}) = 12.5 \text{ cm}$ ). The antennas were made with two current loops ( $\Delta\theta = 65^\circ$ ) that could be fed so their magnetic fluxes added ( $m = 0$  mode,  $I_T$  out of phase with  $I_B$ , push-pull drive), or cancelled ( $m = 1$  mode,  $I_T$  in phase with  $I_B$ , push-push drive). All three antennas also had internal RF current probes near the voltage minimum, thus making possible direct calculation of loading resistance, an advantage not usually seen in other experiments.

A high power 20 foot, 1200 lb. coaxial resonator system <sup>72</sup> was built

to match, balance, and electrically isolate the antenna (Figure 4). The extremely versatile resonator could match almost any reactive or resistive load down to a fraction of an ohm, from 45 to 360 MHz, and up to the megawatt power level. Figure 5 is a simplified sketch of the vacuum antenna feeder. The  $m = 1$  capacitive coupler is shown in solid lines, while the  $m = 0$  magnetic coupler is shown in dashed lines. The vacuum breaks are shown with their  $\text{CCl}_4$  and teflon sleeves. A late version, two stage dc break is also shown. The resonator and coupler box are at ground potential while the antenna and vacuum feeder hardware are at machine potential. Note also how the antenna feeder center conductor is supported only at each end, the antenna current maximum ground, and the vacuum breaks.

Considerable time was spent developing satisfactory high power vacuum electrical breaks. Many materials and geometries were tested at very high RF voltages using a modified panel at the voltage maximum in the resonator. Liquid carbon tetrachloride was found to have superior dielectric and cooling properties for high power, low loss RF insulation.

Finally, two innovative antenna structures and matching systems,  $A_3$  and  $A_6$ , were proposed and designed (Section III-2.6). These last RF launching systems could be a full  $360^\circ$  around the plasma,  $m = 0$  or  $m = 1$ , all metal, and shielded or unshielded. The resonator would be only a half wavelength long at Alcator vacuum, and the vacuum breaks would be at a real power feed point. The  $A_6$  launching system would, theoretically, have superior power handling capability and loading and versatility characteristics.



Alcator A ICRF Antennas

Figure 1

# "A Boat in the Bottle" Technique

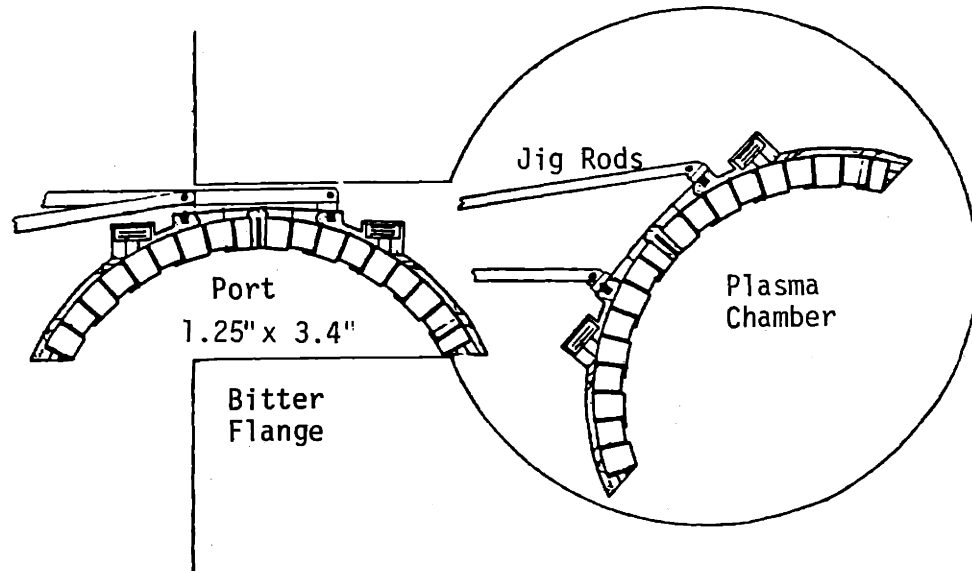


Figure 2

## Nomenclature and Coordinate System

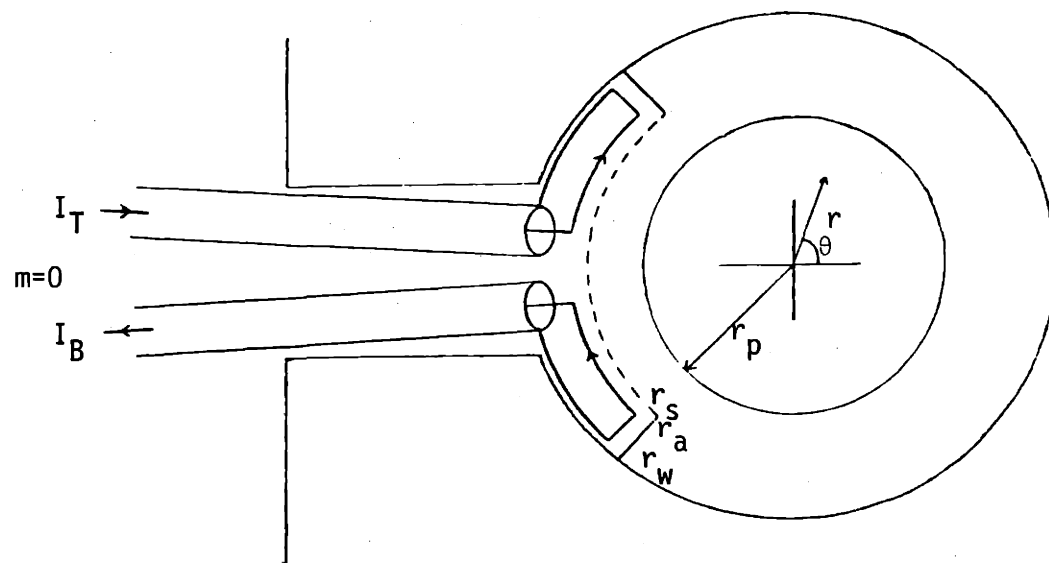
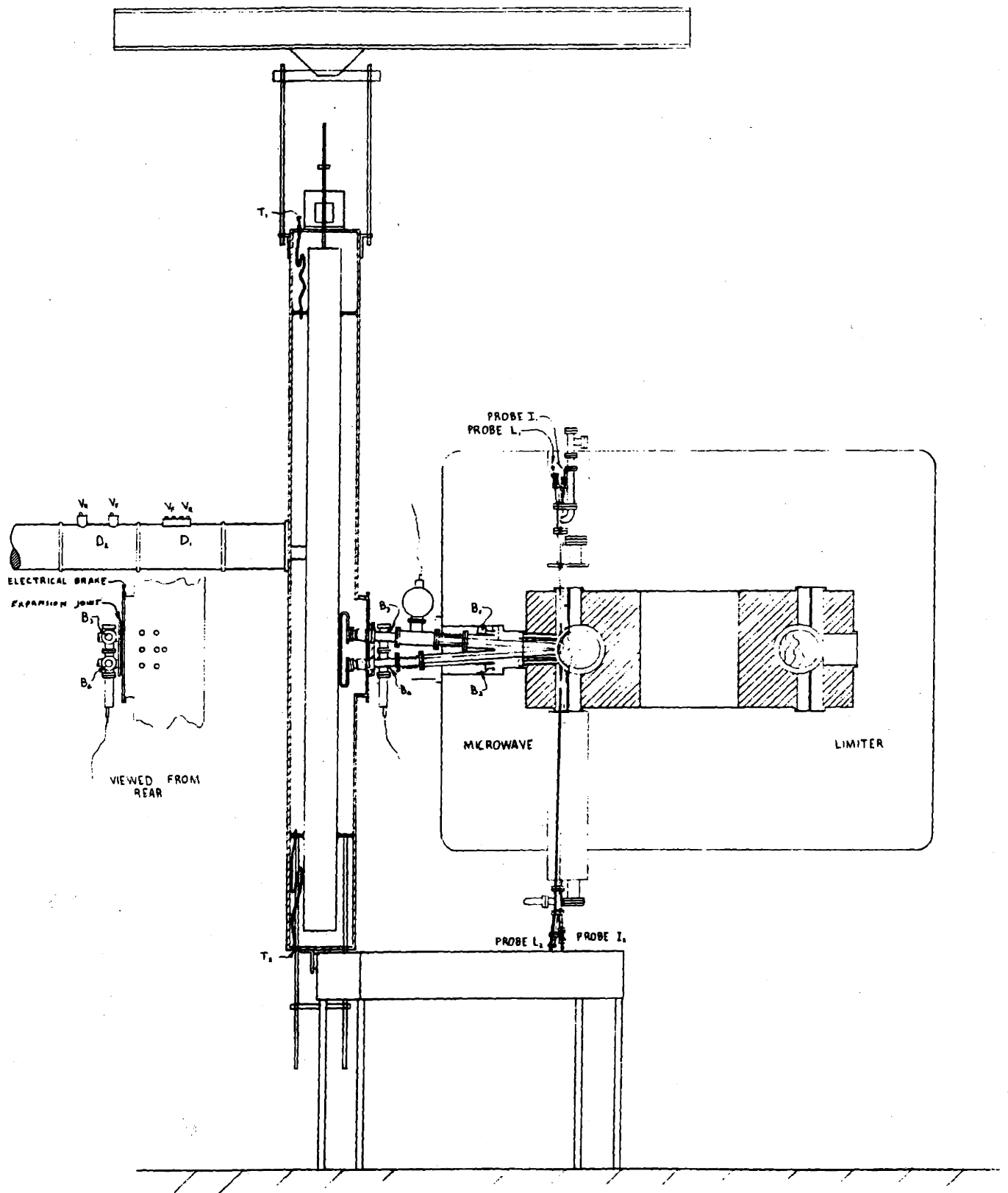


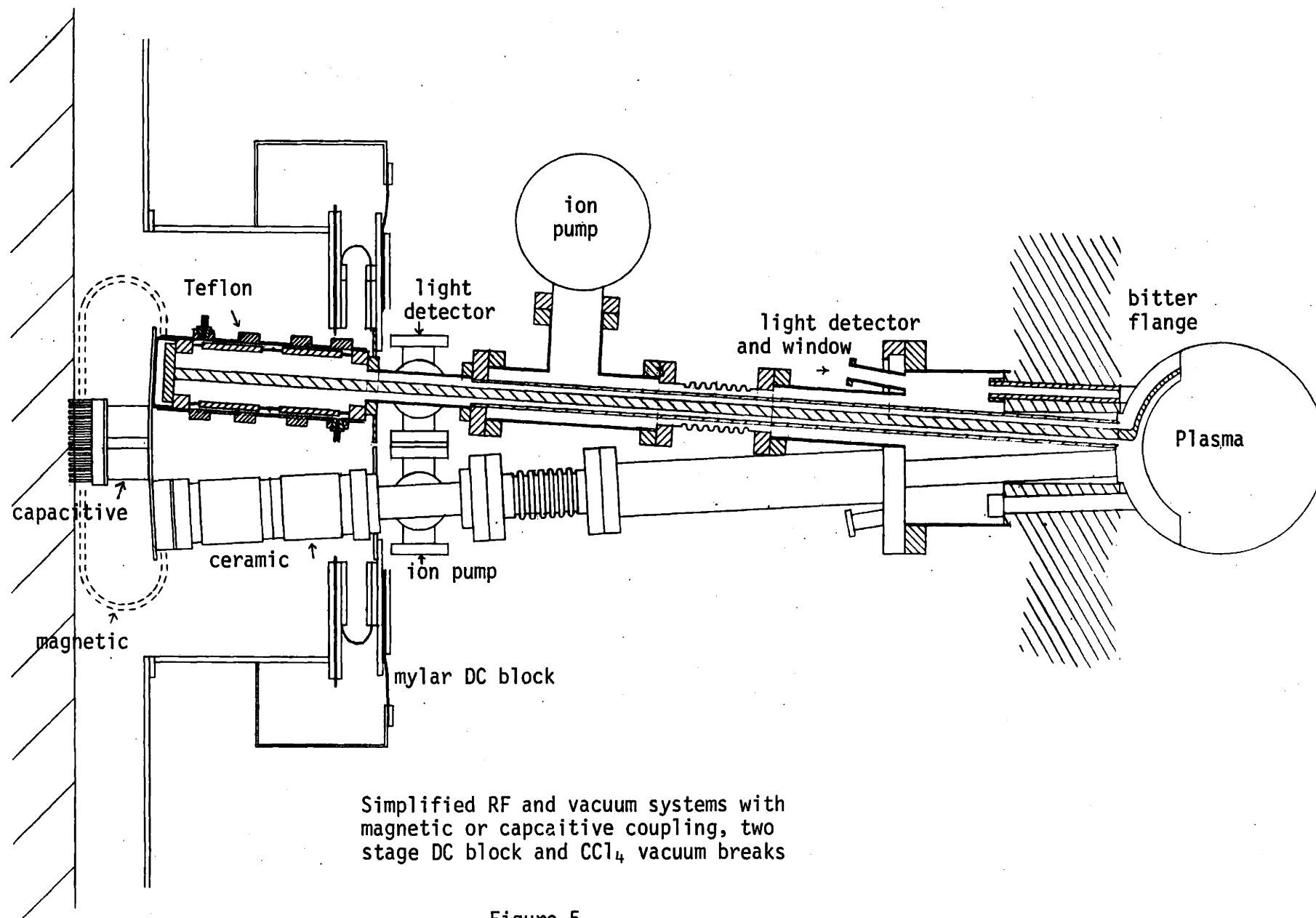
Figure 3



Alcator A ICRF Matching and Coupling System

Figure 4





Simplified RF and vacuum systems with magnetic or capacitive coupling, two stage DC block and  $\text{CCl}_4$  vacuum breaks

Figure 5

### III-2.2. A<sub>1</sub> antenna system and engineering constraints

The Alcator A access ports were made by milling holes and slots in a 3" thick solid block of 304 stainless steel (Bitter flange), which is part of the high compressional strength Bitter magnet. The plasma is actually located in a ten inch hole in this stainless steel block. Figure 1 is a mechanical drawing of the A<sub>1</sub> antenna mounted in the Bitter flange. Two hollow pins hold the antenna firmly against the vacuum wall. The treaded ends that screw in the antenna are locally weakened, so the pins can be broken in case of galling. Light inside the antenna can be monitored through a line of sight path between the antenna center conductor and a window mounted on the main 10 inch spool piece.

The feeder outer conductor is one inch in diameter, and has a "valve seat" fit to the back of the antenna. For comparison, this outer conductor is smaller than the TFR and PLT antenna center conductors, and A<sub>1</sub> is small enough to fit inside the center conductor of nine inch coax. The inside conductor is only half an inch in diameter, giving an impedance of about 40 ohms. 40 ohms is optimum for power carrying capability<sup>18</sup>, and is a good mechanical compromise between a sagging small center conductor, and a large stiff one requiring too much dimensional stability.

The A<sub>1</sub> antenna was "carved" out of a solid block of 304-L stainless steel, as can be seen from Figure 2. Only the center conductor and front Faraday shield clips were welded in afterwards. Five almost symmetric pairs of holes were drilled in the backplane of the antenna. (Figures 2 and 3). Starting at the center of the antenna, we have the one inch feeder coax seat, then the insertion rod holding threads, the holding pin threads, the Langmuir probe seat and finally, at the end of the antenna, the antenna current probe seat.

The rugged Faraday shield is an integral part of the back plane, and was formed by slicing the side limiters at a 60° angle. The front overlapping clips were then electron beam welded to the back plane (Figure 4). Note how the vacuum vessel radius is only 12.5 cm, while the plasma limiter is 10 cm, so the Faraday clips are less than 1 cm from the relatively (to TFR and PLT) high power density plasma edge.

Figures 5 and 6 are mechanical drawings and assemblies of the vacuum RF antenna feeder. All components are standard 4.5 and 2.75 inch stainless steel hardware with knife edge and copper gasket seals. The ceramic vacuum breaks are high grade alumina ( $\text{Al}_2\text{O}_3$ ) with high temperature brazing, so the whole antenna assembly is of ultra high vacuum quality ( $10^{-9}$  torr) and fully bakable.

The two 4.5 inch bellows enable the outer coax conductors to press firmly on the antenna seats. Two 30 liter/sec high-Q ion pumps, mounted on the 2.75 and 4.5 inch "T's" pump the coax feeders and the large trapped volume in the port behind the antenna. Note from Figure III-2.1.(4) how the main ten inch spool piece is at the bottom of a deep tunnel inside the Alcator liquid nitrogen dewar, considerably limiting access to the Bitter flange.

All RF carrying surfaces are electroplated with .001 - .003 inches<sup>71</sup> of pure silver (i.e., no organic brighteners). The skin depth at 200 MHz is<sup>134</sup>

$$(1) \quad \delta = \frac{\sqrt{2}}{\sqrt{\omega \mu_0 \sigma}} = 4.6 \times 10^{-6} \text{ m} = 1.8 \times 10^{-4} \text{ inches}$$

where, for silver,  $\sigma = 6.1 \times 10^7$  mho/meter, and  $\delta$  is much less than the silver thickness.

About half of the easily calculatable system resistive losses comes from the two meters of half inch diameter center conductors

$$(2) \quad R_L = \frac{L}{\sigma \delta 2\pi r} \approx .18 \text{ Ohms}$$

With 200 amperes in these conductors, some

$$(3) \quad P/\text{cm} = \frac{I^2 R_L}{L} \approx 36 \text{ watts/cm}$$

$$P/\text{cm}^3 = \frac{P/\text{cm}}{\pi r^2} \approx 28 \text{ watts/cm}^3$$

and a final temperature of 830°K would be reached after ( $T_o = 270^\circ\text{K}$ )

$$(4) \quad \Delta t = \frac{(T_f - T_o) \rho c_p}{P/\text{cm}^3} \approx 80 \text{ sec}$$

where for stainless steel,  $\rho = 7.9 \text{ g/cm}^3$ ,  $c_p = .504 \frac{\text{J}}{\text{g } ^\circ\text{K}}$  and  $k = .16 \frac{\text{W}}{\text{cm } ^\circ\text{K}}$

This corresponds to about two hours of antenna conditioning, with sixty 10 msec pulses a minute. The 830°K was chosen for a  $10^{-8}$  torr silver vapor pressure calculated from the approximate empirical formula<sup>128</sup>

$$(5) \quad \log_{10} p(\text{torr}) = 8.865 - 14058/T(^{\circ}\text{K})$$

From the one dimensional heat diffusion equation in a solid

$$(6) \quad \frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}$$

where for stainless steel,  $\alpha = \frac{k}{\rho c_p} = .04 \text{ cm}^2/\text{sec}$

we can calculate the approximate heat penetration depth for a 200 msec RF pulses as<sup>139</sup>

$$(7) \quad \Delta x \approx \sqrt{\alpha \Delta t} \approx .09 \text{ cm} \ll r$$

and the heat is deposited only on the surface of the conductors. In fact, the surface temperature of a semi-infinite solid exposed to a heat flux is<sup>140</sup>

$$(8) \quad \Delta T = \frac{P/cm}{2\pi r} \cdot \frac{1}{k} \sqrt{\frac{4 \alpha \Delta t}{\pi}} \approx 5.7^\circ K$$

Similarly, the heat diffusion time out each end of the center conductors can be estimated from equation (7)

$$(9) \quad \Delta t = \frac{\Delta x^2}{\alpha} \approx 17 \text{ hours}$$

which is much longer than any run, and the heat simply piles up as in equation (4).

For sufficiently large surface temperature ( $\approx 830^\circ K$ ), black body radiation<sup>141</sup>

$$(10) \quad P/cm^2 = \sigma \epsilon T^4 (^\circ K) \approx .27 \text{ W/cm}^2$$

where  $\sigma = 5.67 \cdot 10^{-12} \frac{W}{cm^2 \cdot ^\circ K^4}$  and  $\epsilon \approx .1$ , radiation becomes important ( $P/cm^2 \approx .09 \text{ W/cm}^2$  during conditioning), and the surface temperature is not likely to reach the  $10^{-8}$  torr vapor point.

Three basic types of electrical breakdowns can occur in the vacuum feeder: electron cyclotron, multipactor, and avalanche breakdown. Electron cyclotron breakdown occurs in high electric fields where

$$(11) \quad f_o = \frac{2\pi eB}{M} \approx 100 \text{ MHz at } B_o \approx 36 \text{ gauss}$$

The Alcator fringe fields are greater than 36 G, but in any case, a simple coil could easily be wound around the feeders (B parallel to the feeders) to increase the magnetic field. 200 dc amperes pulsed could also be injected in the center conductors (from an RF feedthrough in the resonator) to produce a "poloidal" field, also possibly increasing the threshold of avalanche and multipactor breakdown. The drawback is the significant  $IXB$  force on the antenna center conductor. For 200 amperes, 60 kG, and 10 cm, the force is

$$(12) \quad F = IB\ell \approx 120 \text{ newtons} \approx 27 \text{ lbs}$$

and the average poloidal magnetic field is

$$(13) \quad B_p = \frac{I \mu_0}{2\pi r} \approx 42 \text{ gauss}$$

The time varying tokamak magnetic fields also induce voltages and currents in the antenna feeders,

$$(14) \quad V = -A \frac{\partial B}{\partial t} \approx .03 \text{ Volts}$$

$$(15) \quad I = \frac{V}{R} = \frac{V \sigma \pi r^2}{L} \approx 2.0 \text{ amperes}$$

where we assumed  $A \approx 50 \text{ cm}^2$ ,  $\frac{\partial B}{\partial t} \approx 6 \text{ T/sec}$ ,  $\sigma_{ss} = 1.1 \times 10^6 \text{ mho/meter}$ , and therefore, are unimportant.

Multipactor occurs when the accelerated electron path length in an RF electric field becomes comparable to the electrode spacing.

$$(16) \quad m\ddot{x} = -e E_0 \cos \omega t$$

$$x = \frac{e E_0}{m\omega^2} \cos \omega t$$

$$(17) \quad E_0 = \frac{d m \omega^2}{2e} \approx 900 \text{ V/cm}$$

$$P = 160 \text{ watts}$$

where we assumed  $d = 2 \text{ cm}$ ,  $200 \text{ MHz}$ ,  $50 \Omega$  coax, and  $R_R = 1 \Omega$ . The electrode must also have a secondary emission coefficient greater than one at about  $W \approx 2 E_0 d$ , which is generally true for insulators, and in particular, for  $\text{Al}_2\text{O}_3$ , which has a peak coefficient of about 5, around  $1 \text{ keV}$ .<sup>142</sup> At low energy, the electrons cannot "knock out" secondary electrons, and at high energy, the electrons are buried too deeply in the material. Conductors and, in particular, silver, do not secondary emit easily, and the problem is most likely to occur only in the ceramic vacuum breaks.

For multipactor coefficients greater than one, the electric field<sup>18</sup> need not be in phase with the electron motion, and a whole range of electric fields are possible. In practice, multipactor can occur over several decades in power ( $\propto E^2$ ), especially with long electrical breaks where several different multipactor regions are possible. Figure 8 is a dramatic example of how the antenna current can be almost totally suppressed by slightly lowering the power into the multipactor regime. Figure 7 compares the approximate experimental results with a more sophisticated multipactor model that takes into account the electron phase and the multipactor coefficient. A static magnetic field can greatly modify the electron motion, and thus multipactor. Insulating coatings with low secondary emission coefficient have been developed but were not necessary, since the effect could be circumvented with reasonably low ( $< 1$  watt) and high ( $> 1$  kW) powers.

The more commonly known avalanche breakdown can occur either inside or outside the vacuum, when electrons or ions are literally pulled out of the surfaces, forming large quantities of highly conducting ionized gas. The worst pressure region is in the high millitorr range, slightly above the Alcator backfill pressure. To ensure high vacuum integrity and keep as much as possible of the RF feeder at high vacuum, a 30 liter ion pump was installed near the electrical breaks. The coax gas conductance in the molecular flow regime<sup>80</sup>

$$(18) \quad \lambda_{mfp} \approx 50 \text{ cm} \gg D \text{ at } 10^{-4} \text{ torr}$$

$$(19) \quad U_c \approx 12.2 \frac{D^3}{L} \approx .44 \text{ l/sec} \ll 30 \text{ liters/sec}$$

where  $D \approx 1.9 \text{ cm}$  = average diameter,  $L \approx 100 \text{ cm}$ , and therefore the vacuum break pressure will be about an order of magnitude below the torus pressure.

On the other hand, the Faraday shield conductance is large

$$(20) \quad U_s \cong 11.7 A_s \text{ liters/sec} \cong 100 \text{ l/sec} \gg U_c$$

and the antenna remains at the torus pressure.

Figure 9 shows typical arcing conditions inside and outside the vacuum feeders. Arcs outside the vacuum are usually "stiff", and quickly crowbar the antenna current. Vacuum arcs can be either "stiff" or "soft", and are accompanied by large amounts of light inside the feeders.



A<sub>1</sub> Antenna Assembly in Alcator

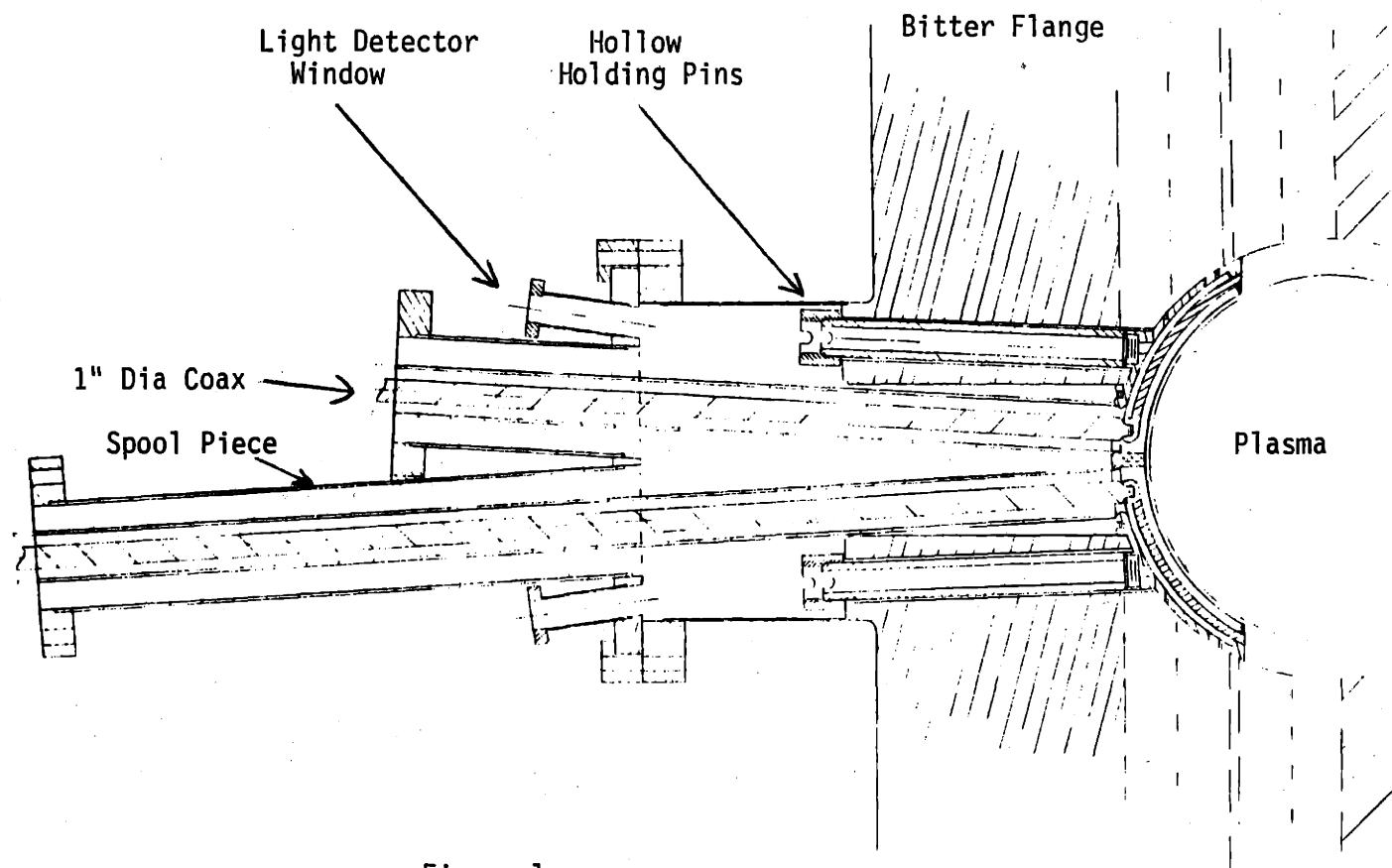
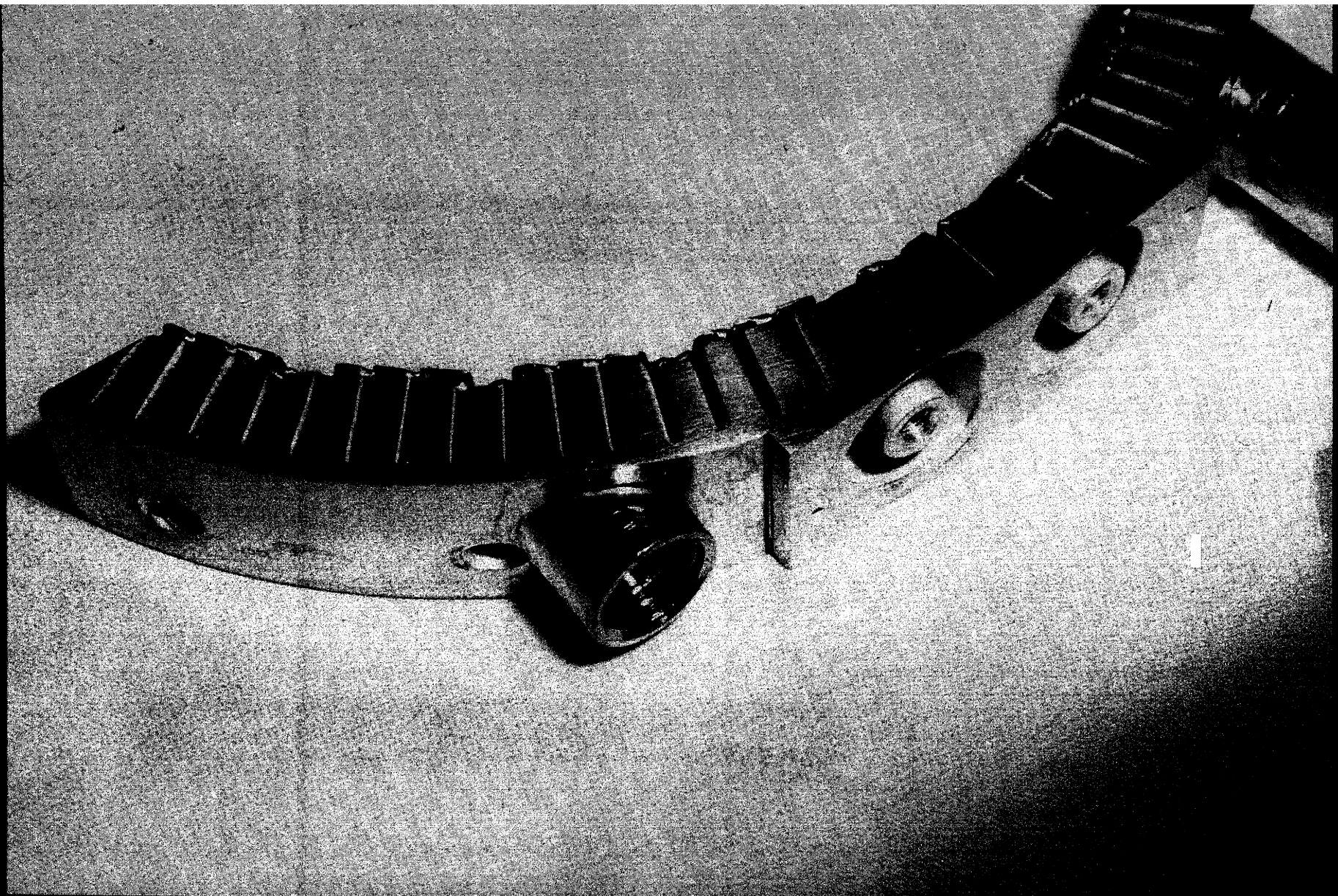


Figure 1

- 124 -

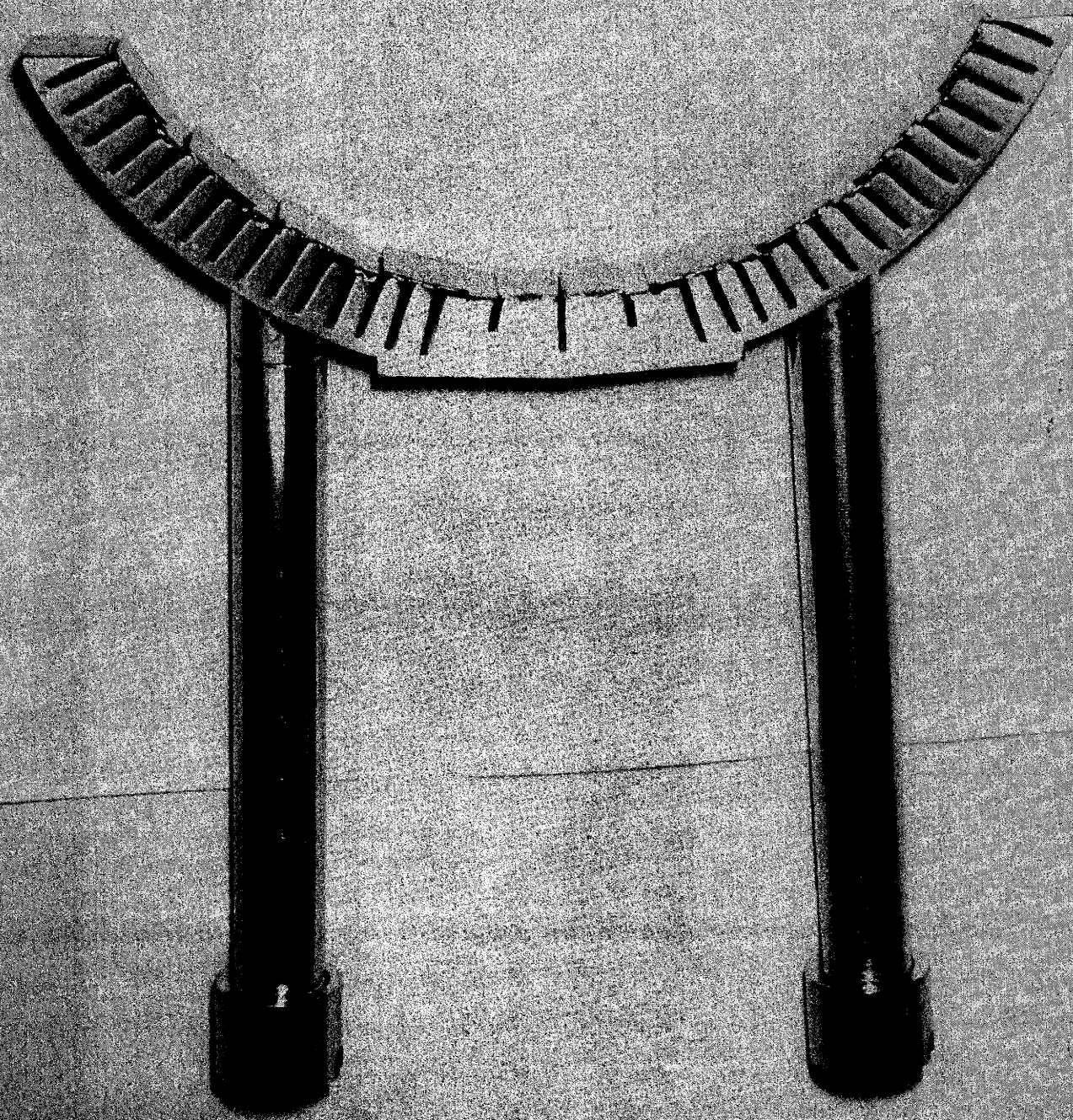




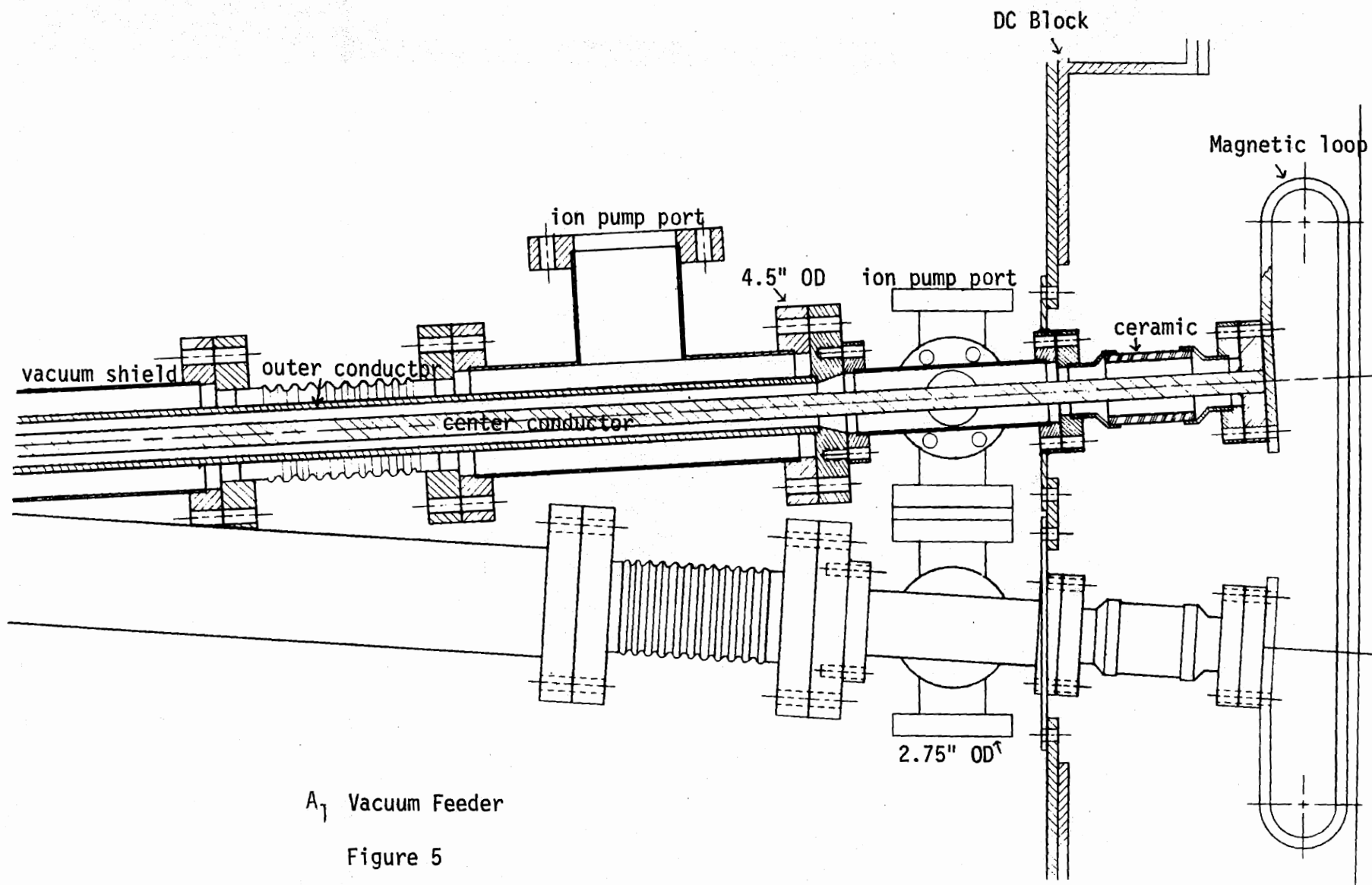
Backview of A<sub>1</sub> Antenna.

Figure 3





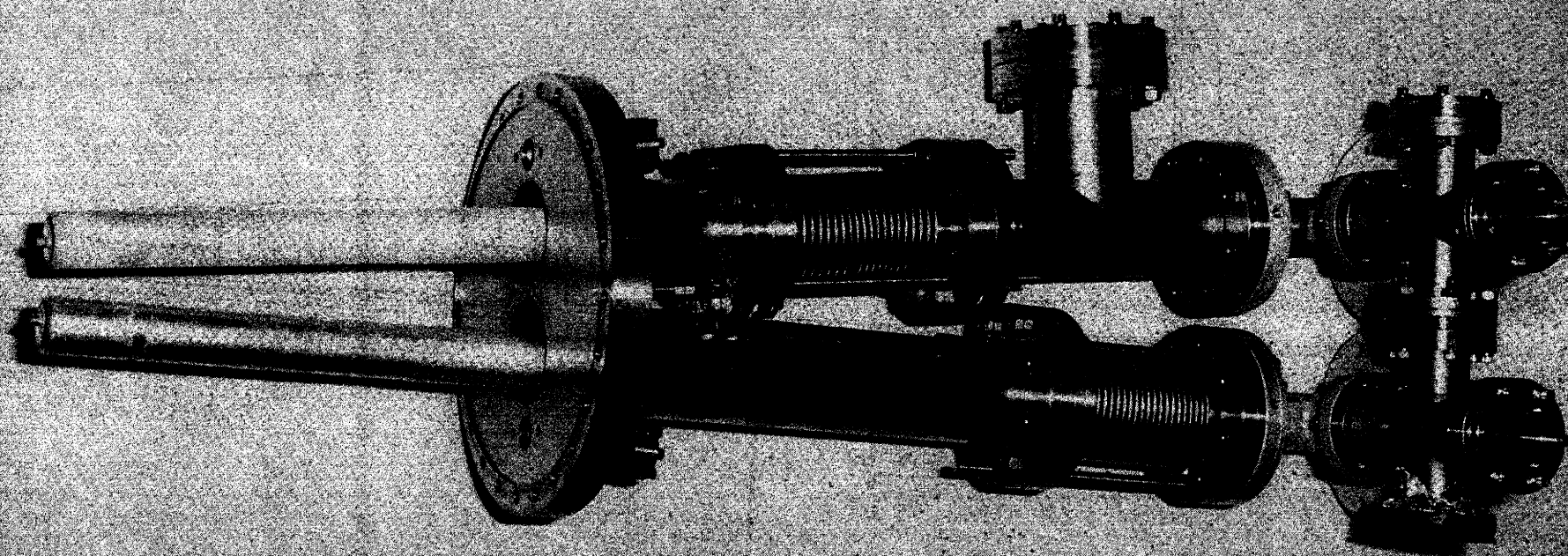
Side View of A<sub>1</sub> Antenna and Holding Pins.  
Figure 4



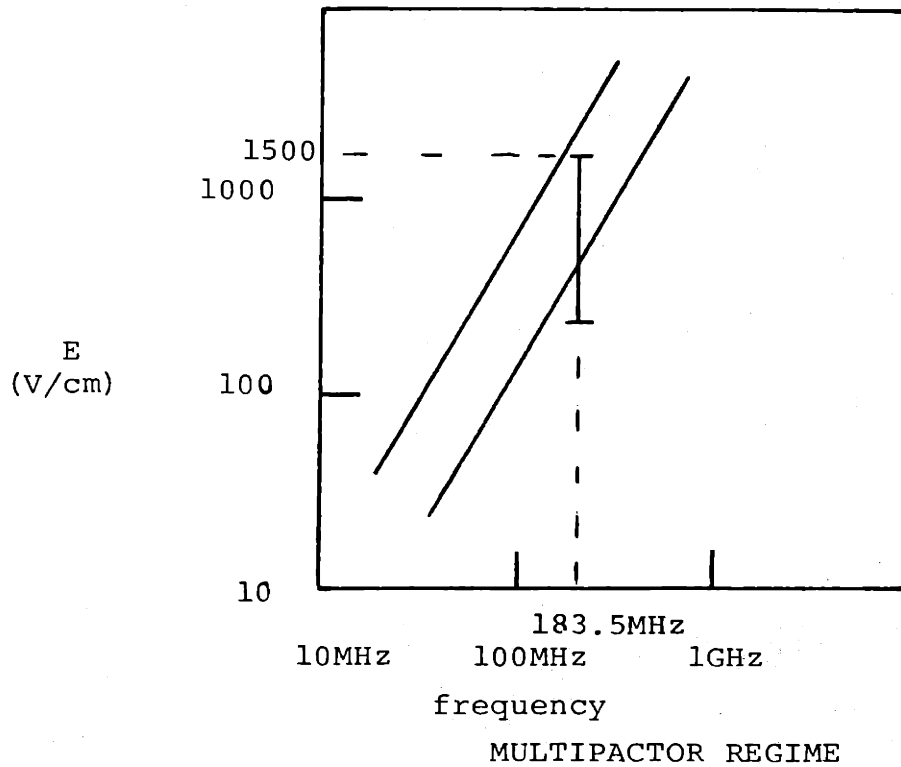
A<sub>1</sub> Vacuum Feeder

Figure 5





A<sub>1</sub> Vacuum Feeder Without Electrical Breaks.  
Figure 6

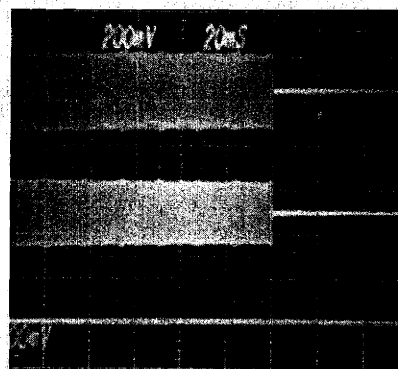


from TEPR-1

PPPL 1977

Figure 7

ABOVE MULTIPACTOR  
REGIME

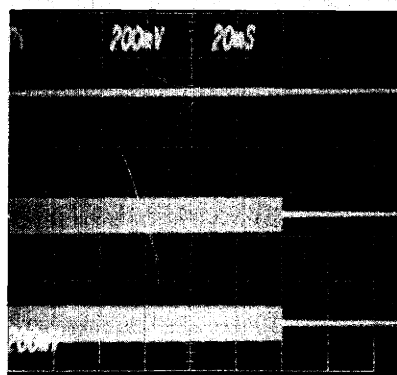


$I = 32$  amperes rms

$V_F = 350$  watts

$V_R$

BELOW MULTIPACTOR  
REGIME



$I_A$

$V_F$

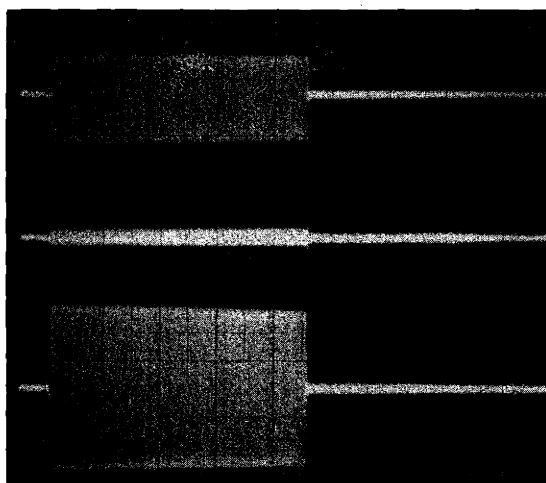
$V_R$

Figure 8

Electrical Break  
Arcs with New  
Long Breaks

3/20/80

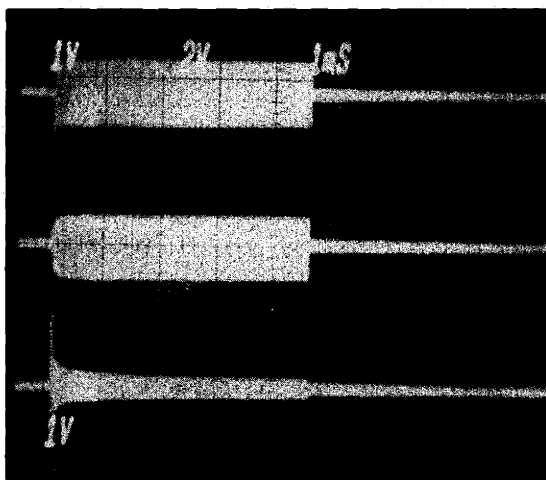
Normal Operation



$V_F$

$V_R$

$$I_a = 6V_{pp} \times 32.5 A_{rms} / V_{pp} \\ = 195 A_{rms}$$

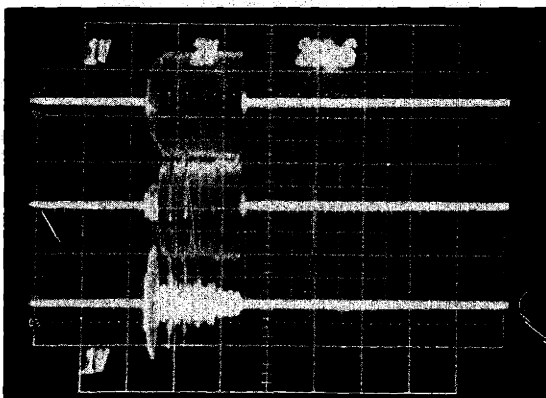


Arc Outside Coax

$V_F$

$V_R$

$I_a$



Arc Inside Coax

$V_F$

$V_R$

$I_a$

Figure 9



### III-2.3. Matching resonator

The Alcator A matching system used a simple high power resonator (Figure 1).<sup>72</sup> Since space was not a constraint, the 13" square coaxial resonator was built large enough to accommodate almost any frequency (45-360 MHz), any resistive or reactive loading resistance (.5  $\Omega$  to 50  $\Omega$ ), and either  $m=0$  or  $m=1$  phasing for powers up to the megawatt level. The resonator was also used as a high RF voltage test stand for electrical break development.

The basic matching system can be illustrated with a simple, half wavelength resonator (Figure 2) with a load  $R$  at  $z = \lambda/2$ . The field components are then<sup>72</sup>

$$(1) \quad V(z) = V_o \sin kz \cos \omega t$$

$$(2) \quad I(z) = \frac{-V_o}{Z_o} \cos kz \sin \omega t$$

At resonance, the impedance at the tap point  $z = d$  is purely resistive. From conservation of power and proper match,

$$(3) \quad \frac{V^2(d)}{Z_o} = I^2 (\lambda/2) R$$

and substituting (1) and (2) into (3), we have

$$(4) \quad \frac{V_o^2 \sin^2 kd}{Z_o} = \frac{V_o^2 R}{Z_o^2}$$

and the tap point angle

$$(5) \quad kd = \sqrt{\frac{R}{Z_o}} \approx 8^\circ \text{ for } R = 1 \Omega \text{ and } Z_o = 50 \Omega \\ \approx 26^\circ \text{ for } R = 10 \Omega \text{ and } Z_o = 50 \Omega$$

The actual resonator used was a combination of three overcoupled resonators as shown in Figures 3 and 4. At 200 MHz, the vacuum feeders are about a wavelength long, and a voltage minimum in the main resonator is tuned to half way between the antenna feeders. The input tap angle is calculated again from (5). Three conditions must be met for proper tuning. First, the system must be resonant, second, the top and bottom currents must be roughly equal, and third, the tap angle must be set for proper match. These three conditions can be met by the three moveable components; top plate, bottom plate, and tap angle. Unfortunately, the three degrees of freedom are not orthogonal, and expertise must be developed for quick, accurate tuning. A number of diagnostic current loop and capacitive probes are located in the matching system, in particular, on the top and bottom tuning plates, near the vacuum feedthrough and, of course, in the antenna.

At 90 MHz, the feeders are only  $\lambda/2$ , and the resonator is  $3/4 \lambda$ , with the bottom shorting plate removed. Other similar resonant modes are also possible, especially at the higher frequencies.

Figure 5 is a much more general model of the resonator system that is valid, even off tune. Using the general lossless transmission line impedance transformation formula

$$(6) \quad Z_x = Z_o \frac{Z_L + i Z_o \tan k\ell}{Z_o + i Z_L \tan k\ell}$$

the impedance of each half of the antenna,  $Z_L = Z_1 = R_R/2$ , is transformed over a distance  $\ell = A$ , to a new, complex impedance,  $Z_x = Z_2$ . Similarly, the lower circuit  $Z_L = 0$  is transformed to  $Z_3$ .  $Z_3$  and  $Z_2$  are combined to give  $Z_4$ , which, in turn, is transformed to  $Z_5$ , and so on until  $Z_9$ , which will be pure, real 50  $\Omega$  if the system is in tune.

The vacuum feeders can also be run in the push-push mode by tying the two feeders together, coupling capacitively, and displacing the lower voltage node in the resonator. Balance is guaranteed by symmetry, only two degrees of freedom are necessary, and the system becomes much easier to tune.

Figure 6 shows the basic mechanical layout of the resonator box, coupler box, and the one stage capacitive DC break between the vacuum feeders and the grounded resonator. Figure 7 shows the top plate and center conductor tap angle remote control drive mechanisms. Figure 8 shows the top shorting plate and tap angle center conductor bellows. The resonator is built of 1/4 inch machined and silver plated copper plate. The resonator must be extremely RF leak tight, since for 200 amperes of push-push antenna current 400 amperes circulate in the resonator, corresponding to 20 kV RF and 8.0 megawatts of reactive power. Even under these circumstances, as we calculated earlier in Section III-1.2, the RF leakage is much less than  $10^{-6}$  watts/cm<sup>2</sup>.

# Side View of 13 inch Diameter Coaxial Resonator

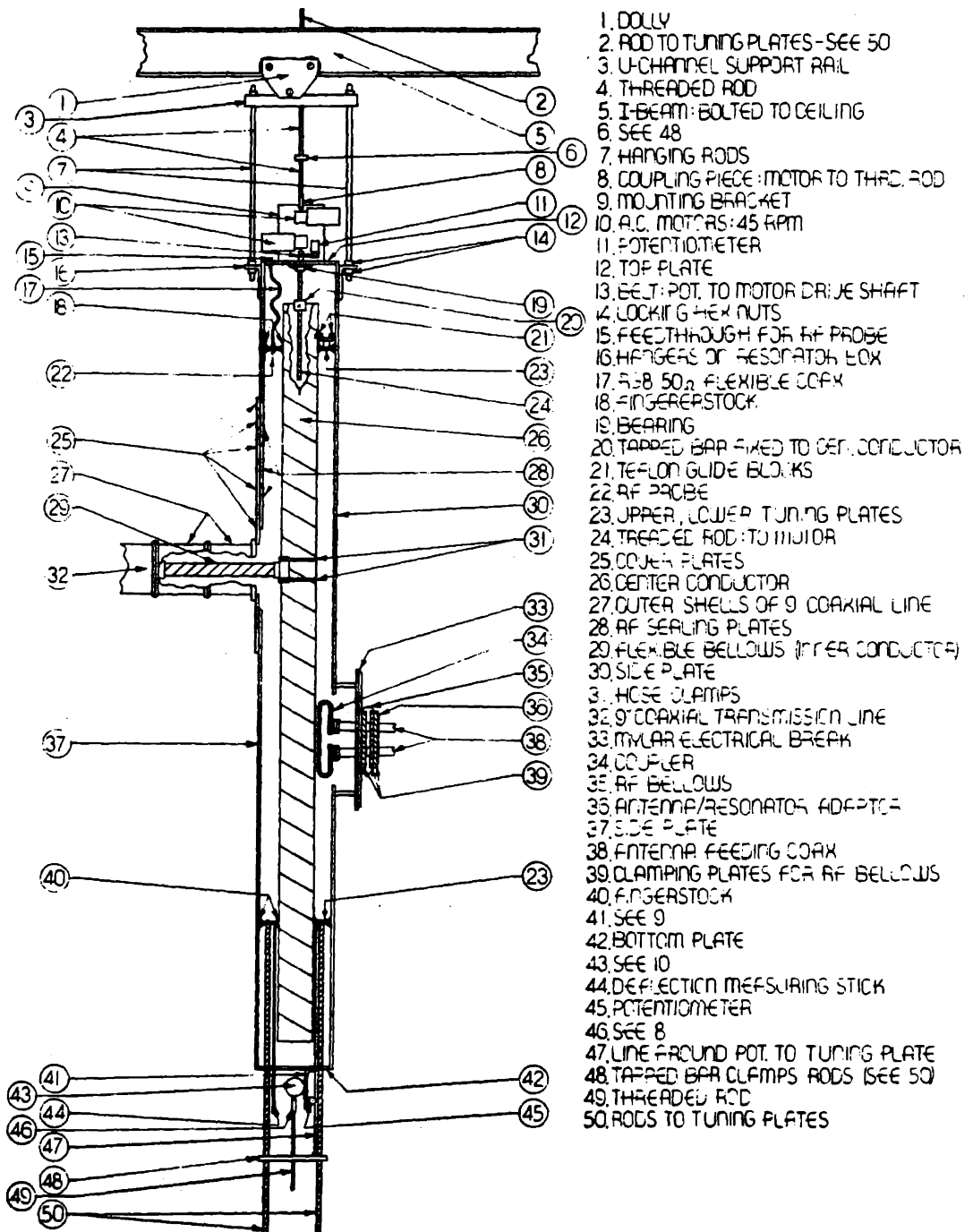


Figure 1

Resonator Waveform

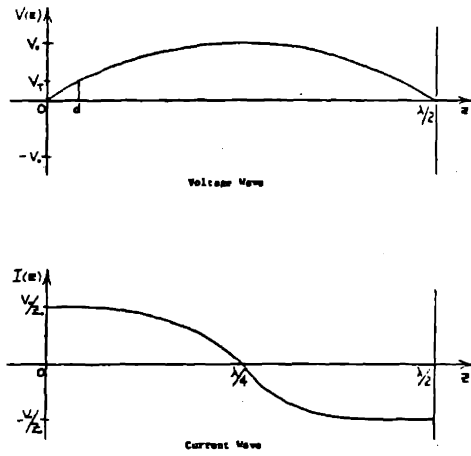


Figure 2

180 MHz Current Waveform

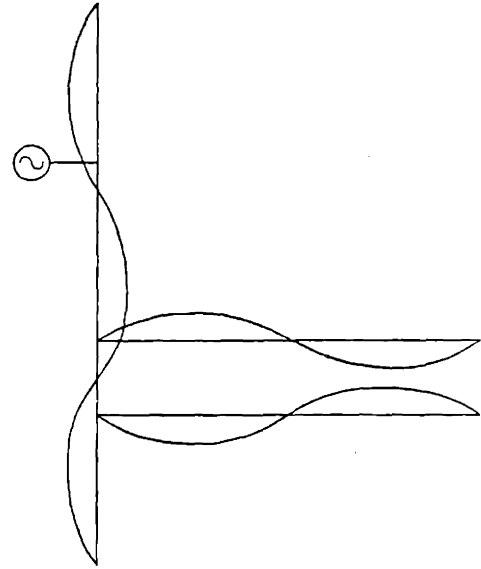


Figure 3

90 MHz Waveform

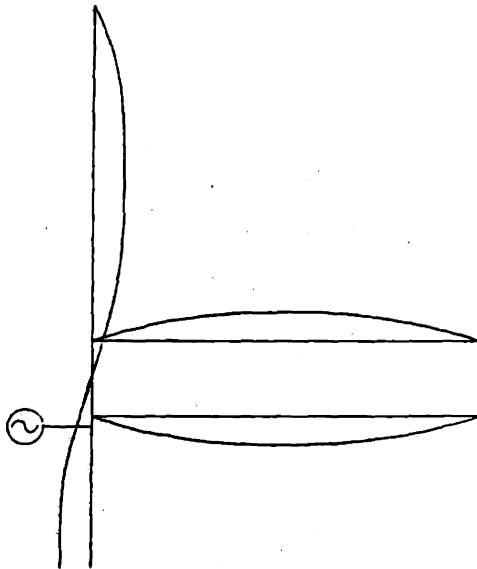


Figure 4

General Resonator Model

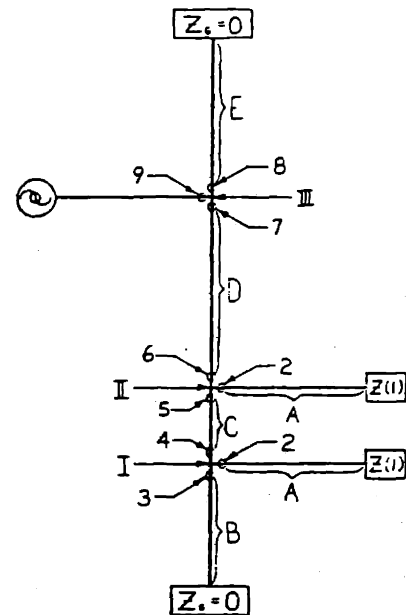
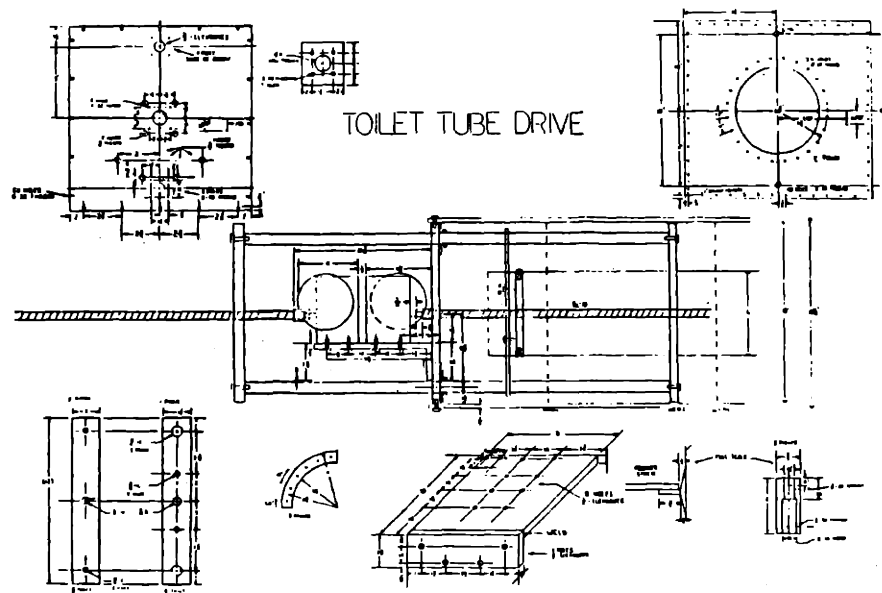
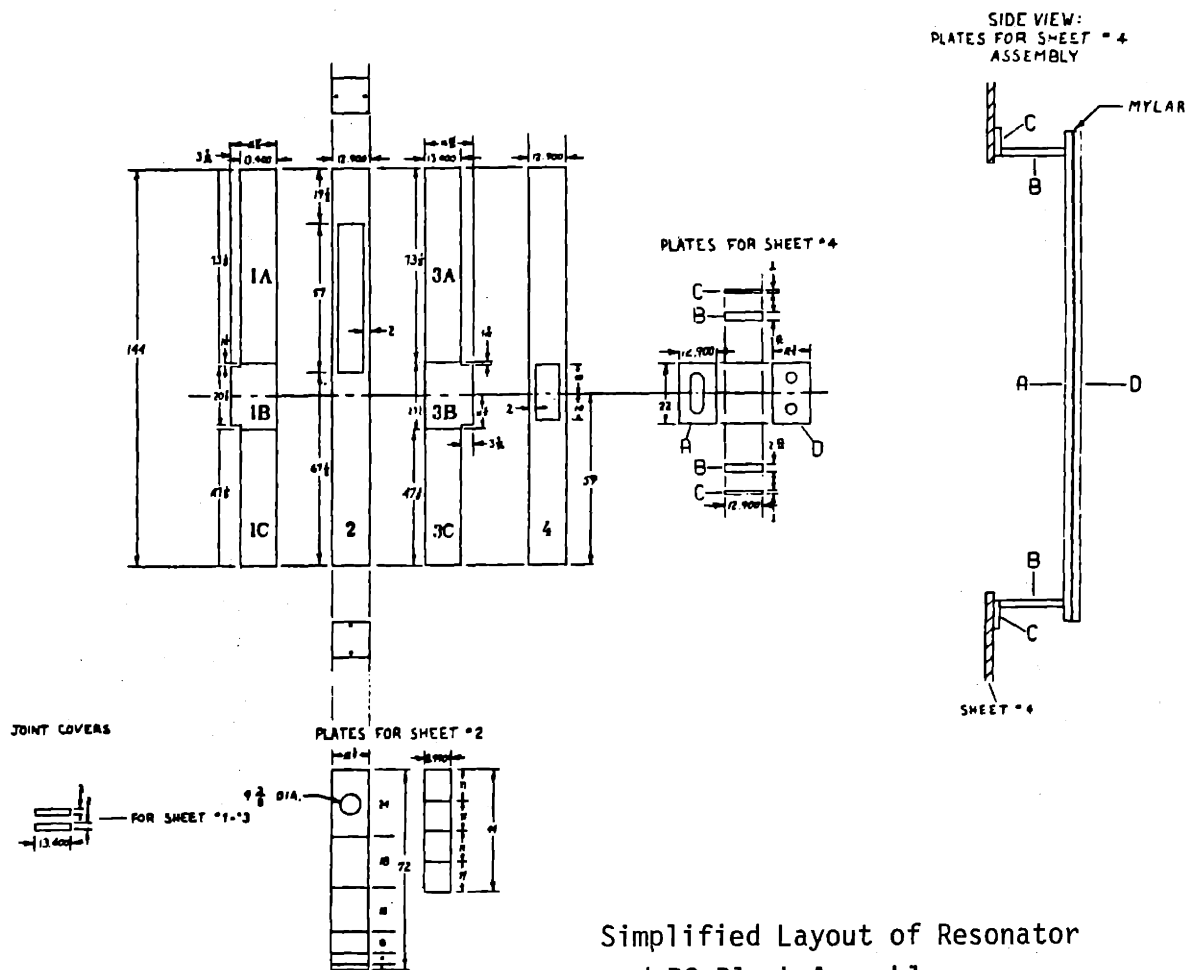


Figure 5



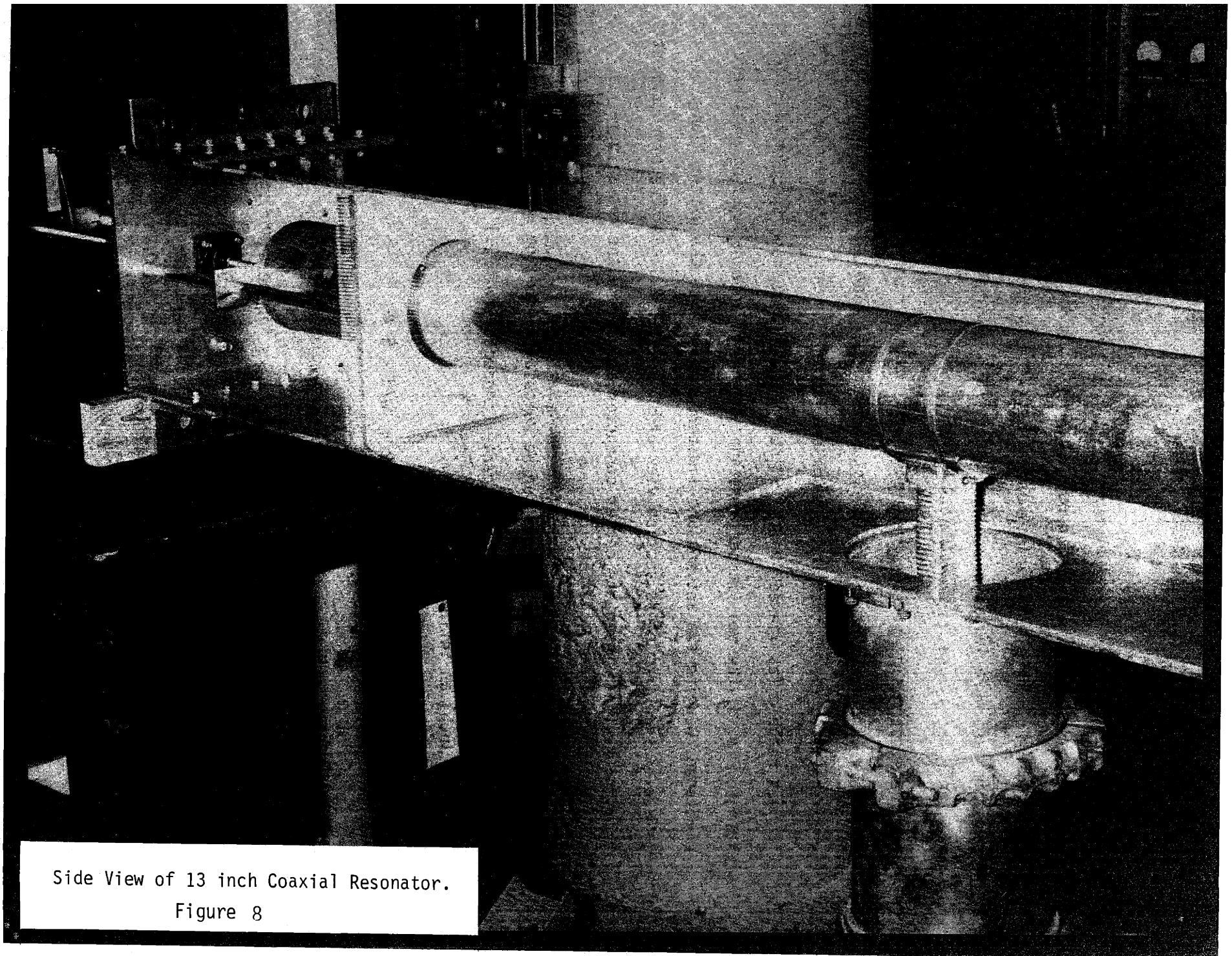
Resonator Drive Mechanism

Figure 7



Simplified Layout of Resonator  
and DC Block Assembly

Figure 6



Side View of 13 inch Coaxial Resonator.  
Figure 8

#### III-2.4. A<sub>2</sub> antenna system

To increase power coupling capability, a new antenna, A<sub>2</sub>, was built with the same backplane and diagnostics as A<sub>1</sub>, but with a loop area 2.5 times larger. The plasma radius was reduced from 10 cm to 9 cm, and slotted side limiters were installed on each side of the antenna, as can be seen in Figures 1, 2, 3, and III-2.1.(1).

The slotted side limiters were similar to the A<sub>1</sub> Faraday shield, and were held in place by thin flexible stainless steel arms. These limiters were installed first, then the antenna and current probes. When proper fit was reached (as viewed from a .5 inch flexible boroscope through the top keyhole Fig. 3), the two limiter arms and two hollow antenna holding pins were tightened, wedging the antenna and limiters securely to the Bitter flange, as can be seen from cross section view of Figure III-2.1.(1). The high current carrying region between the center conductor and back plane was silver-plated as A<sub>1</sub>.

The impedance of the antenna can be calculated from the approximate semiempirical formula for a narrow center conductor ( $w \approx 1.1$  inch) over ( $d \approx .54$  inch) an infinite backplane.<sup>135</sup>

$$(1) \quad Z \approx 125 - 116 \log_{10} (w/d) \text{ for } .2 < w/d < 5 \\ \approx 90 \, \Omega$$

To calibrate the antenna current loop probes, the test set schematically shown in Figure 4 was used. The antenna with its limiters and probes were mounted in an aluminum cylinder simulating the vacuum vessel. A 50 $\Omega$  terminated oscillator, directional coupler and coax were then connected to the antenna coax seat through a special adaptor.



The currents and voltages at the antenna are (Figure 4)

$$(2) \quad I_b = I_a \cos \theta_a$$

$$(3) \quad V_b = I_a Z_a \sin \theta_a$$

where  $\theta_a = k \ell/2 \approx 22.5^\circ$  at 200 MHz.

Similarly, on the coax side of the antenna-coax boundary

$$(4) \quad I_b = I_o \cos \theta_o$$

$$(5) \quad V_b = I_o Z_o \sin \theta_o$$

Equating (2) and (4), (3) and (5), and dividing the two results, we have

$$(6) \quad Z_a \tan \theta_a = Z_o \tan \theta_o$$

$$(7) \quad \theta_o \approx 36.7^\circ$$

From (2), (4) and (7), we finally have

$$(8) \quad I_A = \frac{\cos \theta_o}{\cos \theta_a} I_o = .86 I_o \approx I_o$$

For practical purposes, we will assume

$$(9) \quad I_a \approx \frac{V_f + V_r}{50} \approx \frac{2V_f}{50}$$

and the calibration factor

$$(10) \quad C_p = \frac{I_a}{V_p} \approx \frac{V_f}{25 V_p} \approx 6.7 \text{ amps/volt at 200 MHz (typical)}$$

To increase the voltage and power capability, a much larger (compared to the first version (Figure 6)) pair of electrical breaks was also installed (Figure 7). Although several times longer, these new feedthroughs could only carry about twice the power, since the thin, long exposed center

conductors in the electrical breaks had a very high impedance, and unnecessarily increased the local RF potential. It is believed that the power capability could have been increased by simply enlarging the center-conductors of the new electrical breaks. The electrical breaks were "effectively" shortened (as in Figure 8) by shorting the first half of the electrical break with aluminum foil, resulting in an almost identical current carrying capability.

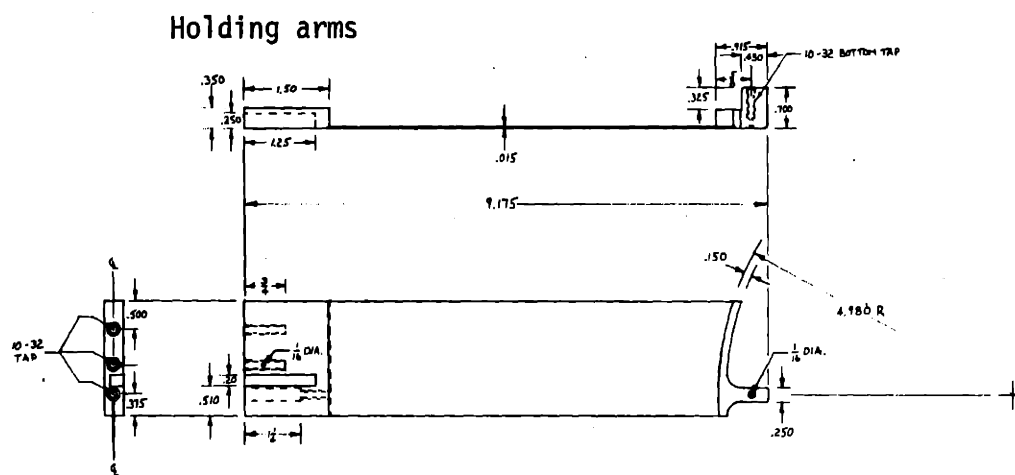
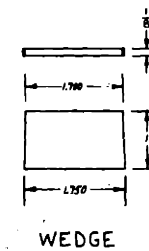
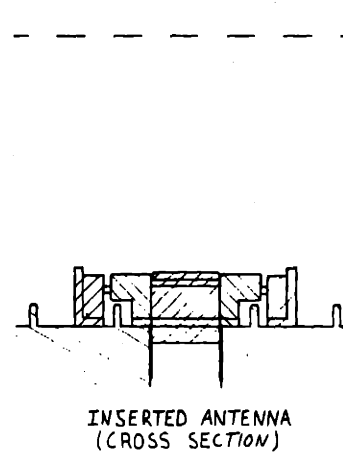
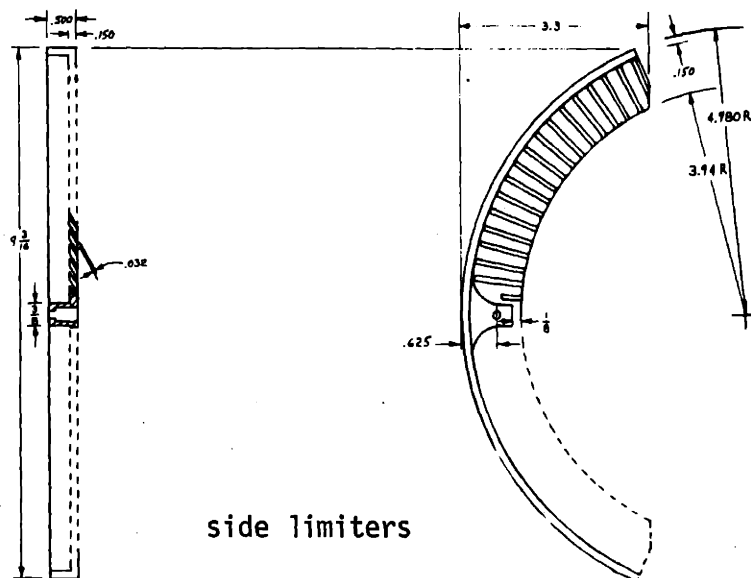
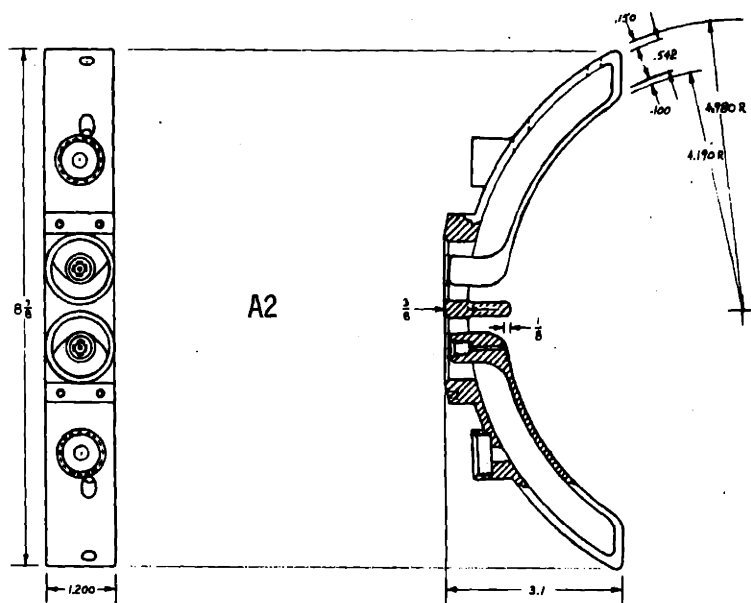
After elaborate testing of many electrical breaks, materials and geometries, it was consistently found that RF breakdown was several times more severe than DC (arcs 6 inches long between feeders were common), and always occurred on sharp edges. The most common and difficult to solve problem was at the metalized surface of the insulator, where the brazing thins out to a sharp knife edge. Several epoxies and corona rings were tested with some success (factors of 1.5 - 2 in antenna current) to cover these edges, but the main drawback was that, once breakdown occurred, the voltage standoff was greatly decreased, and the breaks were very difficult to clean.

Another important factor is keeping the electric field at the metalized joints, into the ceramic, so that particles cannot acquire kinetic energy before colliding. Figure 9 shows a ceramic break compatible with the  $A_1$  and  $A_2$  feeders using this principle, and where the brazing is down in a groove at the end of the ceramic sleeve. High power RF electron tubes are usually built this way.

Several liquids were also tested to wet these edges, or even completely submerge the ceramic electrical break inside a leak tight teflon sleeve. The liquid had to be of very high dielectric strength, low Z, volatile solvent (in case of contamination of the tokamak), lossless (which, for all practical

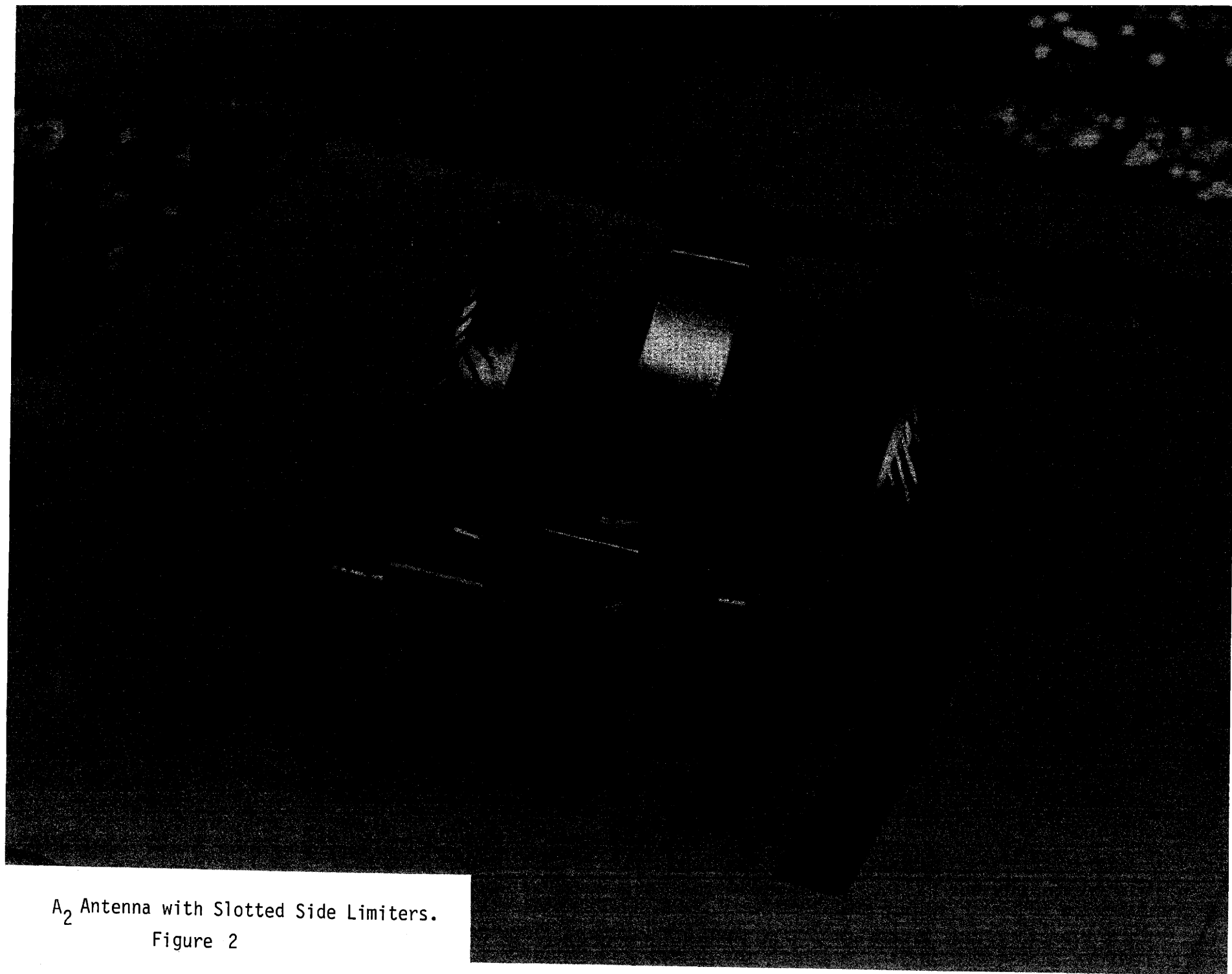
purposes, meant nonpolar), and nonflammable. Carbon tetrachloride met all these specifications with no major drawbacks, except that it is a known carcinogen, and great care must be taken to avoid personnel exposure.

Figure 10 is a simplified sketch of a flexible version of  $A_2$  that could be used for low power tests, and inserted in any of the much smaller HCN or Thompson port side keyholes.

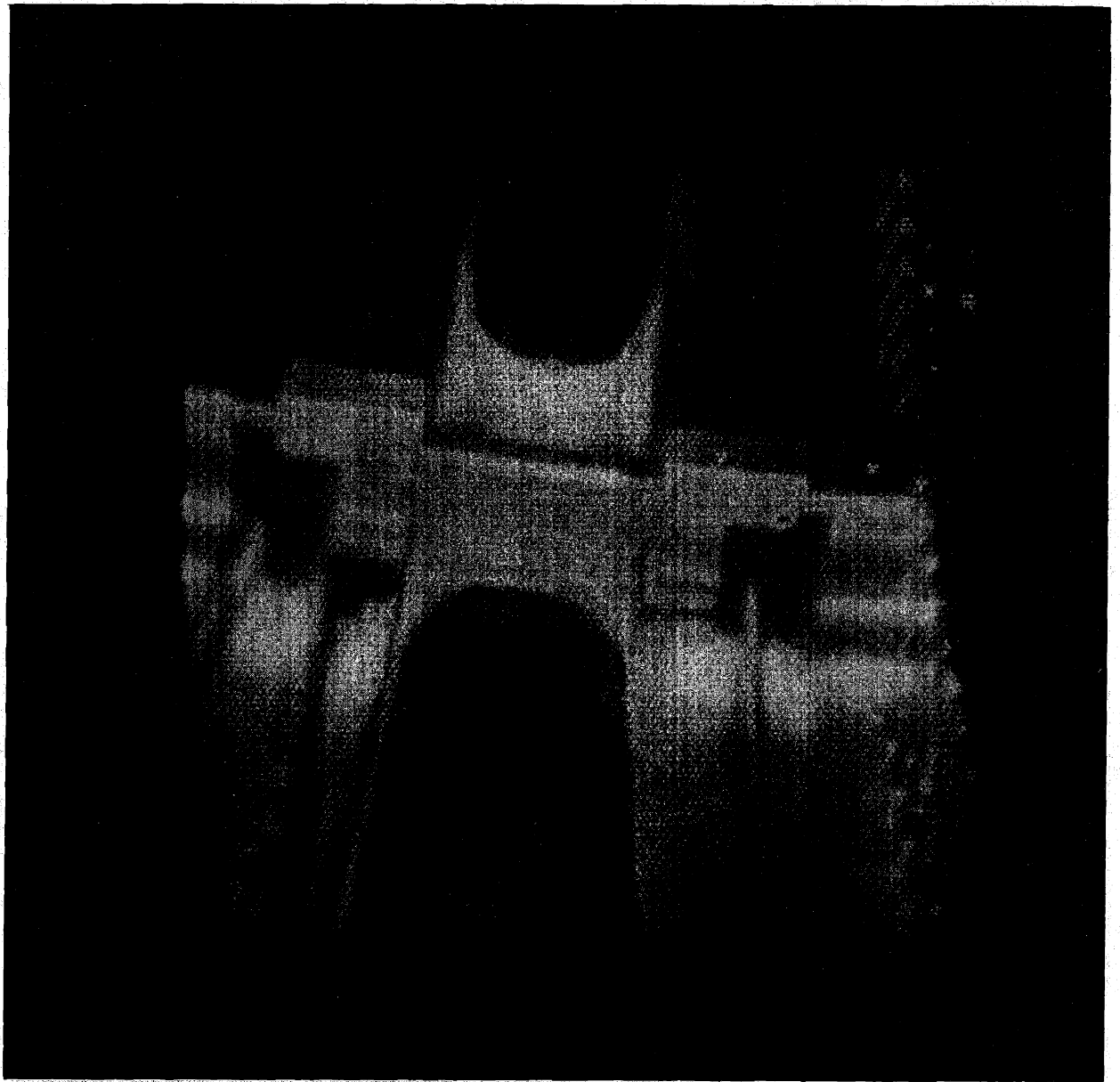


A<sub>2</sub> antenna with slotted side limiters

Figure 1



$A_2$  Antenna with Slotted Side Limiters.  
Figure 2



Boroscope View of A<sub>2</sub> Bridge Assembly Inside Alcator.

Figure 3

# Antenna test stand model

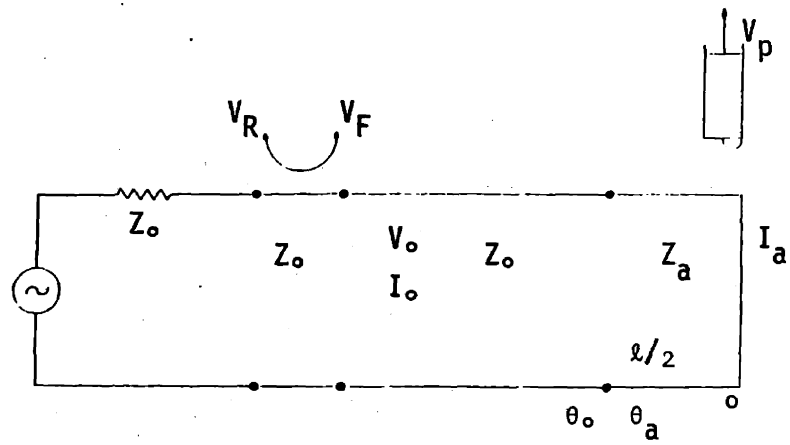


Figure 4

# Probe test stand assembly

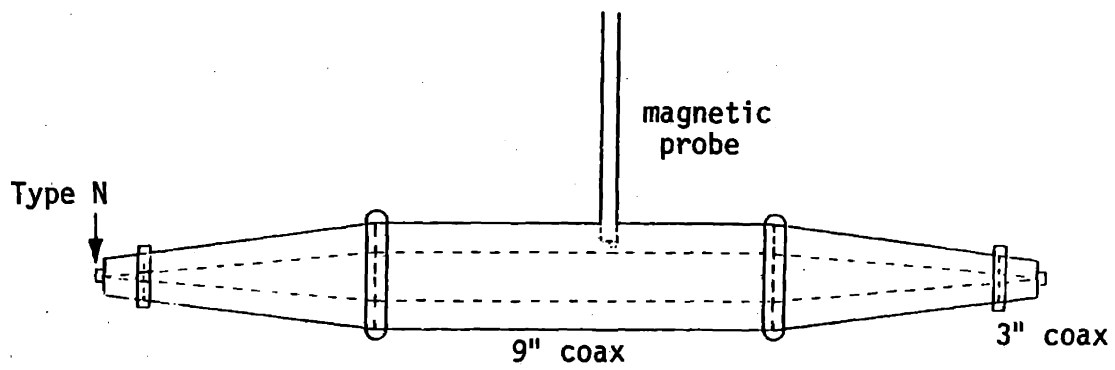
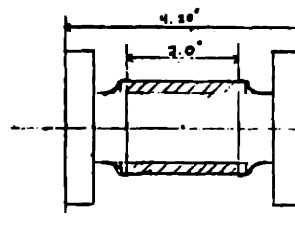
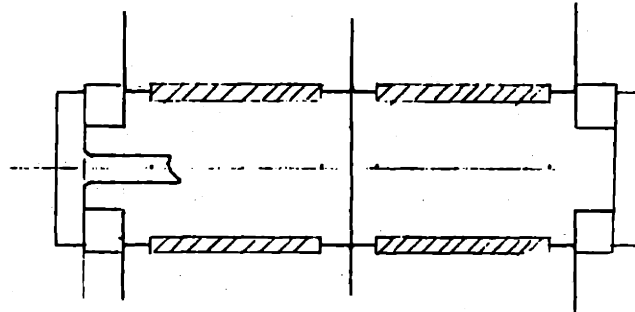


Figure 5



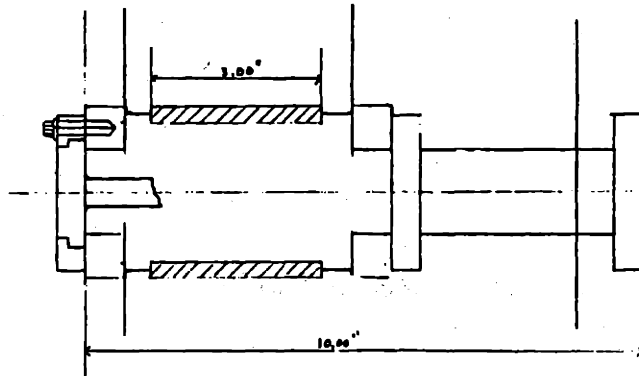
A<sub>1</sub> Vacuum Break

Figure 6



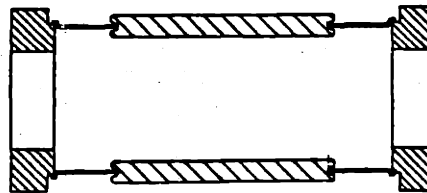
A<sub>2</sub> Vacuum Break

Figure 7



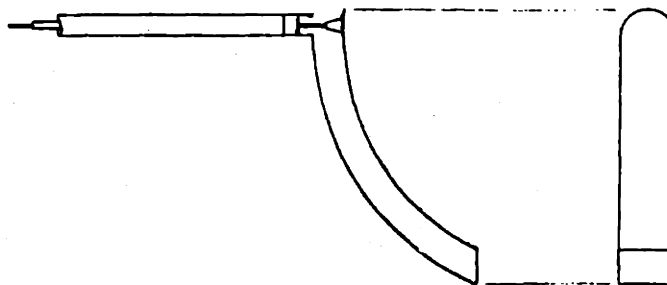
Short Vacuum Break

Figure 8



Improved Vacuum Break

Figure 9



Flexible A<sub>2</sub> type antenna

Figure 10



### III-2.5. A<sub>4</sub> antenna system and Faraday shields

The third antenna tested, A<sub>4</sub>, is shown in Figures 1, 2 and III-2.1.(1). The backplane and side Faraday shield are from the old A<sub>1</sub> antenna. The center conductor was moved out like A<sub>2</sub>, and a thick, dome-type Faraday shield front was added instead of the thin A<sub>1</sub> clips. Slightly higher (than A<sub>2</sub>) slotted limiters were also added.

A radially scannable Langmuir probe, peeking through the front of the Faraday shield, was installed to measure the tenuous plasma near the antenna (Figure 1). The probe connections were made through an additional port on the already overcrowded 10 inch side port.

The purpose of the Faraday shield and side limiters is twofold. First, it keeps the tenuous plasma and, in particular, the particles streaming along the field lines out of the high electric field region of the antenna (kilovolts/cm), and second, it shorts out the unwanted E<sub>z</sub> component of the antenna without affecting the E<sub>θ</sub> components.

Two general rules of thumb can be formulated for the design of a Faraday shield that will not significantly shield out the desired B<sub>z</sub>. First, the shield clip width should be much less than the center conductor to backplane distance. Careful examination of the clip geometry and the backplane currents shows that part of the backplane current will weave up the inside of the clip, about a distance equivalent to the clip width. This image current path will be unimportant only if it is low compared to the center conductor, and thus our first rule. In A<sub>4</sub>, the clip width was 2.5 times less than d.

Second, the clip geometry must be designed so the magnetic energy stored in the gap between clips is less than the energy stored inside and outside of the shield. To see this we will, for simplicity, examine the magnetic energy of only the center conductor of a coaxial antenna. The stored energy is

$$(1) \quad W = A \int_{r_1}^{r_2} (1/r)^2 dr = \left[ \frac{1}{r_1} - \frac{1}{r_2} \right] A$$

where for typical Alcator values

inside shield	$W_{in}$	$= 2 A$	$.25" < r < .5"$
at shield	$W_{at}$	$= .33 A$	$.5" < r < .6"$
outside shield	$W_{out}$	$= 1.66 A$	$.6" < r < \infty$

For a good shield, the flux through the shield region is about the same as without the shield. With the same flux and a gap to width ratio,  $G$ , the energy density in the gap is increased by  $G^2$ , but the volume is decreased by  $G$ , so a net factor  $G$  remains. For  $A_4$ ,  $G = .25" / .032" \approx 8$ , and the energy in the shield,  $W_{at}$ , becomes  $8 \times .33 = 2.64 \approx W_{in} \approx W_{out}$ , and a small reduction in flux can be expected as compared to  $A_2$ .

These rules came from a number of experiments on the simple, but very realistic, set up of Figure 3. Several stripline conductors of width  $w$ , length  $\ell$  and height  $d$  above a large ground plane, were energized as in Figure III-2.4.(4). Several types of slotted and unslotted limiters and Faraday shields were also tested. Flux measurements at height  $h$  were

made with a shielded magnetic probe (Figure III-3.2.(8)).

Figure 4 shows how the magnetic field is independent of the center conductor width  $w$ , and decreases as  $1/r^2$  as expected from the quasi-static near-field with  $d \ll r \ll \lambda$

$$(2) \quad B \propto \frac{1}{r_1} - \frac{1}{r_2} = \frac{1}{r-d} - \frac{1}{r+d} \approx \frac{2d}{r^2}$$

Figure 5 shows the expected dramatic effect of Faraday shield clip width. Figures 6, 7, 8 and 9 are similar tests with  $A_1$  and  $A_2$  inside the cylindrical simulator. Figure 6 shows the weak effect of the slotted limiters spacing on  $A_2$ . Figures 8 and 9 show the flux ratio of  $A_1$  to  $A_2$  for carefully controlled conditions. Although the radiation resistance increases as flux squared ( $4^2 = 16$ ), the plasma radius was decreased by a centimeter (from  $A_1$  to  $A_2$ ), and thus for large  $k_{||} \approx .6/\text{cm}$ , the evanescent edge layer is thicker and the resultant factor is only

$$(3) \quad \frac{4^2}{e^{.6}} \approx 8.8$$

$A_4$  had an effective flux about 30% less than  $A_2$ , as measured in the cylindrical simulator.

A side effect of a good Faraday shield is that the antenna becomes a slow wave structure. From transmission line theory,<sup>134</sup>

$$(4) \quad V = \frac{1}{\sqrt{LC}}$$

$$(5) \quad Z = \sqrt{\frac{L}{C}}$$

For a vacuum dielectric,  $A_4$  geometry without the Faraday shield (Figure 1), and from equation III-2.4.(1) with  $w/d = .83$

$$(6) \quad V = c = 3 \times 10^8 \text{ m/s}$$

$$(7) \quad Z = 134 \Omega$$

Combining (4) - (7) we have

$$(8) \quad C = \frac{1}{cZ} = 2.5 \times 10^{-11} \text{ F/m}$$

$$(9) \quad L = \frac{Z}{c} = 4.5 \times 10^{-7} \text{ H/m}$$

Adding the Faraday shield does not significantly change  $L$ , but the shield capacitance from Figure 1 is large

$$(10) \quad C_s \approx 4.3 \times 10^{-11} \text{ F/m}$$

The new phase velocity and impedance are now

$$(11) \quad V = \frac{1}{\sqrt{L(C + C_s)}} \approx .6 c$$

$$(12) \quad Z = \sqrt{\frac{L}{C + C_s}} \approx 81 \Omega$$

The impedance and phase velocity could be even further reduced by mounting large slotted knobs on the back plane or center conductor (the Faraday shield design rules apply here also) so as to increase the capacitance per unit length, but not the inductance and antenna loop flux.

Following the same procedure as in Section III-2.4.

$$(13) \quad \theta_a \approx 37^\circ$$

$$(14) \quad \theta_o \approx 51^\circ$$

$$(15) \quad I_{A4} = .79 I_o = .92 I_{A2} \approx I_o$$

and our current probe calibrations are still within reasonable error.

Figure 7 is the calibration factor for the top and bottom  $A_4$  current probes as a function of frequency. As expected, the calibration factor is nearly inversely proportional to frequency.

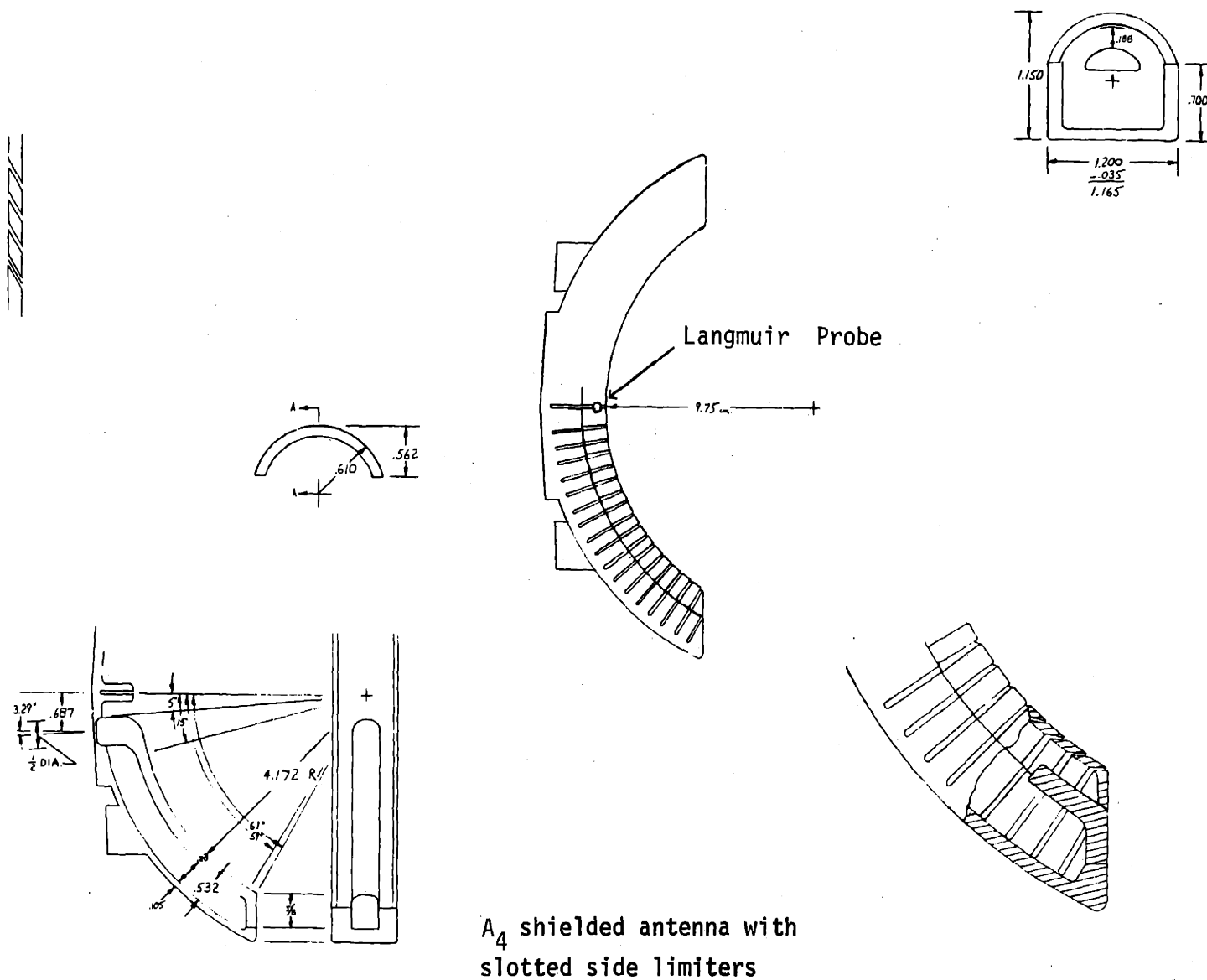
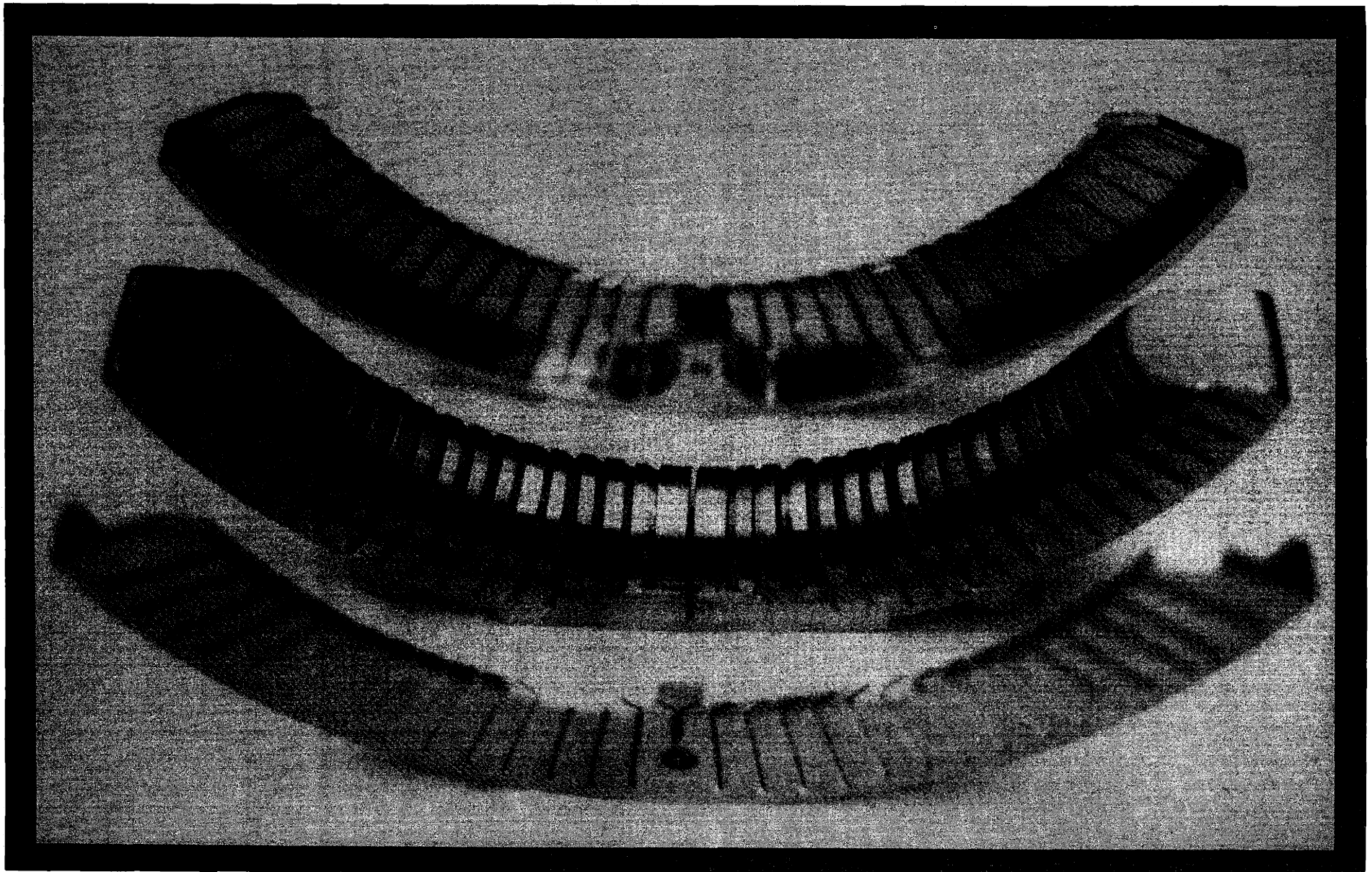


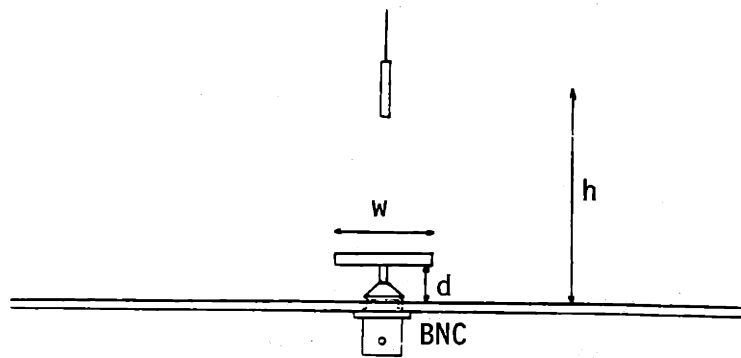
Figure 1



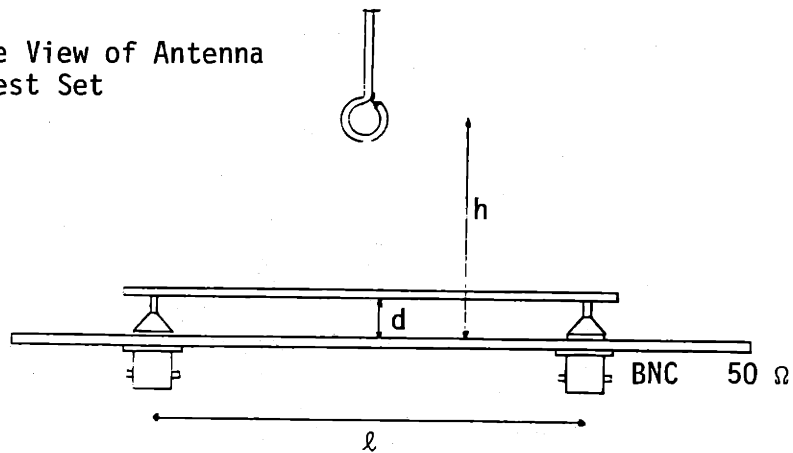
$A_4$  Antenna with Slotted Side Limiters

Figure 2

End View of Antenna Test Set



Side View of Antenna Test Set



End View of Shields and Center Conductors

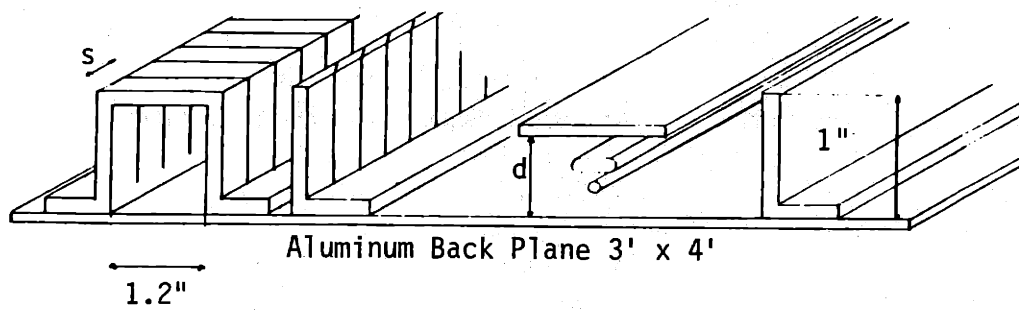


Figure 3



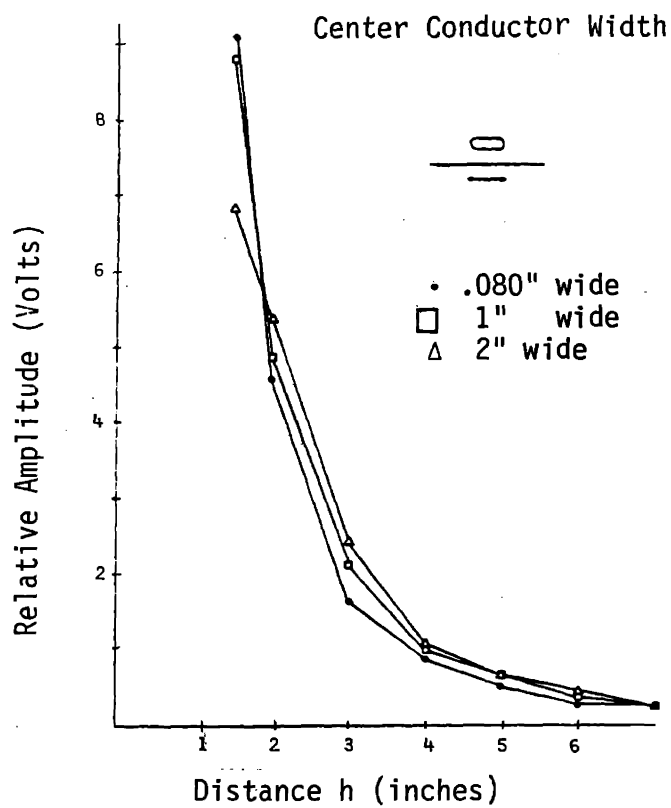


Figure 4

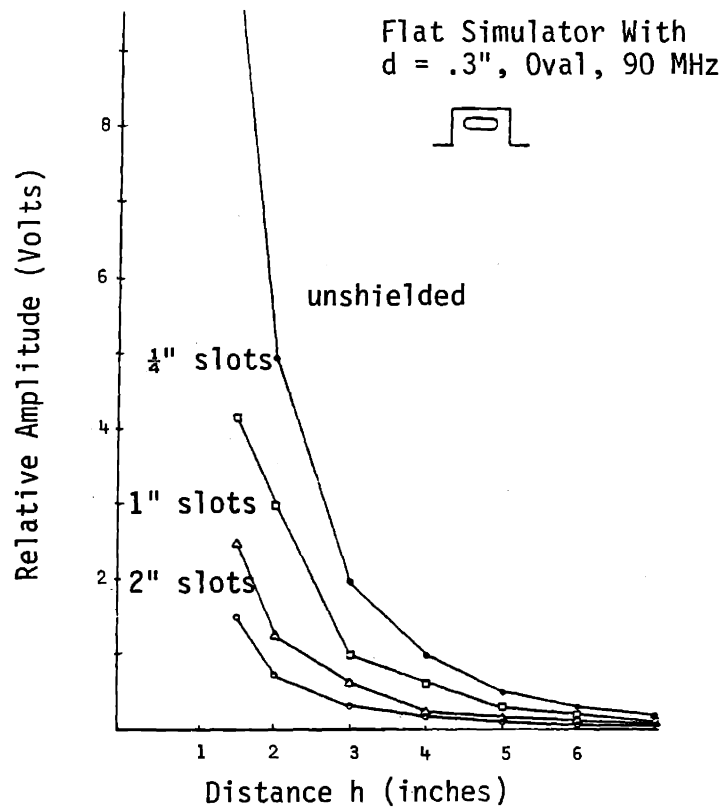


Figure 5

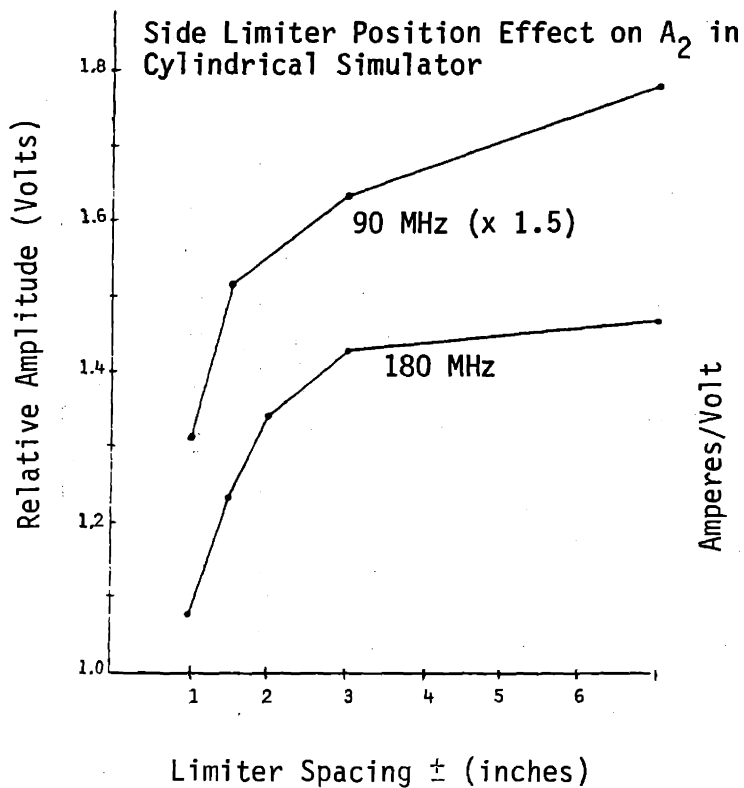


Figure 6

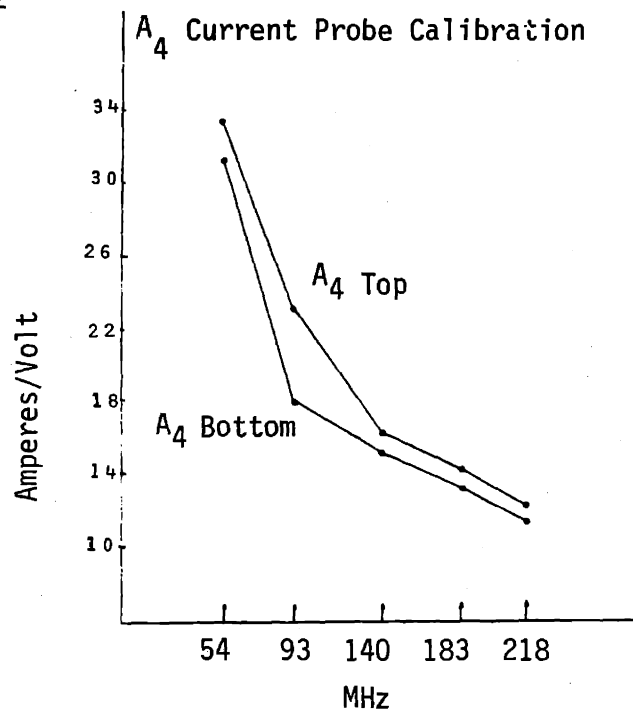


Figure 7

# ANTENNA VACUUM FIELDS VS RADIUS

90MHZ

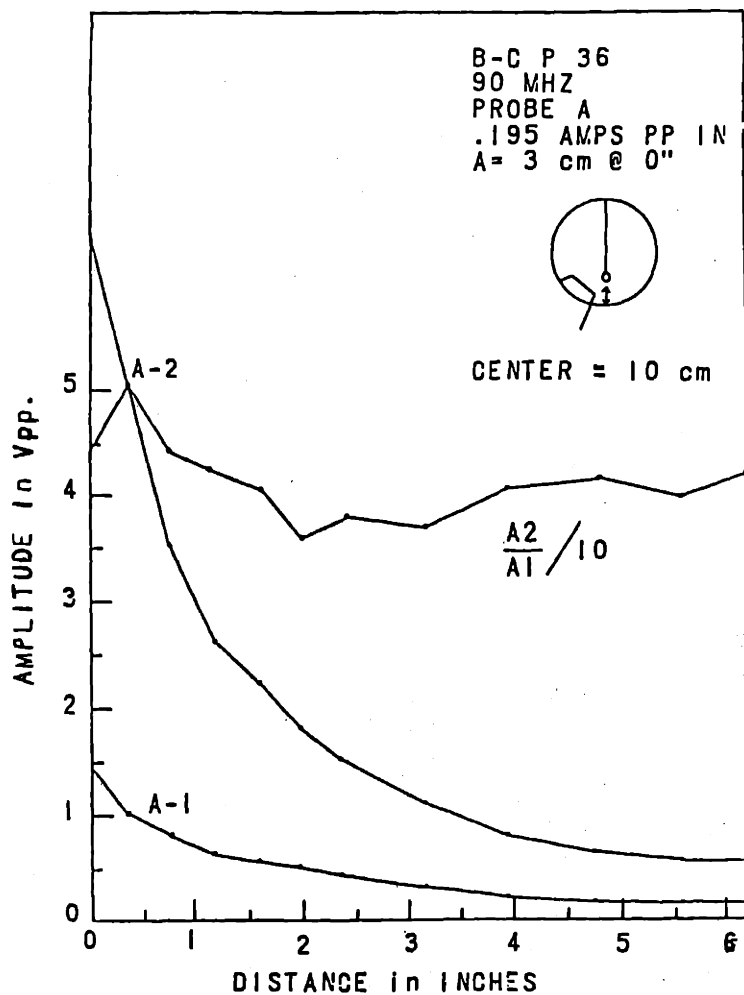


Figure 8

180MHZ

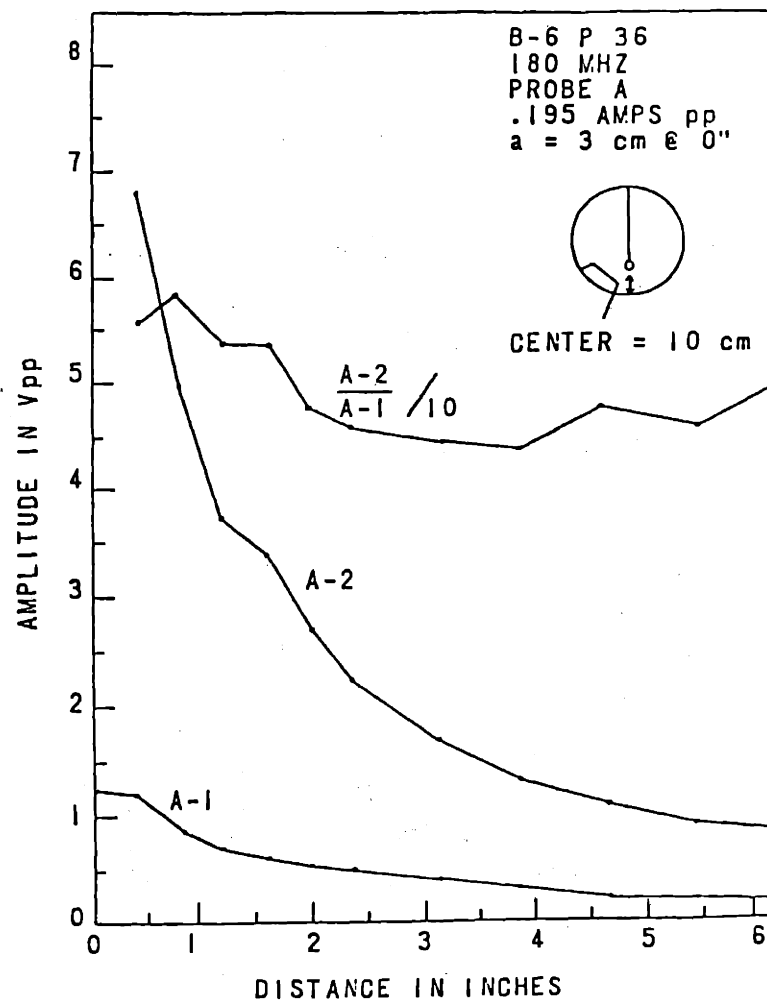


Figure 9

### III-2.6. A<sub>3</sub>, A<sub>5</sub>, A<sub>6</sub> antenna systems

To further improve the power coupling capability of the A<sub>2</sub> unshielded type antenna, a longer and much higher current antenna system, A<sub>3</sub>, was designed (Figure 1). The main new feature is that the matching is done in a vacuum, so the ceramic feedthroughs are at a pure real power point. Thus for a conservative 200 amperes, 800 kW can be coupled.

Figure 1 is a preliminary mechanical design of A<sub>3</sub>. The antenna itself could be almost twice as long as A<sub>2</sub>, and very easy to insert. Both antenna center conductors are hinged to the large, four inch diameter, hollow feeder conductor. Image currents run directly on the vacuum vessel, and the end of the conductors are held in place by threaded rods through the vertical keyholes. The large, stiff, hollow feeder is very rigidly cantilevered from the outer conductor behind the sliding tuning plate. The tuning plate is positioned by motor driven hollow control rods (at atmospheric pressure) with current loops at the plate surface. The entire matching and antenna system is the same size as the A<sub>2</sub> feeder, ( $\lambda/2$  at 90 MHz,  $\lambda$  at 180 MHz). All parts are stainless steel, bakable and silver plated, as is A<sub>1</sub>. Electrical breakdown can be monitored through windows mounted on the tuning and matching control flanges.

Tuning is done through a variable vacuum capacitor in a six inch "T" near the voltage maximum. From conservation of power when the system is in tune, and  $R_R \ll Z_o$ , we can approximately write

$$(1) \quad P_{in} = I_{in} V_{in} = I_o^2 R$$

which can be rewritten as

$$(2) \quad \frac{V_{in}^2}{Z_o} \approx \frac{V_o}{X_c} \cdot V_{in} \approx \frac{V_o^2}{Z_o^2} R$$

and solving for capacitor reactance

$$(3) \quad X_C = Z_0 \sqrt{\frac{Z_0}{R}} \approx 160 \Omega \quad \text{for } Z_0 = 50 \Omega \text{ and } R = 5 \Omega$$

we find  $C \approx 5.0 \times 10^{-12}$  F at 200 MHz. This corresponds to a rather large capacitor disk four inches in diameter with a half inch spacing.

Two other major advantages of the  $A_3$  system are that tuning and matching controls are nearly orthogonal, and the smaller and simpler matching system cannot contain as many parasitic modes.  $A_3$  is thus much easier to tune and model accurately. A major possible disadvantage is that the high current carrying part of the antenna intersects the cyclotron resonance layer.

Figure 4 is a very preliminary conceptual design of a fully shielded 360° antenna similar to  $A_4$ . This antenna would be wider (1.6" instead of 1.2"), and of lower impedance than  $A_4$ .  $A_5$  could be inserted in either (or both) the HCN or Thomson horizontal ports. The keyhole bridges in these side ports would be milled in place without lubricants and without dismantling the tokamak, a tricky but nevertheless feasible task.

Finally, a no compromise realistic antenna was designed with all the technology developed through the  $A_1$ ,  $A_2$ ,  $A_3$ ,  $A_4$  and  $A_5$  designs. Figure 2 is a simplified schematic of the  $A_6$  RF components. The  $A_6$  antenna is a slow wave, low impedance, 360° shielded antenna similar to  $A_4$  and  $A_5$  (Figure 4). Each half of the antenna is independently matched to 50  $\Omega$  by two short  $A_3$  type vacuum resonators (Figure 3). The antenna halves can be easily and arbitrarily phased by a variable delay line (Figure 6). The 9" diameter coaxial power splitter (Figure 5) is of the long, linearly tapered type, which is relatively broadband (90 - 200 MHz) and free of parasitics. The 9 inch DC block is three stage, coaxial and similar to the two stage  $A_2$  version.

Matching is mechanically most easily achieved by changing the frequency, since the  $A_3$  tuning capacitor would be too large for the more compact  $A_6$ . The tap point (near the operating frequency) can be easily calculated and experimentally verified under bench test at atmospheric pressure. For gross tap changes, the standard 4.5" "T's" can be interchanged. The vacuum feedthroughs can be custom built, or the already available (FPS-17) high power 9 inch coaxial TR (Transmit-Receive) switch tube vacuum barriers could be used. The power splitter and delay line could also be built from available 9 inch "T's" (FPS-17 up-down switches) and already motorized  $A^2$  feedback phase shifters.

The antenna itself could be secured in place by any combination of different ways, in particular, by horizontal back plane pins as  $A_1$ ,  $A_2$ ,  $A_4$ , vertical rods as  $A_3$ , or even screws into the Bitter flange as  $A_5$ .  $A_6$  could also have the full line of RF, plasma and breakdown diagnostics. 360° slotted side limiters similar to  $A_2$  and  $A_4$  would also be installed.

For a sufficiently slow wave structure ( $V \approx V_{A4} \approx .6 c$  for Alcator C and 200 MHz), a voltage null in the antenna can be located slightly on the low field side of the major radius. This produces a nearly ideal field pattern for the single perpendicular pass regime ( $R_{TIIH} < R_{\omega CH}$ ) and a small near-field edge heating at the resonant layer.

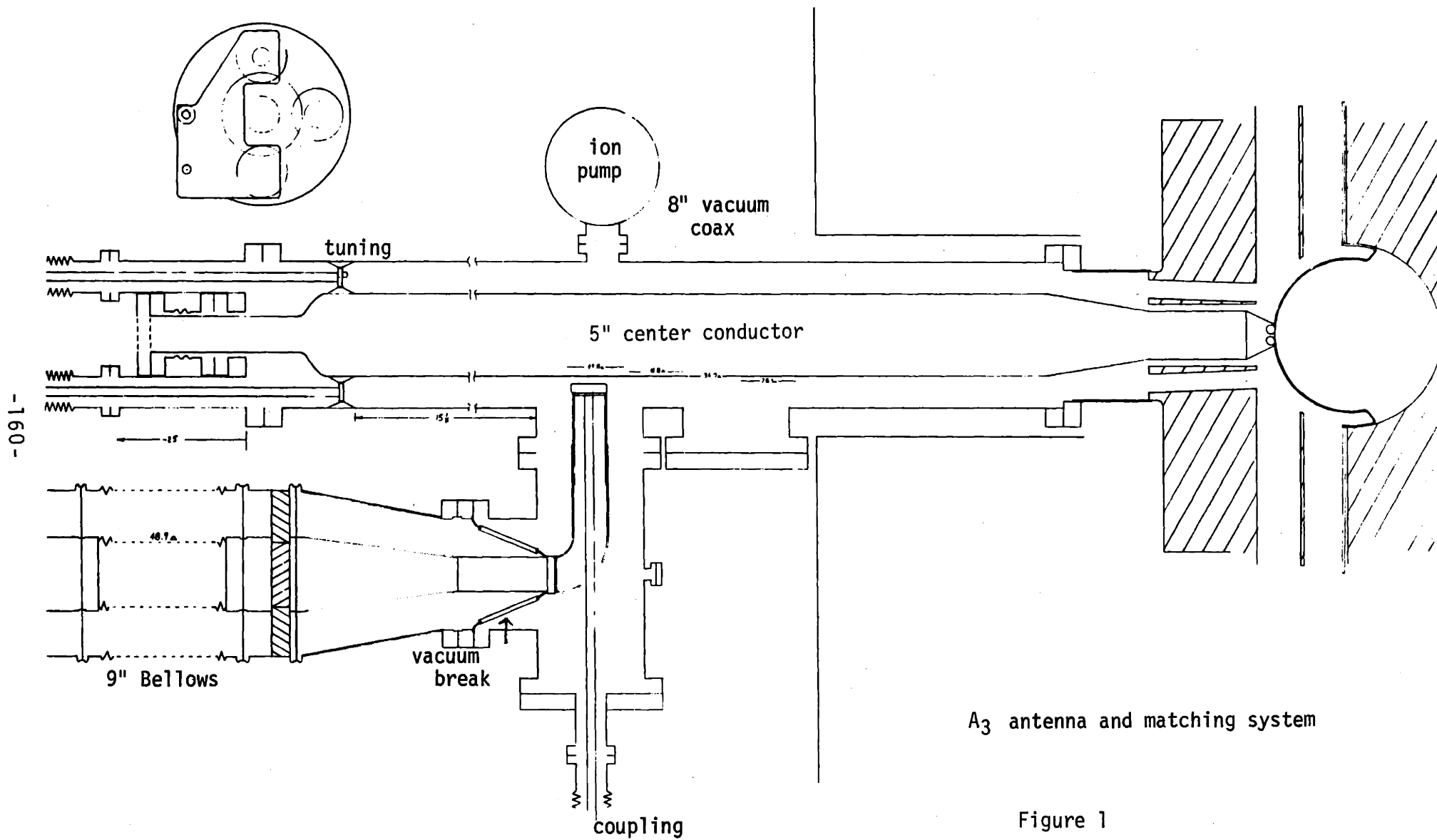


Figure 1

### Simplified Schematic of A<sub>6</sub> System

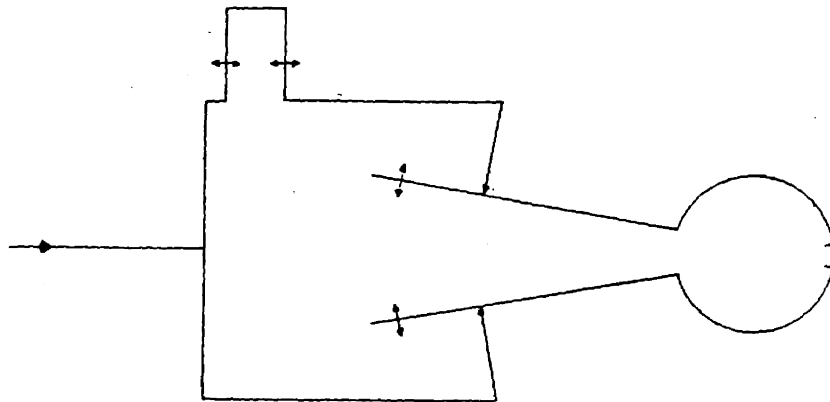


Figure 2

### Simplified Mechanical Layout of A<sub>6</sub> System

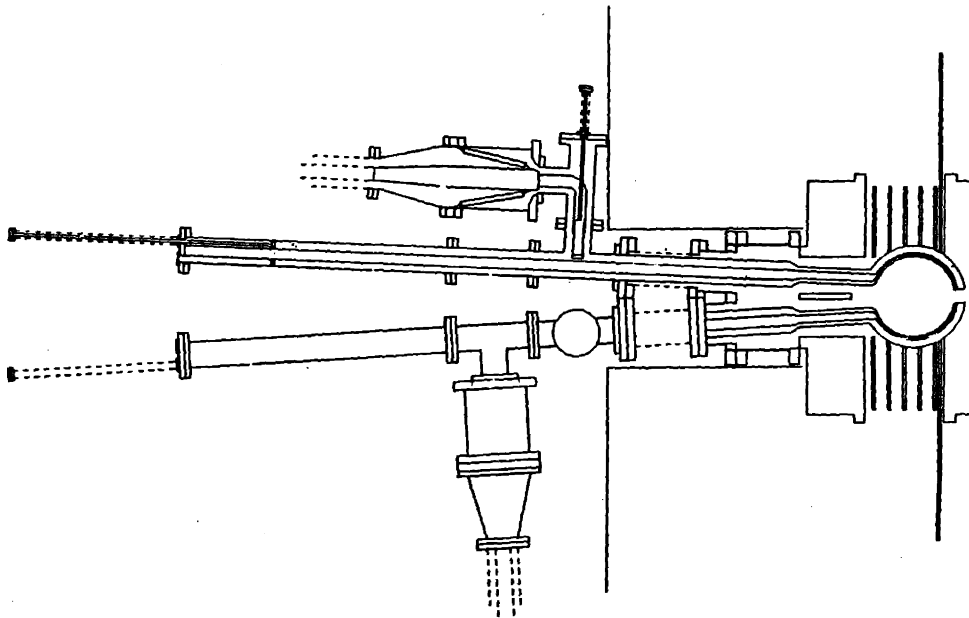
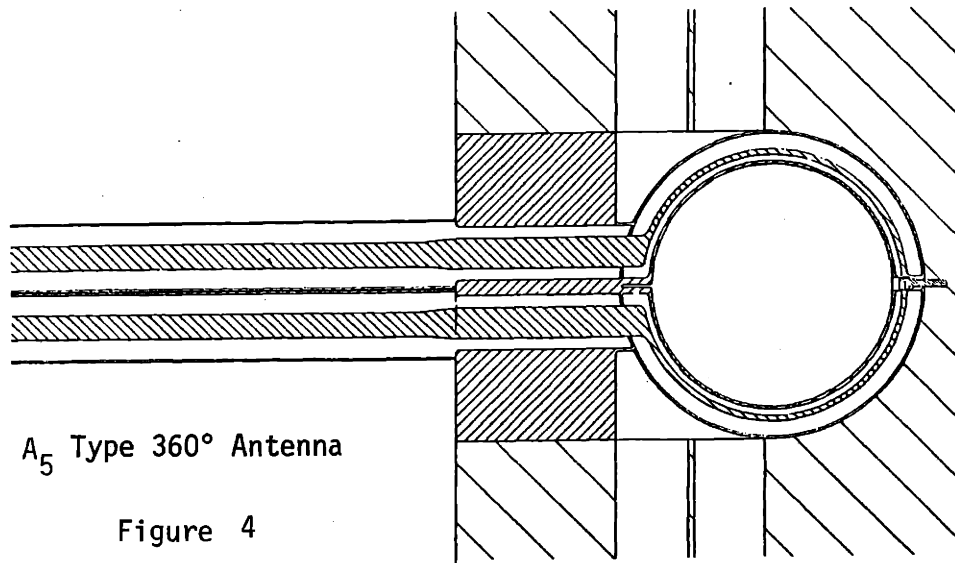
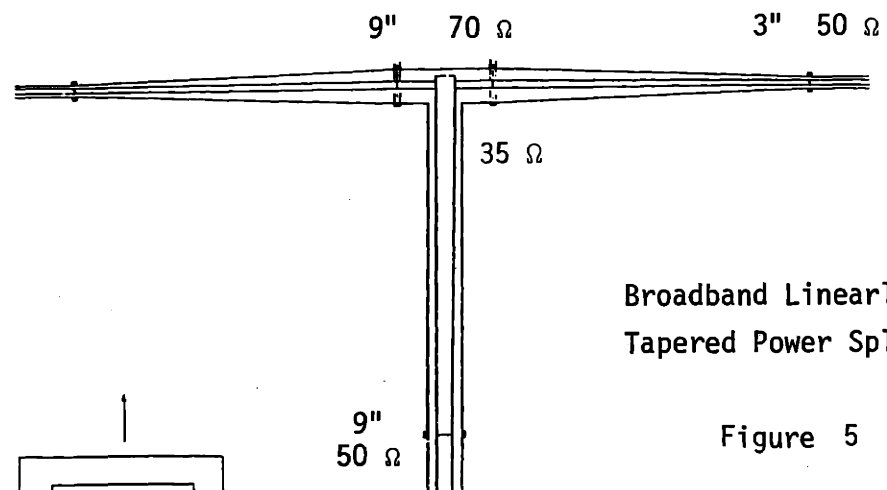


Figure 3



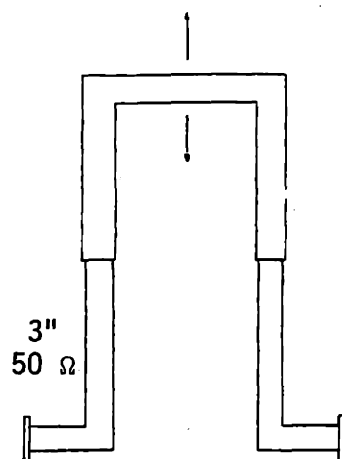
A<sub>5</sub> Type 360° Antenna

Figure 4



Broadband Linearly  
Tapered Power Splitter

Figure 5



A<sup>2</sup> 3" Coax Phase Shifter

Figure 6



### III-3. Extensive Plasma and RF Diagnostics

One major objective of the experimental apparatus was to have as extensive RF diagnostics as possible. For this purpose, almost all easily implemented RF diagnostic schemes were used.

Forward and reflected power were measured at different points in the system with 9" coax directional couplers. Top and bottom resonator balance and antenna currents were measured with magnetic probes. A high speed dedicated analog computer calculated  $R_R$  in real time. Many unshielded RF probes were located around the torus, in particular, in the near-field of the antenna, in the opposite limiter port, and poloidally a quarter of the way around the torus. Ceramic thimbles were also built to house shielded probes and also a three component,  $B_\theta$ ,  $B_r$ , and  $B_z$  RF probe. Two  $k_{||}$  arrays of probes were used to measure the parallel wave length and field profile at the port.

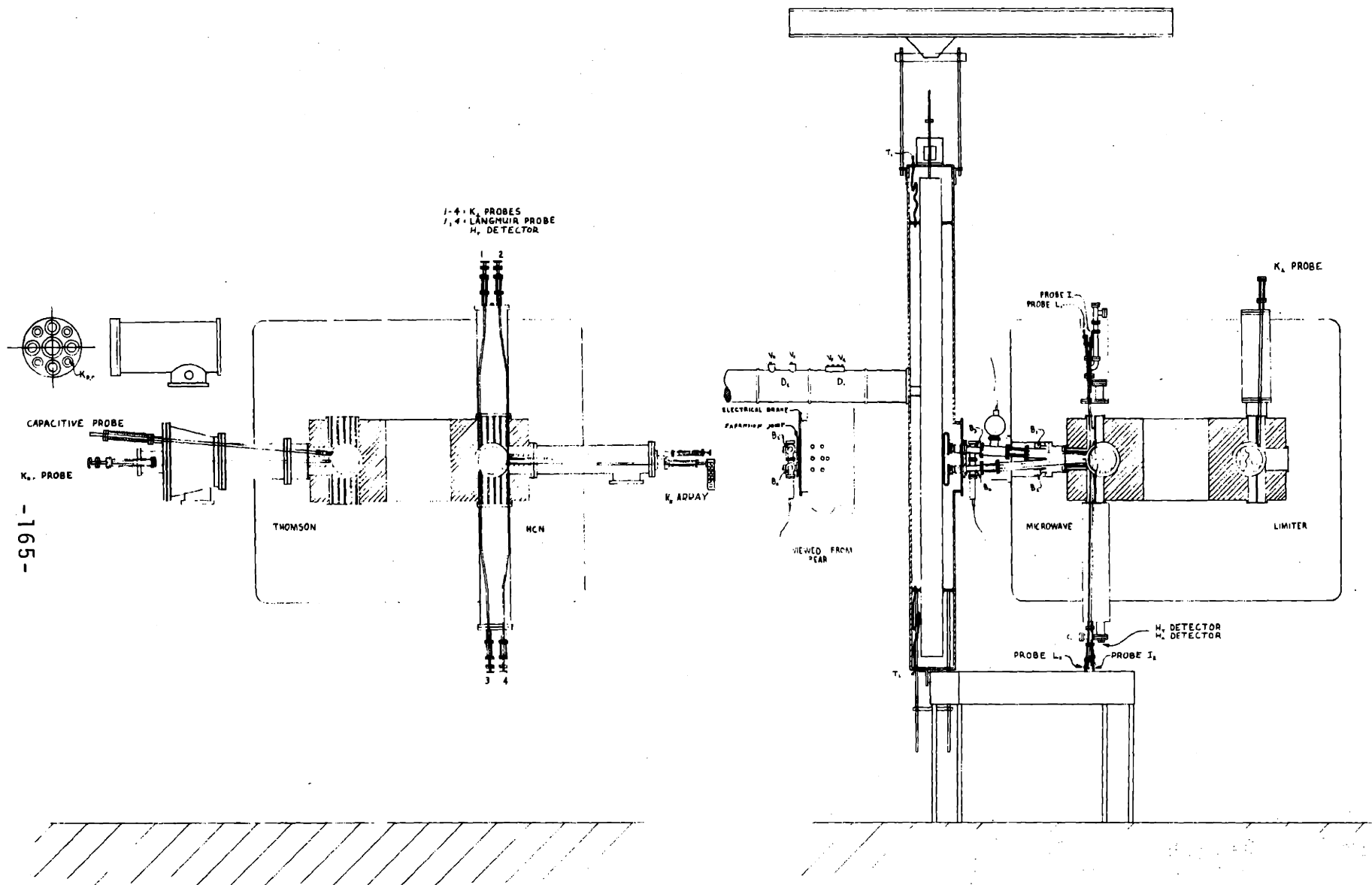
Several increasingly sophisticated phase detection schemes were used to measure parallel and perpendicular mode numbers. These schemes include a high speed compressor and mixer, single sideband generated fringe correlators, and a sine cosine phase detector.

A number of plasma and RF breakdown diagnostics were also designed and used, among which were several high sensitivity light detectors monitoring the RF antenna feeder coaxes, the electrical breaks, and the inside and outside of the antenna (Figure 1). An  $H_\alpha$  light detector, Langmuir probes, high speed bolometer and trapped particle detector were designed for measurements even in very high RF fields.

Finally, all the tokamak support diagnostics were used and, in particular,

the alcohol laser interferometer<sup>73\*</sup>, mass selective charge exchange energetic neutral spectrometer<sup>33-38\*</sup>, soft X-ray detectors<sup>39\*</sup>, neutrons<sup>39\*</sup>, as well as all the usual magnetic diagnostics<sup>2\*</sup>.

\* Stephen Wolfe and Jeffrey Parker were responsible for density measurements, Martin Greenwald and Catherine Fiore for charge exchange, Robert Granetz for soft X-ray, David Gwinn, Daniel Pappas and William Fisher for neutrons, and David Overskei and Bruce Lipschultz for magnetic diagnostics.



Alcator A ICRF Diagnostics

Figure 1

### III-3.1. RF power, radiation resistance and data acquisition

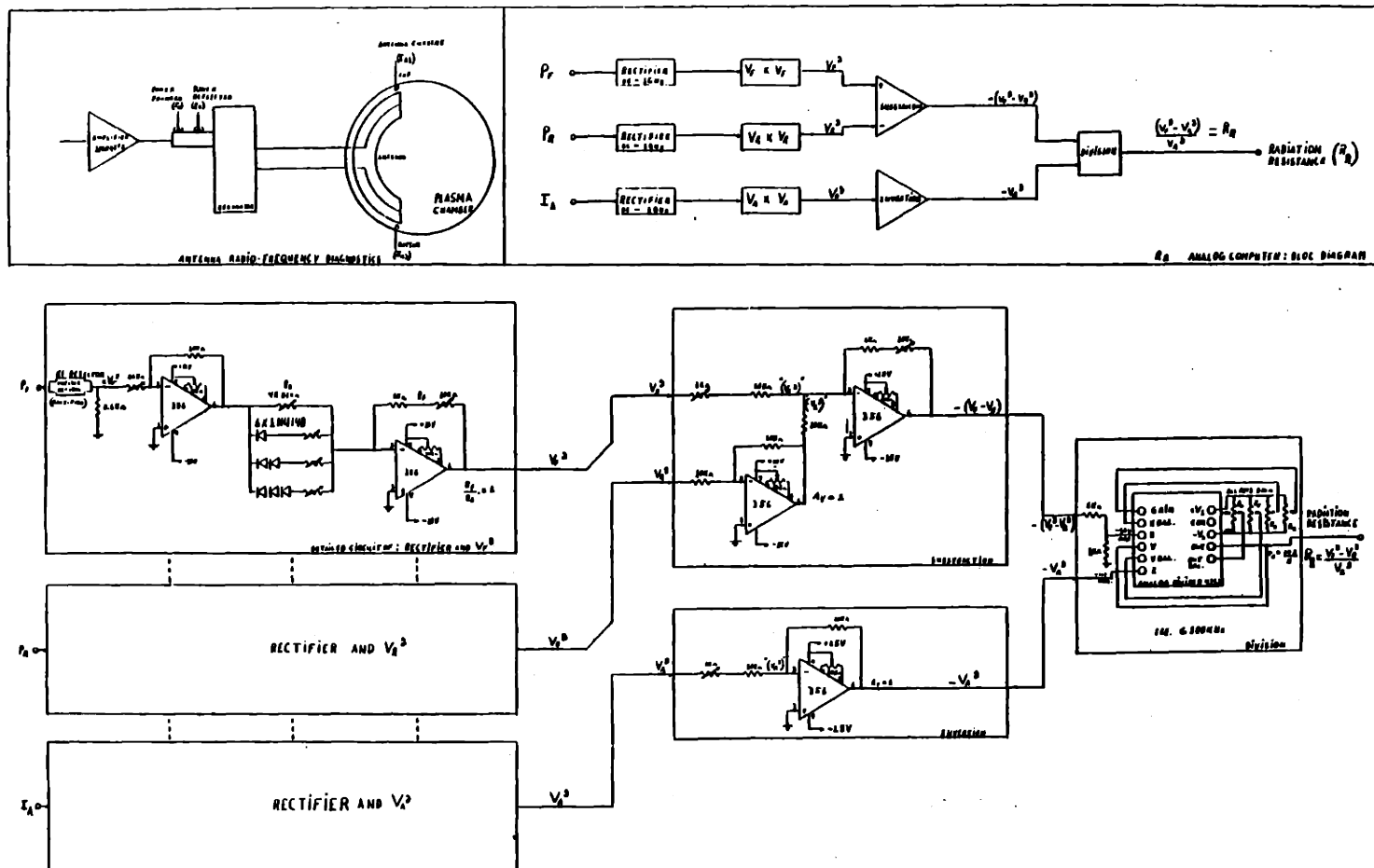
Forward and reflected power were measured right at the matching resonator by 60.2 db (at 200 MHz), 9" directional couplers (Figure III-3.(1)). Top and bottom antenna current were measured by calibrated current loops inserted through the top and bottom MW port.

Double shielded RG 55 coax ( $\approx 7$  db/100 ft at 200 MHz) was used for all RF diagnostics. All lines were properly terminated to  $50\ \Omega$  and shunted with high pass .2 microhenry inductors ( $X_L \approx 250\ \Omega \gg 50\ \Omega$  at 200 MHz). Custom high voltage ( $>1$  kV) DC blocks were installed at all RF sources.

Absolute RF measurements were made with 7A24, 400 MHz,  $50\ \Omega$  terminated Tektronix plug-ins in two (7834), 400 MHz storage mainframes. Relative measurements at the base frequency and absolute measurements at lower frequencies were made with 7A18 plug-ins in five 7623A, 100 MHz mainframes (slew rate limited,  $\approx 3$  db down at 100 MHz, 14 db at 200 MHz). RF voltages were usually measured at the base frequency as peak to peak measurements off oscillograms, and rarely by ordinary square law diode detectors.

Figure 1 shows the 100 kHz bandwidth on-line radiation resistance analog computer. Forward and reflected power and either top or bottom antenna current signals were 3 db split, displayed at the base frequency and square law detected. The diode signals were corrected for true square law over a 20 db range by diode networks, and then subtracted and divided to calculate loading resistance.

Most of the plasma diagnostic signals were recorded either on storage oscilloscopes, or on a slower CAMAC digital memory storage system interfaced to a PDP-11/55 computer.



High Speed, On-line Radiation Resistance Analog Computer

Figure 1

### III-3.2. RF wave probes and arrays

Many small unshielded RF wave probes were installed around the torus, as can be seen from the cross section view of Figure III-3.(1). Most of the probes were unshielded and similar to the ones shown in Figure 2 and the bottom of Figure 1. These probes were mounted on 3/8" OD pipe, with atmospheric pressure on the inside. Bakable .125" OD hardline coax and high temperature brazed alumina feedthroughs were used for high vacuum integrity. The feedthrough welds were electron beam welded and protected by side limiters. The Langmuir probe,  $A_2$  flux model probe and  $k_{||}$  array of Figures 5, 4, and 1 were of similar construction. The array probe spacing was about 2 and 1 cm. A simple  $k_{||}$  array of 3 probes similar to Figure 1 was also used.

Figure 6 and top left of Figure 1 show a ceramic thimble that was used to house more complicated wave probes near the plasma, in an atmospheric environment. Among these probes were unshielded (Figure 7), shielded (Figure 8), differentially shielded (Figure 9), and three axis ( $B_\theta$ ,  $B_r$ ,  $B_z$ ), magnetic loop probes.

The inductance of the small probe loops can be approximated as<sup>136</sup>

$$(1) \quad L \approx \frac{R_p^2}{9 R_p + 20 r_p} \mu H \approx 4 \times 10^{-9} \text{ H}, \quad R_p \approx .065", \quad r_p \approx .025"$$

The effective area of the probe  $A_e$ , is larger than the estimated  $A_m$  (due to the reentrant geometry of the probe tip), and can be accurately calibrated in the nine inch diameter coaxial field simulator of Figure III-2.4.(5). The fields could be pure electric, magnetic, or both, depending on termination and location in the simulator.

$$(2) \quad I = \frac{V_o}{Z_o} \text{ for } z = z_o$$

$$(3) \quad H = \frac{I}{2 \pi r_c}$$

$$(4) \quad V_p = A_e \omega \mu_o H$$

$$(5) \quad A_e = \frac{V_p r_c z_o}{V_o f_o \mu_o} \approx .19 \text{ cm}^2 > A_m = \pi r^2 = .086 \text{ cm}^2$$

Capacitive coupling effects by the unshielded probes were similarly investigated. For simplicity, we will assume the probe tip has an effective capacitance area  $A_c$  exposed to a wave electric field. The current and voltage induced in the probe can be estimated as

$$(6) \quad I_c = A_c \omega \epsilon E$$

$$(7) \quad V_c = L \dot{I}_c = L \omega^2 A_c \epsilon E$$

and can be neglected even in the high dielectric plasma edge since

$$(8) \quad E \approx \frac{\omega}{k} B$$

$$(9) \quad \frac{V_c}{V_p} = \frac{A_c}{A_p} L \frac{\omega^2 \epsilon}{k} \approx \frac{\omega^2 \epsilon}{k} L \approx .013 \quad \text{for } k_o, \epsilon_o, 200 \text{ MHz}$$

$$\approx .016 \quad \text{for } k \approx 1/\text{cm}, 30 \epsilon_o$$

Each half of the unshielded antenna can be modeled as shown in Figures 11 and 12. Power can be dissipated by four basic different mechanisms; First and second, the conventional inductively coupled  $R_R$  and circuit losses  $R_\ell$ , third and fourth, resistive  $R_p$ , and capacitively coupled  $R_c$  losses, primarily at the high voltage ( $V_a$ ) feeder area ( $A$ ) of the antenna. The many possible pitfalls are best illustrated by a plausible, realistic numerical example.

$$(10) \quad P_{\ell/2} = I_o^2 (R_R/2) \approx 20 \text{ kW} \quad \text{for } I_o = 100 \text{ amps and } R_R = 4 \Omega$$

$$(11) \quad E_{\theta w} = \frac{I_o R_R}{\ell} \approx 20 \text{ V/cm} \quad \text{for } \ell = 20 \text{ cm}$$

$$(12) \quad \theta_{\ell/2} = 2\pi \frac{\ell/2}{\lambda_o} \approx 24^\circ \quad \text{for } f_o = 200 \text{ MHz}$$

$$(13) \quad V_a = I_o Z_o \sin \theta_{\ell/2} \approx 3.7 \text{ kV for } Z_o = 90 \Omega$$

$$(14) \quad E_z \approx \frac{V_a}{d} \approx 1.8 \text{ kV/cm} \gg E_{\theta w} \quad \text{for } d = 2 \text{ cm}$$

$$(15) \quad C_p \approx \frac{\epsilon_o A}{r_a - r_p} \approx 10^{-12} \text{ F} \quad \text{for } A \approx 10 \text{ cm}^2 \text{ and } r_a - r_p \approx 1 \text{ cm}$$

$$(16) \quad |X_C| = \frac{1}{\omega C} \approx 800 \Omega$$

$$(17) \quad |X_L| \approx \frac{V_a}{I_o} \approx 37 \Omega$$

$$(18) \quad P_C \approx \frac{(V_a/2)^2}{R_C} \approx 4.3 \text{ kW} \quad \text{for } R_C \approx |X_C|$$

$$(19) \quad P_p \approx \frac{V_a^2}{R_p} \approx 4.6 \text{ kW} \quad \text{for } R_p \approx 3000 \Omega$$

The root of the problem comes from the fact that the antenna electric near-field (eq. 14) is two orders of magnitude larger than the  $E_{\theta w}$  (eq. 11) wave field (five orders larger than  $E_{zw}$ ). This high voltage (eq. 13) combined with even large impedances, can give rise to substantial losses (eq. 18 and 19).

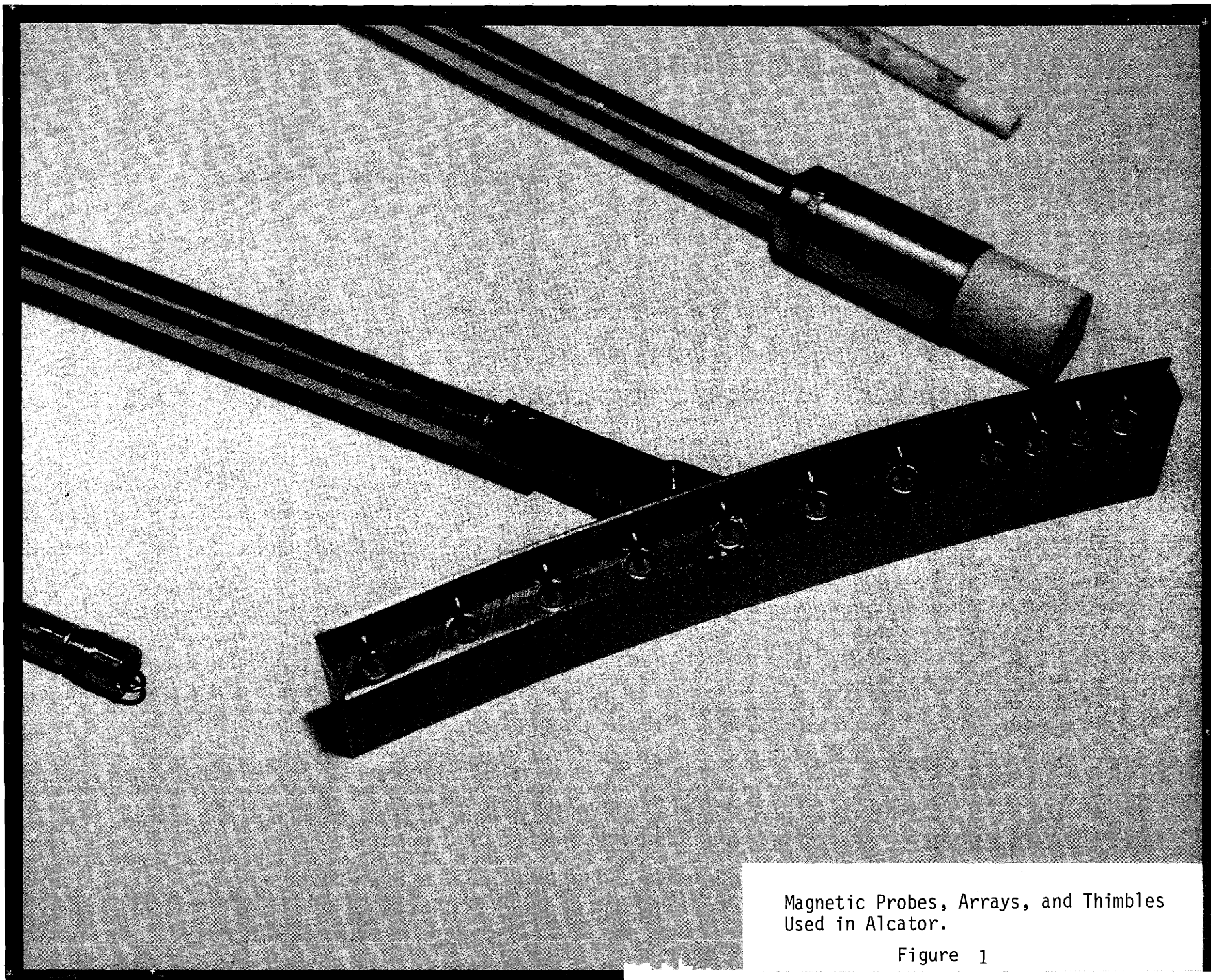
In equation (15), we assume a vacuum dielectric  $\epsilon \approx \epsilon_o$ . If instead, we assumed a more tenuous plasma with  $\epsilon \approx 30 \epsilon_o$  then  $|X_C| \approx |X_L|$ , and substantial current could be diverted away from the antenna. Note also that  $\epsilon_{||} \approx 10^3 \epsilon_{\perp}$ , and the problem is even more severe along the magnetic field lines.

To investigate these possible pitfalls, a medium power ( $<10 \text{ kW}$ ), capacitive antenna was installed. Figures 6 and 13 show this probe with a 1.5 inch diameter



molybdenum electrode. Power was fed through RG 8 coax with custom-made high voltage connectors. The inside of the probe was gas cooled, so the probe could be inserted into the plasma past the limiter radius.

Since the tokamak main limiter radius had to be adapted to each antenna, several partial limiters, similar to Figure 13, and full 360° limiters were built and also energized with RF or used as RF probes.



Magnetic Probes, Arrays, and Thimbles  
Used in Alcator.

Figure 1

RF Probe

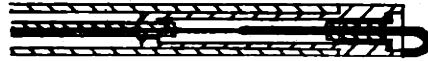


Figure 2

RF Probe

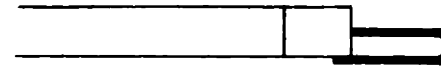


Figure 3

Thimble



Figure 4

Langmuir Probe

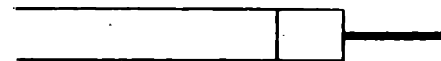


Figure 5

Capacitive Probe

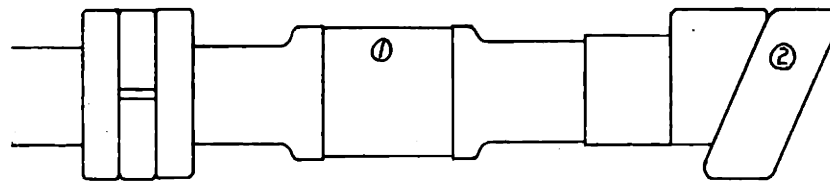


Figure 6

① - Ceramic  
② - Molybdenum

Simplified Mechanical Drawings of Magnetic Capacitive  
and Langmuir Probes Used in Alcator

## Unshielded, Shielded and Shielded Balanced Magnetic Probes

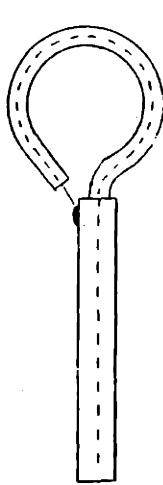


Figure 7

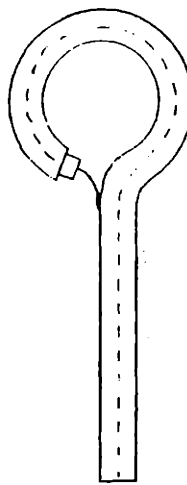


Figure 8

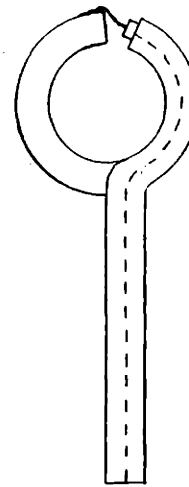


Figure 9

## Top View of $k_{II}$ Array and Access Keyholes in Thompson and HCN Ports

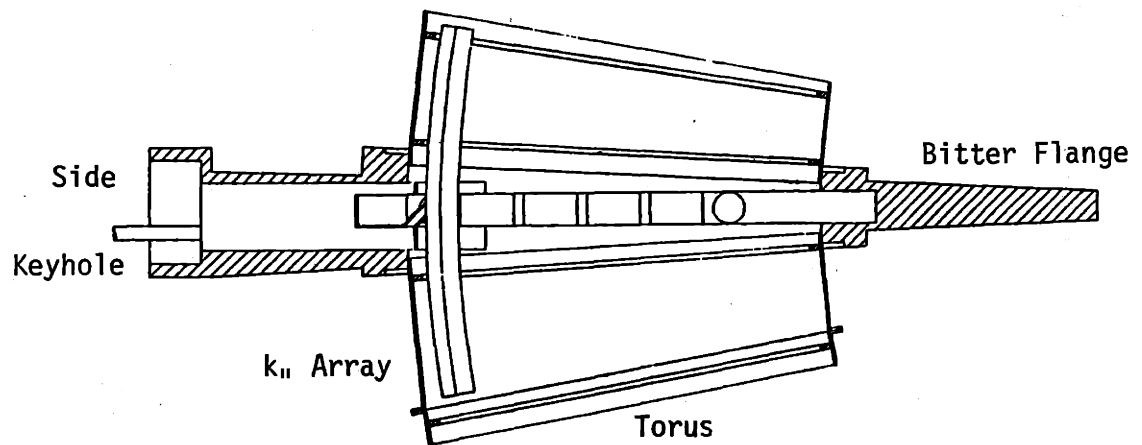
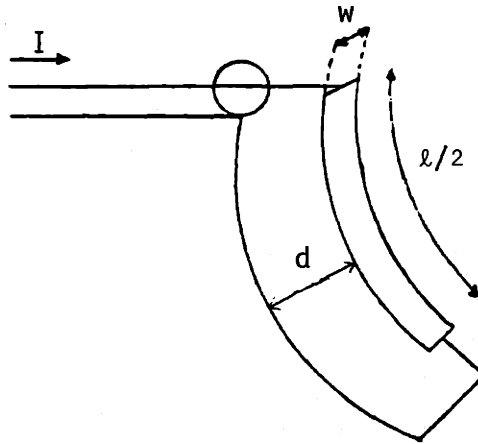
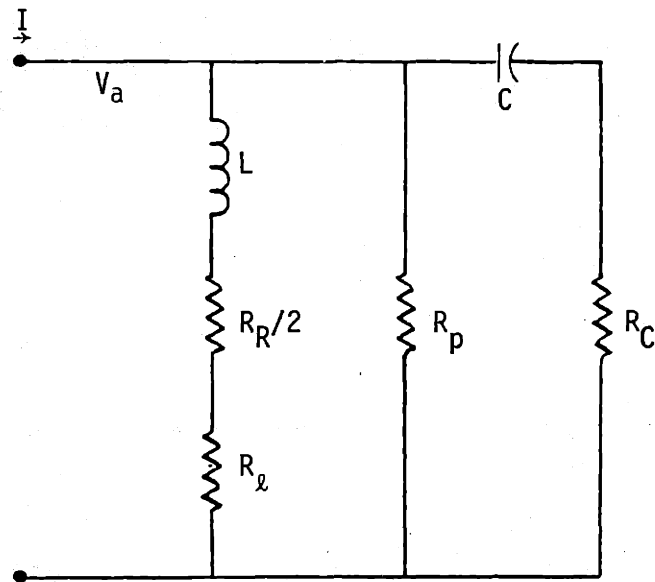


Figure 10



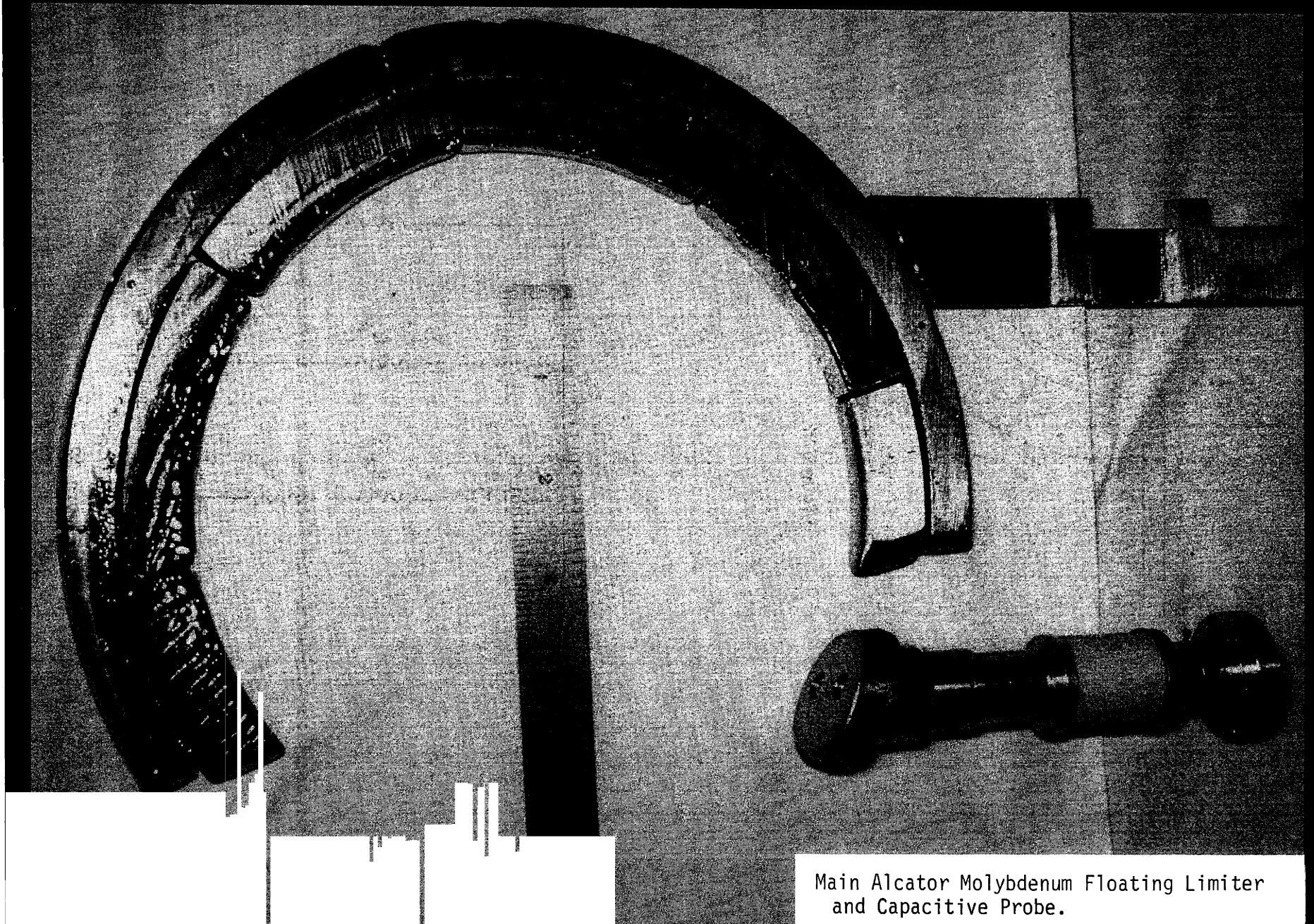
Simplified Model of  $A_2$  Antenna

Figure 11



Simplified Electrical Model of  $A_2$  Antenna

Figure 12



Main Alcator Molybdenum Floating Limiter  
and Capacitive Probe.

Figure 13



### III-3.3. RF wave correlators

Several different schemes were used to measure the phase difference between various RF probes around the torus. The measurement difficulty comes from the large dynamic range necessary to continuously track the phase of the nearly 100% high frequency (up to 100 kHz) AM and PM modulated probe signals.

Figures 1 and III-1.1(5) show the most used basic techniques. Conceptually, the simplest scheme was to actively compress and limit the probe signals with cascaded 28 db RF amplifiers and diode limiters. These signals were then mixed at the base frequency with standard 7 dbm double balanced mixes to give a signal roughly proportional to  $\cos(\phi_1 - \phi_2)$ . The main drawbacks are phase error and ambiguity. The advantage is, on the other hand, unlimited bandwidth. A more clever scheme, the sine-cosine method, did not require limiting, and is based on the simple trigonometric identities

$$(1) \quad A \cos \alpha \ B \cos \beta = \frac{AB}{2} \cos(\alpha - \beta) + \text{USB}$$

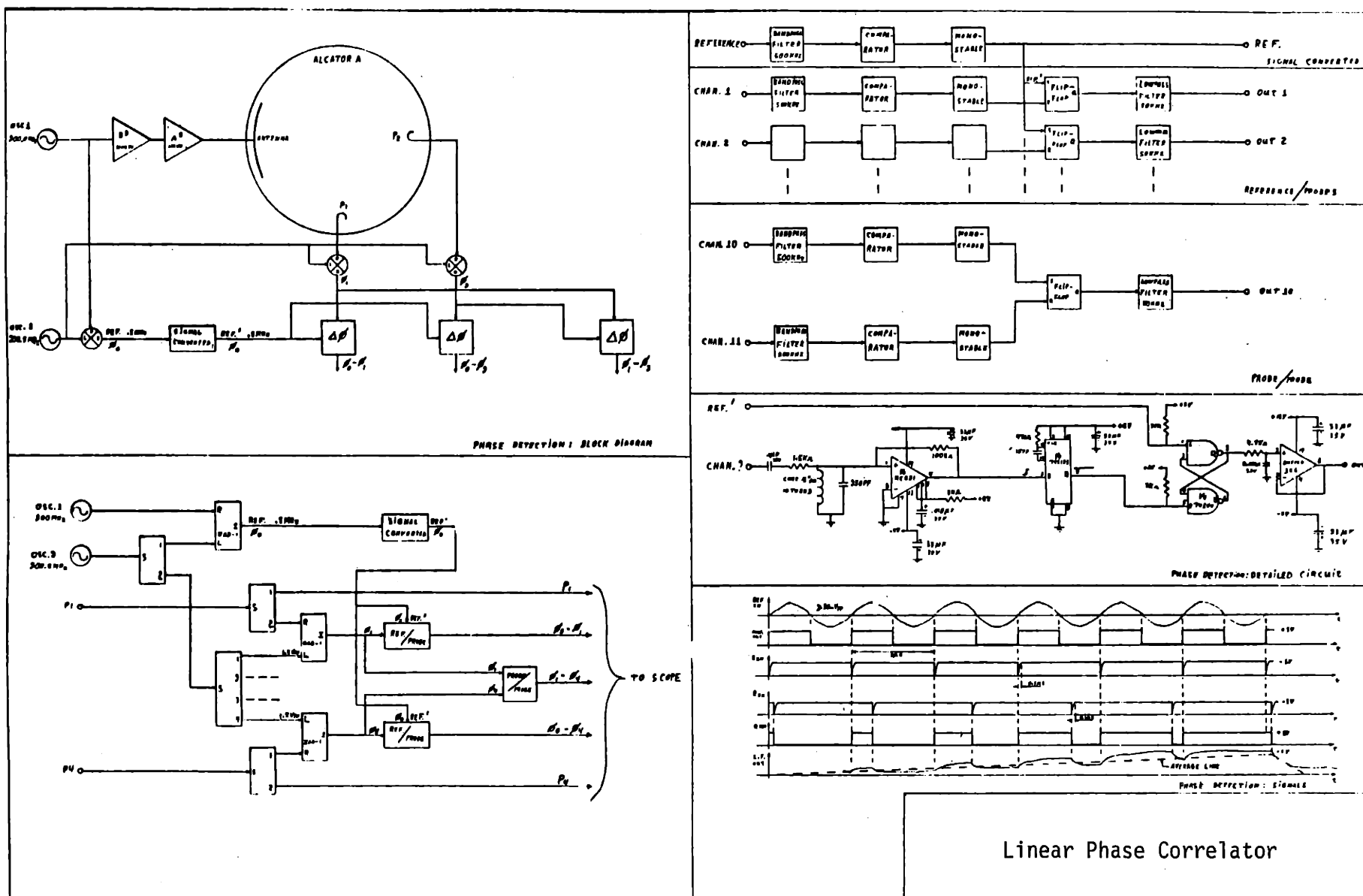
$$(2) \quad A \cos \alpha \ B \sin \beta = \frac{-AB}{2} \sin(\alpha - \beta) + \text{USB}$$

This method is implemented by mixing the A and B signal as in equation(1). The upper sideband (USB) is suppressed by a 16 MHz filter. The B signal also is phase-shifted with a 90° delay line and mixed with A as in equation(2). The phase difference can now be unambiguously resolved, given  $D \cos \gamma$  and  $D \sin \gamma$ , by either the PDP 11 or a high-speed analog computer (4 quadrant divider and  $\arctan(0 - 2\pi)$ ). The main disadvantages of this method are the high data storage rates for off-line digitally processing many probe signals, and the fact that the amplitude of the local port mixer signal

must not be too modulated (the oscillator or antenna current are appropriate, but not an uncompressed probe signal).

All these problems can be circumvented by superheterodining the 200 MHz probe signals down to 500 kHz, and then using a linear set-reset type  $0 - 2\pi$  phase correlator. Figure 1 is a detailed schematic of a later version of this method. The mixers can be operated in the linear range, since the local port signals are derived directly from the 200 MHz transmitter oscillator and a 200.5 MHz local oscillator. The USBs are removed by 16 MHz filters, and all lines, including the out ports, are properly terminated. 60 db amplitude dynamic range (5 mV to 5 V) is obtained by a high speed zero crossing comparator. High phase stability (in terms of local oscillator drift), 500 kHz, input RC filters were used for more accurate but smaller dynamic amplitude range tracking. Both positive and negative feedback were used to control the gain and hysteresis of the comparator. Without feedback and hysteresis, the correlator is unstable without signal. Simple first order and active fourth order 50 kHz output filters were also used to filter out the 500 kHz USB. An up-down counter could also have been used instead of the set-reset flip-flop, but the output can run away if tracking is lost.





Linear Phase Correlator

Figure 1

### III-3.4. Plasma edge and RF breakdown diagnostics

Three small Langmuir probes were installed in the HCN port and in front of the  $A_4$  Faraday shield to estimate the density and temperature of the tenuous plasma edge (Figures 1 and III-3.2.(5)).

Although probe theory can be extremely complicated in the presence of high magnetic or RF fields, the basic physics is nevertheless the same.<sup>77</sup> For highly negative biased probe (with respect to the vacuum vessel and plasma potential), a thin sheath, depleted of electrons, is formed around the electrode. Ions in the nearby plasma acquire an energy

$$(1) \quad -e V_s \approx \frac{K T_e}{2}$$

from the leaking electric field (Debye shielding) before entering the sheath and free-falling onto the probe. Outside the sheath, quasineutrality and Boltzman relation are valid, so that

$$(2) \quad n_e = n_o e^{\frac{e V_s}{K T_e}} \approx n_i$$

The ion saturation current at the sheath boundary is thus simply

$$(A_s \approx A_p \text{ since } r_p \gg \lambda_D)$$

$$(3) \quad I = e n_i v_i A_s = e n_o e^{-1/2} \sqrt{\frac{K T_e}{M}} A_p$$

Slightly above (in probe potential) the ion saturation regime, the  $dI/dV_p$  is mainly due to the electrons reaching the probe, and again from Boltzman relation

$$(4) \quad I_e \propto e^{e V_p / K T_e}$$

and the electron temperature can be inferred from the slope of the  $\ln(I)$  vs.  $V_p$  curve.

The Alcator edge plasma has a very small  $n_0 \sqrt{T_e}$  scale length ( $\approx .2$  inches), so the .050 inch diameter stainless steel probe wire has an effective area  $A_p \approx .2 \text{ cm}^2$ . For a 10 eV,  $5 \times 10^{13} \text{ cm}^{-3}$  plasma, the ion saturation current will be about 3.0 amperes, and the electron temperature can be inferred from a 60 volt peak to peak triangular probe voltage and current trace. Figure 1 shows the basic circuit used, where care was taken to avoid standing waves in the probe coax by proper termination at 200 MHz.

A number of high sensitivity (0.4 A/W) silicon pin diodes were used to monitor visual light between 3,000 Å and 11,000 Å around the torus and antenna system, and in particular, through the antenna hollow holding pins, inside the electrical breaks and vacuum feeders, and in front of the antenna through the bottom MW keyhole.  $H_\alpha$  filters (6520 - 6600 Å) were also used to estimate hydrogen ionization. The detectors were RF shielded, and multiple wave reflections were carefully suppressed to again avoid any possibility of RF pick-up (Figure 2).

To further investigate the intricate arcing and diffuse light production at the antenna, an optical viewer was designed to photograph (and later film) the antenna from the HCN or Thompson ports during a plasma shot. Figure 3 shows how a small pyrex window and polished stainless steel mirror attached to a 5/8 inch diameter pipe could be used with a high resolution, 1/2 inch diameter boroscope to image the plasma cross section (Figure 4). This diagnostic would also be used to monitor plasma edge position near the antenna.

## Langmuir Probe Electronics

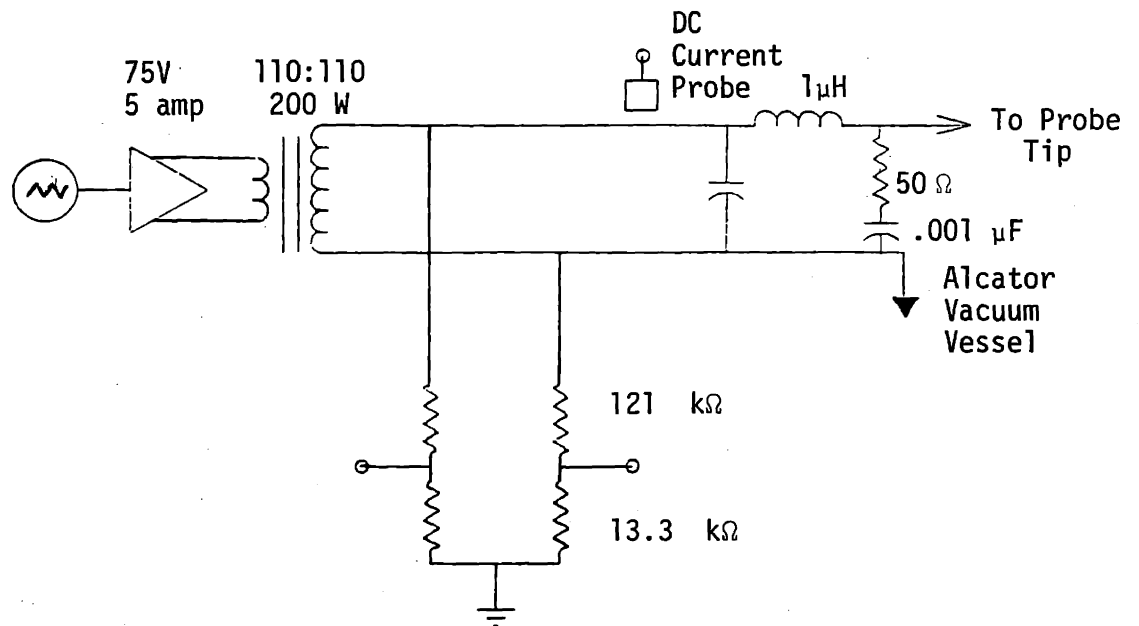


Figure 1

## Optical Detector Electronics

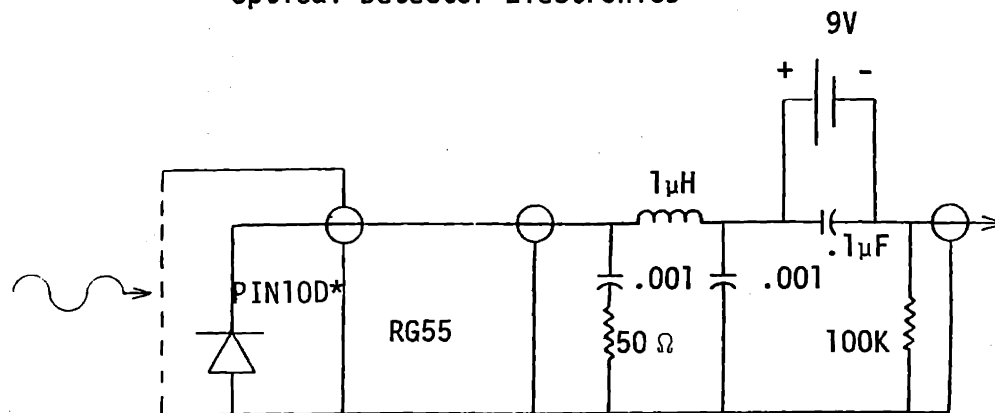


Figure 2

\*United Detector Technology, Inc.

# Top View of Optical Viewer Geometry

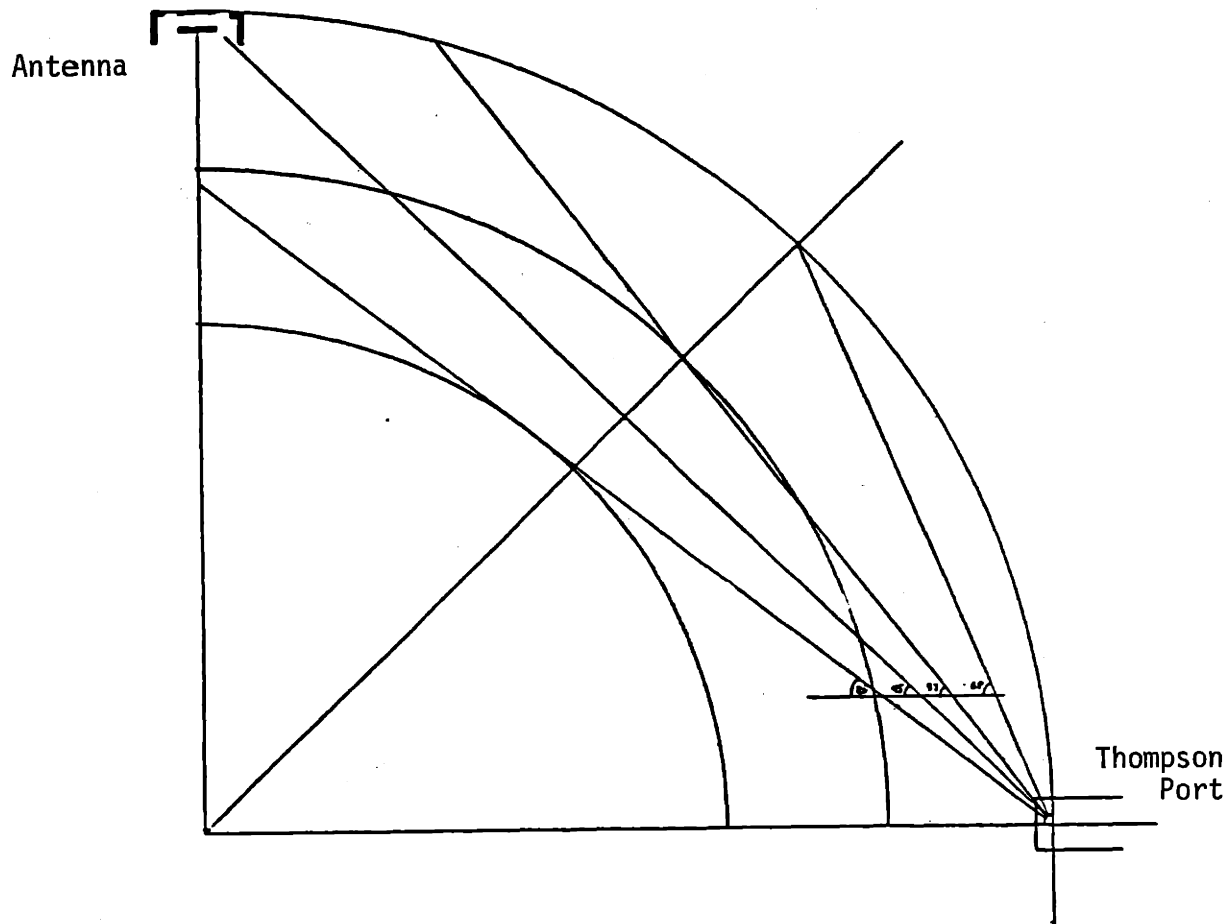


Figure 3

Detail of Stainless Steel Mirror and Boroscope Assembly

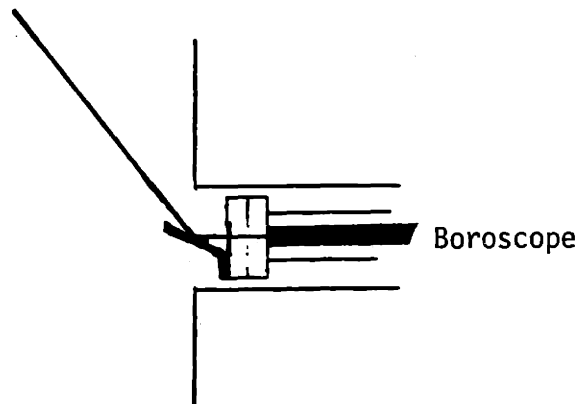


Figure 4

### III-3.5. Charge Exchange Diagnostic

Charge exchange<sup>33-38</sup> was the single most important diagnostic for medium power heating experiments (Figure 1). Figure 2 is a simplified schematic of the fast neutral spectrometer. Energy selection is done by a series of electrostatic capacitor plates  $C_0 - C_5$ . Mass selection is also made possible by replacing  $C_0$  by an electromagnet. At high plasma density, the energetic neutral flux is very small thus requiring high stripping cell efficiency. Either  $10^{-4}$  torr of steady-state nitrogen or  $10^{-2}$  torr of pulsed helium was used, with extensive use of baffles and differential pumps, to limit plasma contamination by the high pressure stripping gas. Figure 3 is a typical high density shot where both steady-state  $N_2$  and pulsed He were used. The first 100 ms of flux is detected with background  $N_2$ , and then He is pulsed, giving a much higher sensitivity. The 20 ms decay time of the He signal is due to the gas pressure decay, and not a change in the neutral flux.

The fast neutral flux arriving at the spectrometer can be written as the product of three factors: collection, production and attenuation.

$$(1) \quad \frac{dN}{dE} = \underbrace{\frac{\Omega}{4\pi} A v_i}_{\text{collection}} \underbrace{\int_{-a}^{+a} \sigma_{cx} v_i n f_i(E) dr}_{\text{production}} \underbrace{\exp\left(-\int_r^a \frac{\gamma_{io} + \gamma_{cx}}{v_i} dr\right)}_{\text{attenuation}}$$

For low density and high energy neutrals, the flux is easy to calculate and interpret, since attenuation (ionization and charge exchange) is small, and since the neutral density in the center is not too many e-foldings lower than at the edge, and high energy ions are only produced in the hot center. The central temperature can then be simply estimated from the slope of the high energy neutral spectra.

At high density, and especially with RF in Alcator, none of the above is true. Attenuation can be several e-foldings, and the central neutral density is very low compared to the edge. Low energy neutrals from the surface ( $n_n \approx 10^{11} - 10^{12} \text{ cm}^{-3}$ ) have e-folding lengths of the order of a cm, and do not contribute to the central neutral density ( $10^7 - 10^8 \text{ cm}^{-3}$ ) which is sustained by electron-ion recombination. At high density, for well-behaved distribution functions and profiles, the high energy spectra will be colder than the central temperature. Figure 4 shows the neutral spectra for different times in a typical plasma shot. Note how the energetic neutral flux dramatically decreases with density. Figure 5 shows a typical ion temperature evolution with time.

Another major pitfall of charge exchange is that Alcator has a comparatively large port ripple (Figure 7), and only perpendicular viewing is possible, so only the  $\nabla B$  drifting trapped particles are observed, and are, in many cases, not representative of the bulk of the distribution function. The mean free path for detrapping these particles out of the magnetic well is approximately (Figure 9)<sup>1,35,85,86</sup>

$$(2) \quad \lambda \approx \frac{v_{\nabla B} \theta^2}{\gamma_{ij}} \approx 5.6 \times 10^{13} \frac{\sqrt{\mu} \theta^2 E^{5/2}}{n R B}$$

where

$$(3) \quad v_{\nabla B} = \frac{v_{\perp}^2}{2 \omega_{ci} R}$$

$$(4) \quad \theta^2 = \left( \frac{v_{\parallel}}{v} \right)^2 = \frac{\Delta B}{B} \approx .04$$

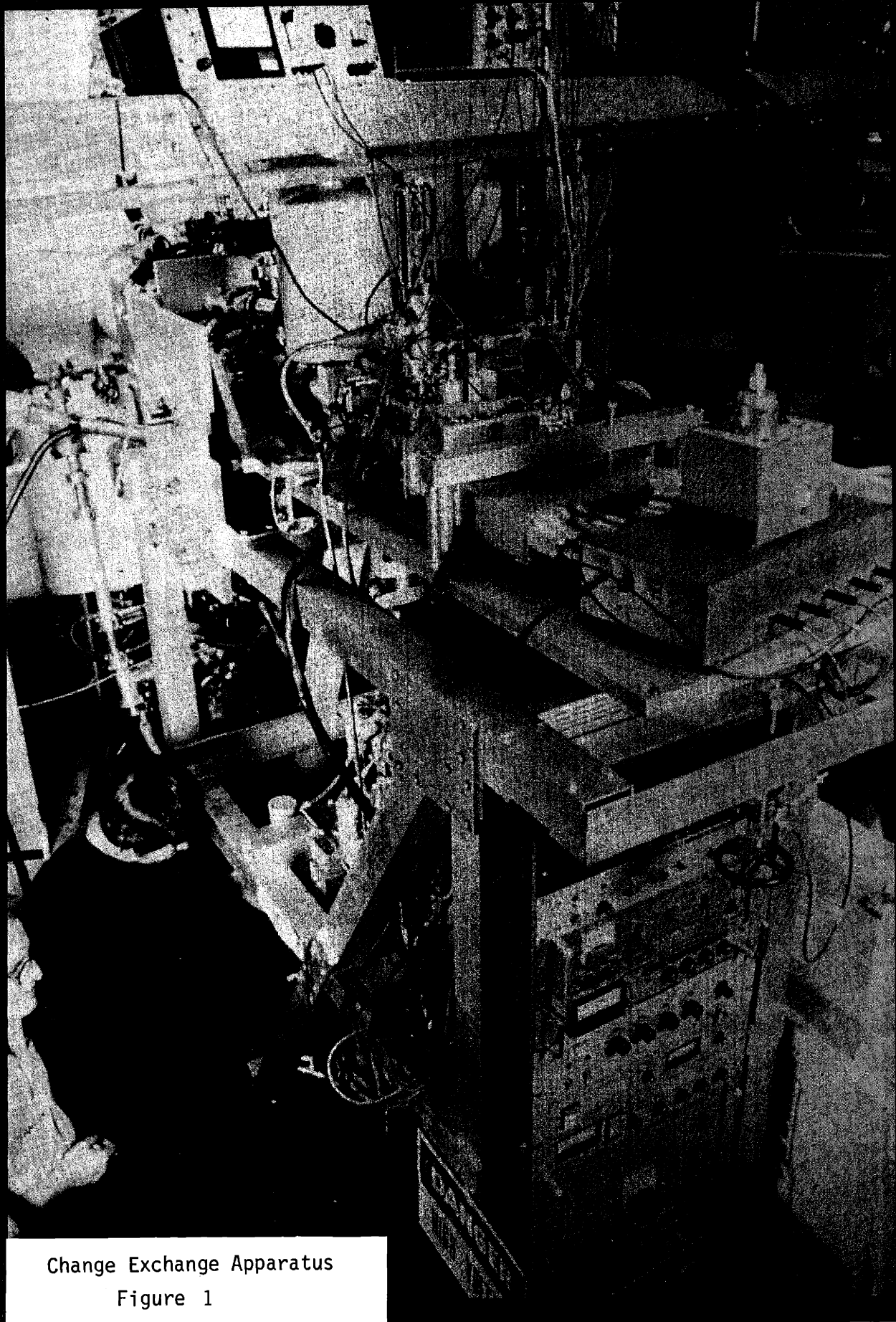
$$(5) \quad \gamma_{ij} = 1.8 \times 10^{-7} \mu^{-1/2} E^{-3/2} \Lambda n$$

This mean free path allows the higher energy trapped particles to drift upward significant distances ( $\lambda_{mfp} \approx a$ ) in the plasma. This effect is

experimentally seen from Figures 6 and 8 where the asymmetric effective temperature and neutral flux are shown. This observed asymmetry is of course, reversed with reversed toroidal field, and closely tied to temperature and density profiles as can be seen from the difference between Figures 10 and 11, where the plasma current was changed from 100 kA to 225 kA. At high currents the profiles are broader, and the high energy production source becomes closer to the high neutral density edge, thus enhancing the asymmetry.

For high enough energies so the mean free path is larger than the plasma radius, the central chord distribution function can be completely depleted. This effect is also observed experimentally by a dramatic drop in neutral flux above 4 - 7 keV in medium density plasmas.





Change Exchange Apparatus  
Figure 1

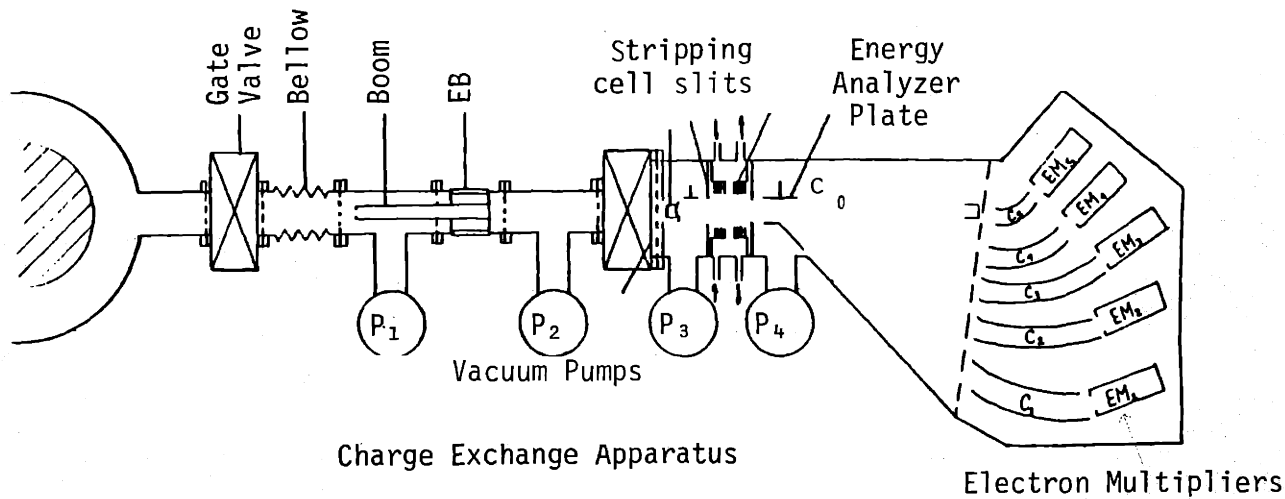
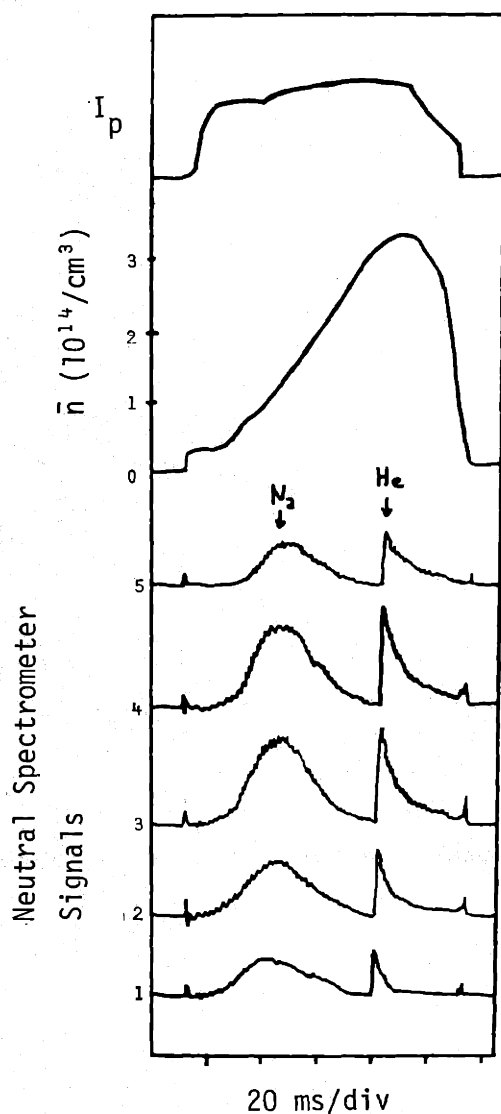
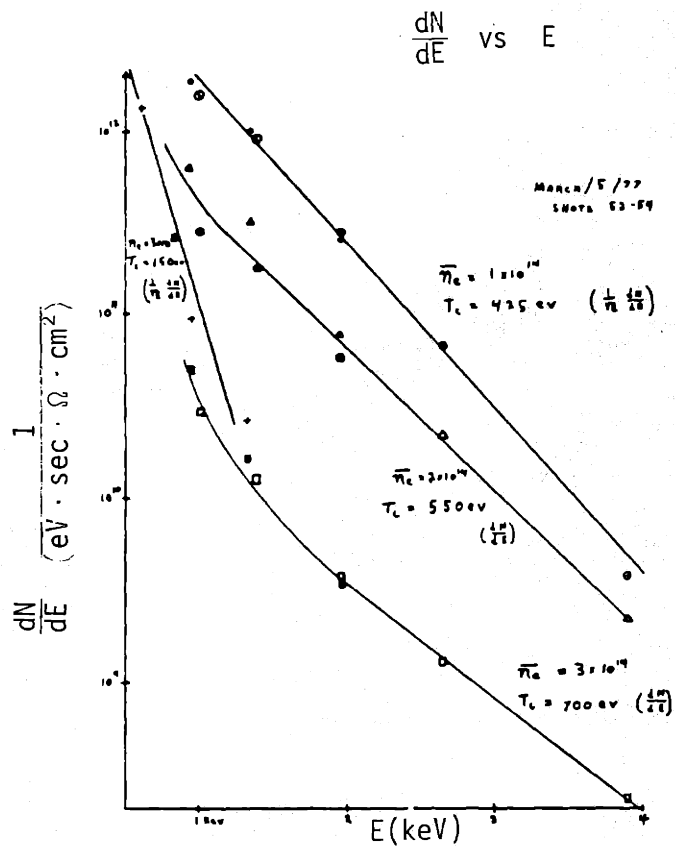


Figure 2



Typical Charge Exchange Data at High Density

Figure 3



Neutral Spectra Change with Density

Figure 4

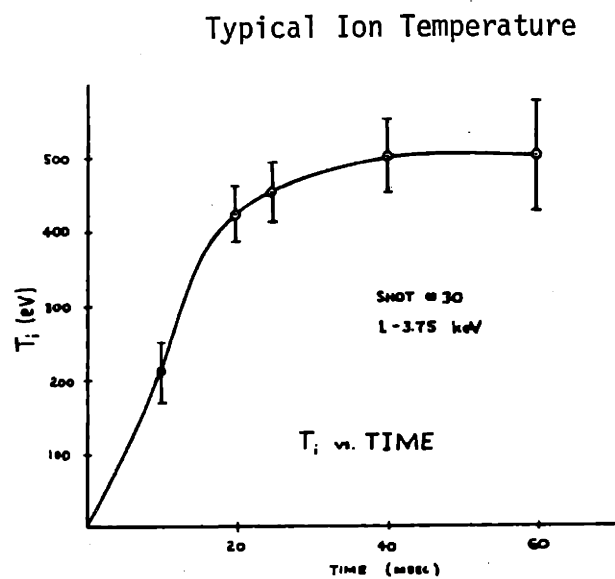


Figure 5

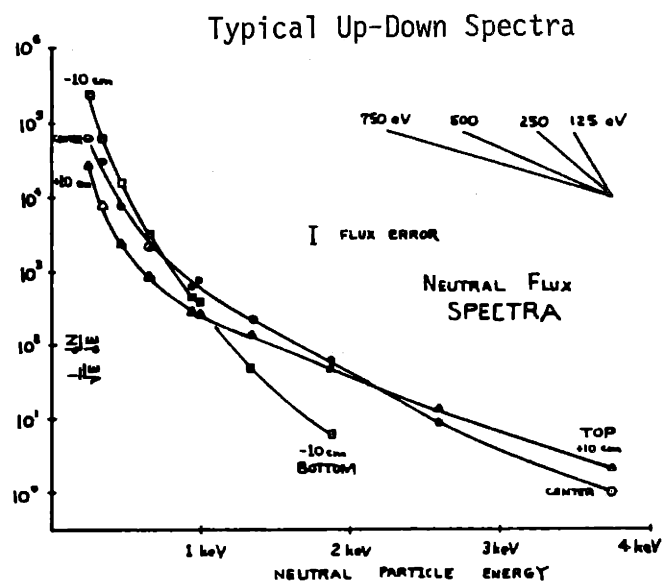


Figure 6

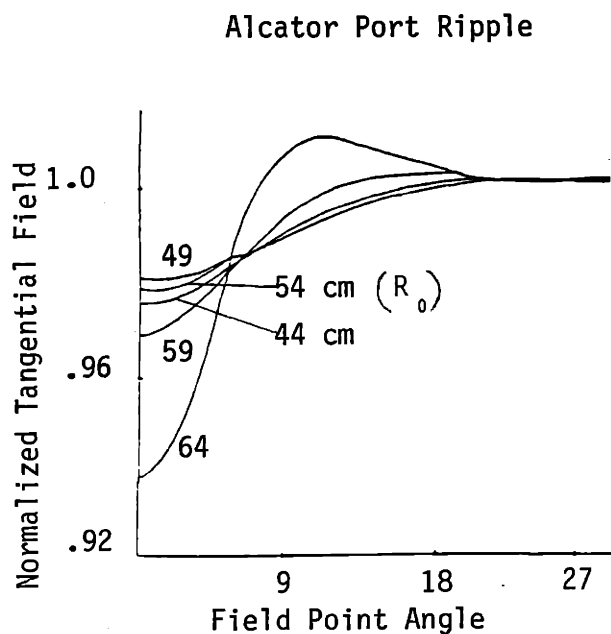


Figure 7

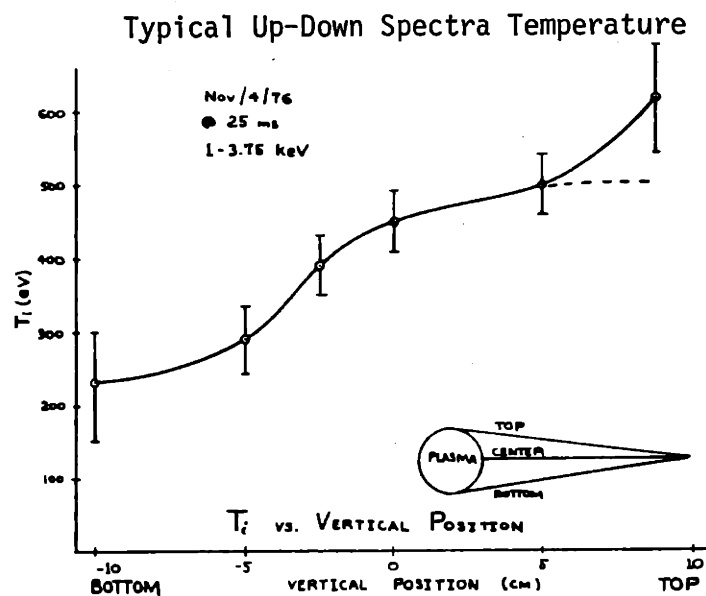


Figure 8

# Ion Mean Free Path as a Function of Ion Energy

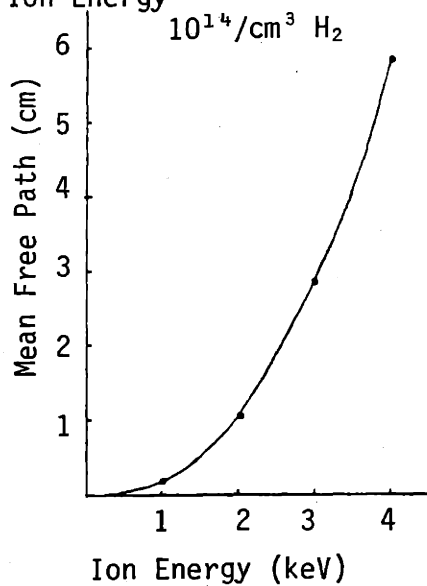


Figure 9

# Neutral Flux vs. Up Down Position at High Plasma Current

3/17/78  
60 kG  
225 kA  
 $3 \times 10^{14}/\text{cm}^3$

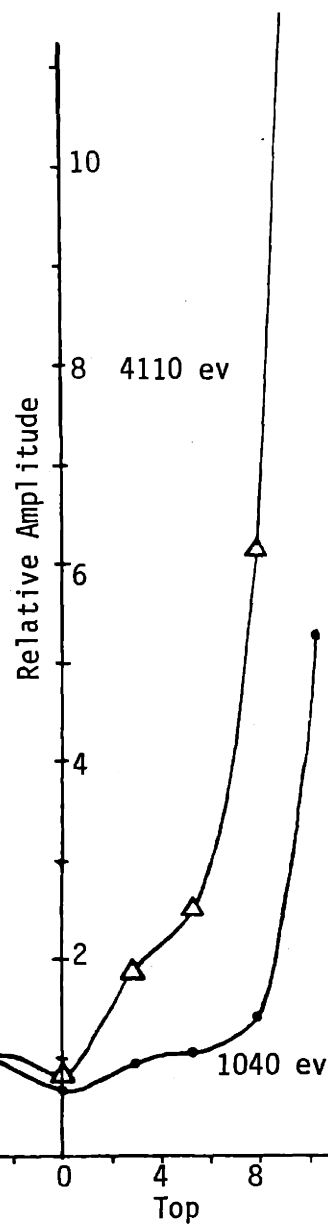


Figure 11

# Neutral Flux vs. Up Down Position at Low Plasma Current

3/18/78  
60 kG  
100 kA  
 $3 \times 10^{14}/\text{cm}^3$

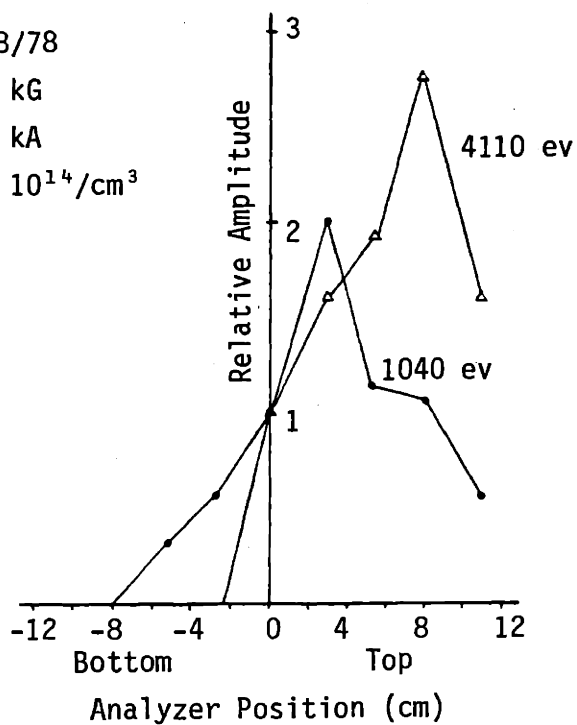


Figure 10

### III-3.6. Bolometers and superbanana trapped particle detectors

In this Section, we present a zeroth order model to illustrate how trapped particles may also be an important power absorption loss. This discussion will be confined to the high energy particles at 5 keV, with  $\omega_o = 2 \omega_{ch} \approx 2 \pi 200$  MHz, 4% ripple, and  $n_H = 10^{14}/\text{cm}^3$ . The ICRF wave interacts with these trapped particles in a volume<sup>103</sup>

$$(1) \quad v_T = 4(\Delta R a \ell_T) \approx 800 \text{ cm}^3 \ll 2 \pi R \pi r^2 \approx 10^5 \text{ cm}^3$$

where

$$(2) \quad \Delta R \approx 1 \text{ cm} \quad (\text{Eq. IV-4.2. (36)})$$

$$(3) \quad a \approx 10 \text{ cm}$$

$$(4) \quad \ell_T \approx \frac{\pm 10^\circ}{360} 2 \pi R \approx 20 \text{ cm} \quad (\text{Figure III-3.5. (7)})$$

The trapped particles drift upward (or downward) at velocity

$$(5) \quad v_{\nabla B} = \frac{v_\perp^2}{2 \omega_{ci} R} \approx 1.4 \times 10^5 \text{ cm/sec}$$

and are confined a time

$$(6) \quad t_T \approx \frac{a}{v_{\nabla B}} \approx 7.1 \times 10^{-5} \text{ sec}$$

before leaving the plasma. A large fraction of velocity space is trapped, so that

$$(7) \quad n_T \approx n_o \frac{v_H}{v} \approx 2 \times 10^{13} \text{ cm}^{-3}$$

Since these trapped particles have large  $v_\perp$  and bounce back and forth through the resonant layer, they can absorb much more power than the statis-

tical average (over the plasma volume) calculated in Section IV-4.2.

If 50 kW were absorbed by these ions, they would reach an energy

$$(8) \quad W = \frac{P t_T}{n_T v_T} \approx 1.4 \text{ keV}$$

before leaving the plasma. These ions would have a parallel velocity

$$(9) \quad v_{||} \approx \sqrt{\frac{\Delta B}{B}} \quad v_{\perp} \approx 2 \times 10^7 \text{ cm/sec}$$

and pass some

$$(10) \quad N = \frac{v_{||}}{v_{TB}} \frac{a}{\lambda_T} \approx 70$$

times through the resonant layer, acquiring

$$(11) \quad \Delta W \approx \frac{W}{N} \approx 20 \text{ eV}$$

per pass. This very crude calculation is in reasonable agreement with the energy kick we calculated from equation IV-4.2.(20) where we can make the second harmonic substitution of equation IV-4.2.(34).

$$(12) \quad \Delta W = \left( \frac{k_{\perp} \rho}{\sqrt{2}} \right)^2 \frac{e^2}{2m} E_+^2 \frac{2\pi}{\omega'} \approx 2.2 \times 10^{-11} \text{ ergs} \approx 14 \text{ eV}$$

where we used<sup>103</sup>

$$(13) \quad E \approx 1 \text{ statvolt/cm (Section IV- 5.3)}$$

$$(14) \quad \omega' \approx 2 \omega_{ci} \frac{v_{||} \Delta B}{\lambda_T B} \approx 5 \times 10^{13} \text{ sec}^{-2}$$

$$(15) \quad \frac{k_{\perp} \rho}{\sqrt{2}} \approx .05$$

Upon the realization of the importance of this possibly large power loss mechanism, a trapped particle detector and bolometer were designed and built.<sup>103</sup> These probes were small (3/8 inch diameter), and could be inserted in any port,

but in particular, at the antenna port (through the top and bottom keyholes), under and over the resonant layer where trapping is most likely to occur.

Figures 1 and 2 show a simple, conceptual design where a poloidal screen and the machine virtual limiters are used to shield out streaming particles, but not the superbanana orbits. The circuitry is similar to Figure III-3.4.(1) and the probe current can be estimated as

$$(16) \quad I_p \approx e v_{TB} n_T l_T w_p \approx 4.5 \text{ amperes}$$

The parallel and perpendicular energy can also be estimated from the probe characteristics

$$(17) \quad W_{||} = W_{\perp} \frac{\Delta B}{B} \approx 200 \text{ eV}$$

if the probe can sustain the high voltage ( $V \approx 2W_{||}/e \approx 400 \text{ V}$ ) needed to repel the ions without causing a plasma discharge in the probe tip.

The bolometer probe is a simple .002" thick stainless steel foil, with thin, flattened thermocouple wires tack-welded to the back of the foil (Figures 2 and 3). The ohmic heating power surface density averaged over the torus vacuum chamber is

$$(18) \quad P/\text{cm}^2 = \frac{I V}{2\pi R 2\pi a} \approx 11 \text{ W/cm}^2$$

and from equation III-2.2.(4), the bolometer temperature increase over a 150 millisecond shot is approximately

$$(19) \quad T = \frac{P/\text{cm}^2 t}{\rho c_p \Delta x} = 41^\circ\text{C}$$

where

$$(20) \quad \Delta x \approx .002" \text{ foil} + .002" \text{ thermocouple}$$

The detector response time can be calculated from equation III-2.2.(9) as

$$(21) \quad \Delta t \approx \frac{\Delta x^2}{\alpha} \approx 2.6 \text{ msec.} \ll t_{RF} \approx 20 \text{ msec.}$$

41°C corresponds to 1.6 mV, with chromel-alumel, type k, .062" OD jacketed Omegaclad thermocouple wire, which is easily measurable during the plasma shot. Localized trapped particle fluxes could give rise to large temperature increases. A short 20 msec. 50 kW pulse, on the other hand, would only produce 36  $\mu$ V, and the measurement becomes extremely difficult due to the large  $\frac{dB}{dt}$  and RF fields near the plasma. Even with a small 1 cm<sup>2</sup> loop in a 1 KG, 30 msec. decay time magnetic field, the pick-up voltage would be 300  $\mu$ V.

To make the measurement of such small signals possible, a triple differential measurement scheme was used, as shown in Figure 3. The input differential amplifiers have a gain of 1000, and were tested to discriminate a 10  $\mu$ V, 1 kHz (-3 db) signal on top of a 1 volt 60 Hz common mode noise. A second reference signal is used to subtract background temperature and pick-up. The second differential stage is high pass ( $\tau > 2$  sec) to avoid saturation (total series gain is 10<sup>5</sup>). The third differential stage is at ground potential in the control room.

The jacketed thermocouple wires are twisted and shielded inside an RF tight stainless steel tube. The electronics is in a RF tight aluminum box that is in a larger, half-inch thick, soft steel magnetic shield box. The steel box is electrically tied to the vacuum vessel, and acts as an electrostatic guard for the aluminum box. A heavy braid shield covers the thermocouple wires between the Conex vacuum feedthrough and the aluminum box. The thermocouples and electronics are oriented so as to have minimum coupling to the tokamak OH and vertical field.



## Electrostatic Trapped Particle Detector

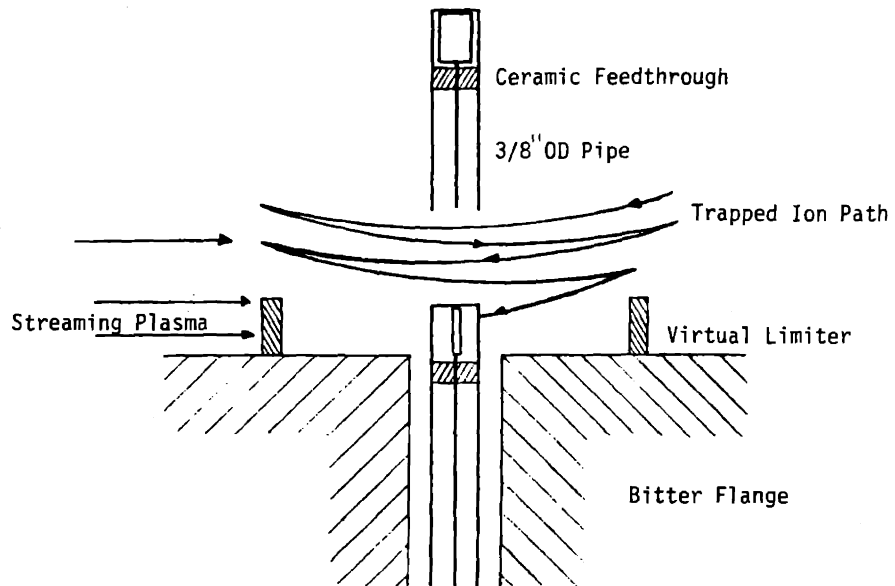


Figure 1

## Electrostatic Trapped Particle Detector and Bolometer

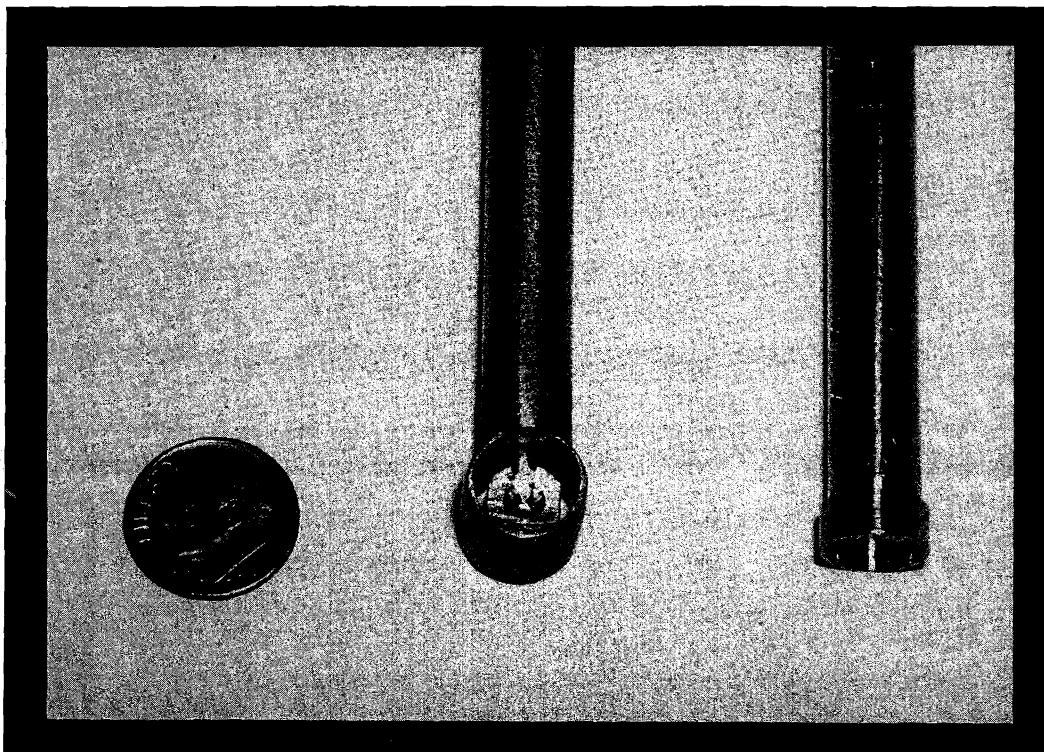
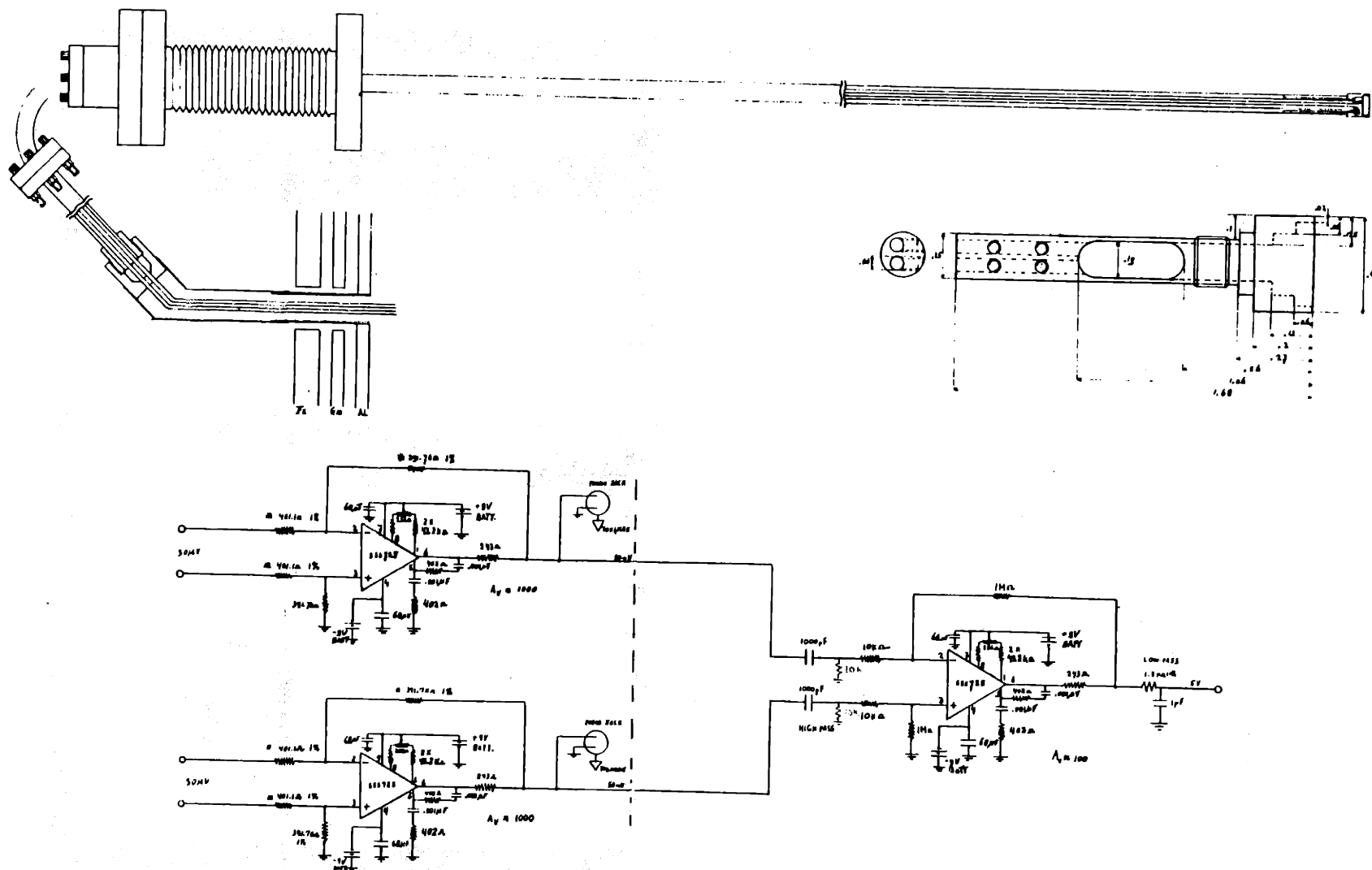


Figure 2



Mechanical and Electrical Drawing of Stainless Steel Foil Bolometer  
Figure 3

#### IV. THEORETICAL WAVE MODELS AND COMPUTATIONS

##### IV-1. Introduction, First Order Models and Approximations

For the present work, many parameters and processes not fully discussed in the literature need to be defined<sup>21,22,30-32</sup> clearly in simple models to avoid confusion in the more complicated analytical or computational models. An intuitive feeling for the order of magnitude of most important parameters is derived from the most elementary foundations of physics. Among the processes that one must understand qualitatively and quantitatively, are Alfvén refractive index and wavelength<sup>1</sup>, experimentally and theoretically derived radiation resistance<sup>32</sup>, and wave quality factors<sup>7</sup>. The homogeneous<sup>46</sup> Cartesian waveguide is then usually sufficient to derive the most important scaling laws of the wave field structure and antenna design.

The reader more experienced in current ICRF work may wish to proceed directly to Sections IV-5., IV-6., and IV-7, where the more advanced models are discussed: inhomogeneous cylindrical plasma model, stochastic mode stacking, and single perpendicular pass regime.

#### IV-1.1. Eigenmode wave field approximations in toroidal geometry

In the toroidal geometry of the Alcator tokamak, the wave is bound by a highly conducting vacuum vessel. For wavelengths short enough to fit in the minor torus cross section, the wave will propagate and bounce around inside the toroidal cavity. Since the minor diameter is much less than the free space wavelength ( $\lambda_o = 150$  cm at 200 MHz), we will have a density cutoff below which the wave cannot propagate. In Alcator we typically have

$$\omega_o \approx 2\pi \text{ 200 MHz}$$

$$B_o \approx 60 \text{ kG}$$

$$(1) \quad n_e \approx 3 \times 10^{14} / \text{cm}^3$$

$$\lambda_{\text{Alfvén}} \approx 4 \text{ cm}$$

$$n_{\text{cutoff}} \approx 10^{13} / \text{cm}^3$$

For wave damping length smaller than the minor radius, no eigenmode is formed, a situation known as the single perpendicular pass regime (IV.7). For damping length of more than the minor radius, but less than the major radius ( $R_o = 54$ ), we have perpendicular eigenmodes, but not toroidal ones. This situation is called the single toroidal pass regime. Since the Alcator aspect ratio ( $R_o/a = 5.4$ ) is large, toroidal geometry maps out well into cylindrical geometry. (IV-3, IV-5, IV-6)

The basic physics of a perpendicular eigenmode can be illustrated with a simple  $TE_{10}$  mode (Transverse electric) in a rectangular waveguide along  $z$ , of width  $2a$ , filled with an isotropic non-dispersive dielectric (Figure IV-1.2 (1)). The electric and magnetic fields (from Faraday's Law) are of the form (MKS)<sup>134</sup>

$$(2) \quad E_y = E_o \sin(k_x x) e^{i(k_z z - \omega t)}$$

$$(3) \quad H_x = -E_y \frac{k_z}{\omega \mu_o}$$

$$(4) \quad H_z = E_0 \frac{-ik_x}{\omega \mu_0} \cos(k_x x) e^{i(k_z z - \omega t)}$$

with boundary conditions

$$(5) \quad E_y \Big|_{x=0, 2a} = 0$$

$$(6) \quad k_x \approx k_\perp \approx \mu_\perp \frac{\pi}{2a}$$

where  $\mu_\perp$  is the perpendicular mode number (typically  $1 \leq \mu_\perp \leq 10$  for Alcator).

Similarly, in cylindrical geometry, using the usual coordinates  $r, \theta, z$ , we have TE or TM eigenmode solutions of the form<sup>134</sup>

$$(7) \quad A = A_0 \cos(m\theta) J_m(k_\perp r) e^{i(k_\parallel z - \omega t)}$$

where  $m = 0, 1, 2, \dots$

For waves of the Alfvén class near the ion cyclotron frequency ( $\omega \approx 2\omega_{ci}$ ), we notice ( Appendix 1 ) two basic differences between a plasma and a "normal" dielectric. First,  $\epsilon_\parallel \gg \epsilon_\perp$ , so  $E_\parallel \ll E_\perp$ , and thus the wave and its resulting eigenmode must be TE. Second, the wave is not allowed to be linearly polarized, so we must let the eigenmodes rotate (elliptically polarized) as they propagate along  $z$ . We then have wave field solutions of the form<sup>46</sup>

$$(8) \quad A = A_0 J_m(k_\perp r) e^{i(m\theta + k_\parallel z - \omega t)}$$

with  $m = 0 \pm 1 \pm 2 \pm 3 \dots$

We must allow  $m$  to be both positive and negative, since a field rotating with the ions "feels" a different dielectric constant than one rotating against the ions. The ion trajectory

$$(9) \quad r_i = r_o e^{i(-\theta - \omega_{ci}t)}$$

and the  $m < 0$  field patterns are left-handed. (The  $m < 0$  electric fields are also predominately left-handed, as will be seen in IV-5.3.) The  $m > 0$  modes are right-handed, and have a larger dielectric constant than the  $m < 0$  modes (Section IV-5.3). The  $+m$  and  $-m$  modes thus have different  $k_{||}$ , are orthogonal, and must be considered separately. For  $m = 0$ , equations (7) and (8) are equivalent since the mode field pattern is not rotating either way. For typical Alcator ICRF wave and plasma parameters, radial mode numbers ( $\mu$ ) up to 5 can be obtained with small  $m$  and  $k_{||}$  (Section IV-5.3).

With damping lengths larger than the major radius, both perpendicular and toroidal eigenmodes can exist. To zeroth order, we have a dispersion relation of the form

$$(10) \quad k_{\perp}^2 + k_{||}^2 = k_T^2 \cong k_A^2$$

combining (6) with (10) fully determines  $k_{||}$ , and toroidal resonances will occur if

$$(11) \quad \begin{aligned} 2\pi R_o &= n\lambda_{||} \\ k_{||}R_o &= n \end{aligned}$$

where  $n$  is the toroidal mode number (typically  $1 < n < 60$  for Alcator).

The orthogonality property of eigenmodes with different mode numbers enables these modes to propagate and be calculated independently without coupling to each other. For our simple TE modes, we can see easily that indeed  $TE_{m0}$  is independent of  $TE_{n0}$ , since for  $m \neq n$

$$(12) \quad \int_0^{2a} \sin \left( m \frac{\pi x}{2a} \right) \sin \left( n \frac{\pi x}{2a} \right) dx = 0$$

Propagation along  $z$  is also orthogonal for  $k_{||1} \neq k_{||2}$ .

Physically, an eigenmode resonance is formed when two or more waves overlap coherently in space. The usual standing wave is found when a wave reflects off a boundary and adds to itself, (i.e. the perpendicular eigenmodes).

$$(13) \quad \cos (kx - \omega t) + \cos (-kx - \omega t) = 2 \cos kx \cos \omega t$$

In toroidal geometry, a toroidal standing wave can be formed by two waves of the same  $k_{||}$  and perpendicular field structure circulating in opposite directions around the torus. If, on the other hand, the two  $k_{||}$  are not the same, but one of them satisfies (11), we have a resonant running mode

$$(14) \quad \cos (kz - \omega t) + \cos (kz - \omega t) + \dots = A \cos (kz - \omega t)$$

Physically, this amounts to a wave leaving the antenna, circulating in one direction around the torus, and adding coherently to itself. In this way, for small damping, and either standing or running waves, the circulating wave power can be many times the power coupled from the antenna.

#### IV-1.2. Theoretical radiation resistance

To calculate the effective radiation resistance ( $R_R$ ) the plasma presents to the antenna, we assume for the moment a simple  $TE_{10}$  eigenmode structure in a rectangular wave guide (Section IV-1.1) as seen in Figure 1, and the single toroidal pass regime.

$$(1) \quad \frac{1}{R} < k_{||i} < \frac{1}{a}$$

Since we have a lossless system except for the plasma, the power into the antenna,  $P_{in}$ , must equal the total wave Poynting flux flowing down the waveguide ( $\pm z$ ),  $2P_{EXH}$ . Thus (MKS)

$$(2) \quad P_{in} = \frac{I^2 R_R}{2} = \frac{IV}{2} = 2P_{EXH}$$

where

$$(3) \quad V = E_A \ell = E_o \ell \sin k_{\perp} d \approx E_o k_{\perp} d \ell$$

$$(4) \quad P_{EXH} = \iint_S \frac{E_y X H_x}{2} \cdot ds = E_o^2 \frac{a^2 k_{||}}{\omega \mu_o}$$

and combining (2), (3), and (4) we have

$$(5) \quad R_R = \frac{V^2}{4 P_{EXH}} = \frac{E_a^2}{E_o^2} \frac{\ell^2}{k_{||} a^2} \frac{\omega \mu_o}{4} = \frac{\beta_a^2 \ell^2}{k_{||} a^2} \frac{\omega \mu_o}{4}$$

where we defined

$$(6) \quad \beta_a = \frac{E_a}{E_o} \approx k_{\perp} d \approx \mu_{\perp} k_{o\perp} d$$

Equation (5) is the most important coupling scaling law. For typical ICRF in Alcator at high density



$$\begin{aligned}
 n_e &\approx 3 \times 10^{14} \text{ cm}^{-3} & d &\approx 1 \text{ cm} \\
 f_o &\approx 200 \text{ MHz} & \ell &\approx 20 \text{ cm} \\
 (7) \quad B_o &\approx 60 \text{ kG} & k_{||} &\approx k_A/3 \approx .5/\text{cm} \\
 \lambda_A &\approx 4 \text{ cm} & \beta_a &\approx .16 \\
 a &\approx 10 \text{ cm} & R_R &\approx .8 \text{ Ohms}
 \end{aligned}$$

Simply setting  $k_{||}$  of order  $k_A^2/3$  is sufficient for rough calculations except at densities near the mode cutoff where  $k_{||} \rightarrow 0$  and  $R_R$  becomes momentarily large. The factor,  $\beta_a^2$ , is much more difficult to estimate correctly, since it can easily vary over more than two orders of magnitude due to the evanescent edge layer, as we shall see in Section IV-5.

Of course, many modes can exist at high density, and the total radiation resistance will be the sum over all the propagating poloidal and radial eigenmode number combinations.

$$(8) \quad R_T = \sum_{\mu} \sum_m R_{\mu m}$$

Calculating  $P_{EXH}$ ,  $\beta_a$  and  $R_T$  accurately will require careful modeling of the antenna, and evaluation of the dispersion relation and wave fields in realistic geometry and profiles. This will be the subject of Section IV-5.3.

If we allow the damping to be very small and  $k_{||}R_o = n$  so as to have a toroidal eigenmode, another almost equivalent approach is to consider the back EMF produced by the wave in the antenna loop,

$$(9) \quad V_{\text{loop}} = - \frac{d\phi}{dt} = \omega \ell d \mu_o H_{\text{edge}}$$

and the usual definition of  $Q$

$$(10) \quad Q = \frac{\omega W}{P_{dis}}$$

where

$$(11) \quad W \approx \left\langle \frac{1}{2} \mu_o H^2 \right\rangle Vol \approx \frac{1}{8} \mu_o H_o^2 Vol$$

We have again from conservation of power

$$(12) \quad P_{in} = P_{dis} = \frac{V^2}{2 R_R}$$

and combining (9) to (12) we have<sup>93</sup>

$$(13) \quad R_R = \left( \frac{H_{edge}}{H_o} \right)^2 (\ell d)^2 \frac{2\omega\mu_o}{\pi^2 a^2 R} Q$$

The importance of the antenna loop area and the evanescent edge are again emphasized, but a new factor,  $Q$ , has appeared. Physically, a high  $Q$  resonant toroidal eigenmode does not change the single pass Poynting flux  $P_{EXH}$  of eq. (4), but increases the antenna electric field,  $E_a$ , by a factor proportional to  $\sqrt{Q}$ . Thus at constant power and increased voltage, one has a large  $R$  impedance,  $R_R$ . Surprisingly, but quite correctly, weak damping produces a larger antenna loading and thus better coupling (Neglecting Parasitics, Section IV-2.6 and IV-4.5).

The above formulations, although simple and physically tangible, are very limited. Either one must be in a narrow window of damping length to have a single toroidal pass regime or to be on top of a narrow high  $Q$  toroidal resonance. We will treat this problem rigorously, with much more general and flexible formulations in Sections IV-5, IV-6 and IV-7.

## Theoretical Radiation Resistance

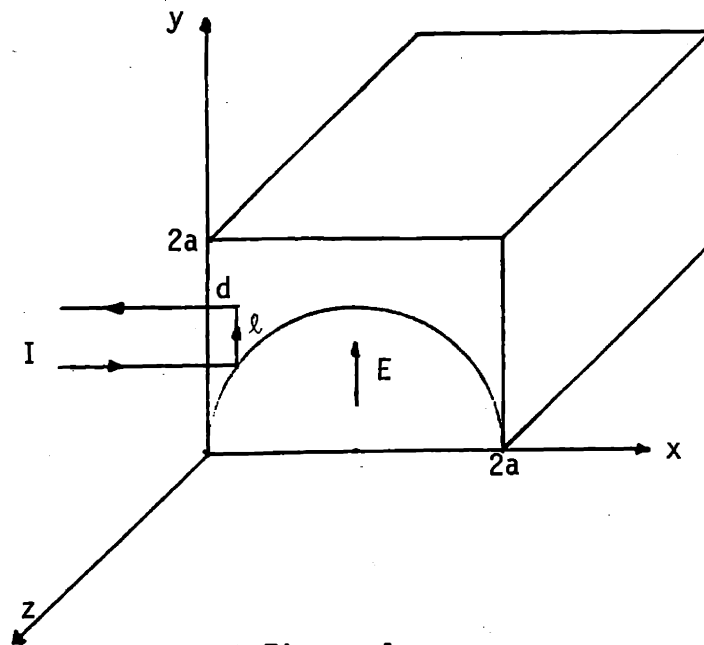


Figure 1

## Experimental Radiation Resistance

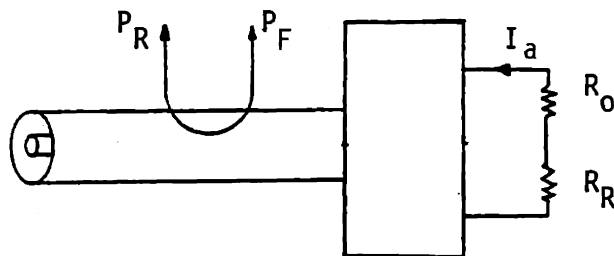


Figure 2

### IV-1.3. Experimental radiation resistance

For typical loop antenna coupling systems, the real radiation resistance,  $R_R$ , is usually combined with a large reactance. To ease  $R_R$  measurements, and to allow efficient transfer of power between the transmitter and the antenna, a matching network system transforms the complex antenna load  $R_R + iX$  into a real load of about the transmission line impedance (Figure IV-1.2.(2)). By measuring the antenna current with simple current loops inside the antenna, and the incoming and reflected power with directional couplers, we can use conservation of power to write:

$$(1) \quad P = I_{\text{rms}}^2 R$$

$$(2) \quad P_{\text{forward}} - P_{\text{reversed}} = I_{\text{rms}}^2 (R_{\text{losses}} + R_{\text{plasma}})$$

Coupling structure losses are easily measured by observing the loading resistance without plasma when the wave cannot propagate in the tokamak. Antenna matching system losses of the order of .5 ohms are common, and thus plasma radiation resistances of at least several ohms are necessary for efficient coupling.

#### IV-2. Cold Plasma Approximation and Cartesian Dispersion Relations.

The next step is to produce a rigorous, quantitative evaluation of the dispersion relation for the high density regime. Many approximations<sup>7</sup> are made, and must be justified quantitatively in view of the accuracy needed later.

The three main steps are the formulation of the dielectric tensor, wave equation, and dispersion relation<sup>7,45</sup>. All three are evaluated at high and low density and for the several harmonic numbers used in the experiment. Both branches, slow and fast<sup>7,22</sup>, of the dispersion relation are also investigated over a range of wave  $k_{||}$ ,  $k_{\perp}$  and frequency, and plasma parameters and profiles.

##### IV-2.1. Cold dielectric tensor wave equation and dispersion relation

To derive the general cold dielectric tensor and wave equation, we proceed exactly as in Appendix 1, except that we let  $E_z$  and  $E_y$  be variables as well as arbitrary  $\vec{k}$ .<sup>45</sup>

Assuming first order quantities proportional to

$$(1) \quad e^{i(\vec{k} \cdot \vec{r} - \omega t)}$$

and a current density

$$(2) \quad \vec{J} = \sum_k n_k z_k \epsilon_k e \vec{v}_k$$

where

$k$  = electrons, ions

$z$  = charge magnitude

$\epsilon$  = charge signs

the equation of motion is

$$(3) \quad m_k \frac{d\vec{v}_k}{dt} = z_k \epsilon_k e (\vec{E} + \frac{\vec{v}_k}{c} \times \vec{B})$$

For an ion, (3) becomes

$$(4) \quad v_x = \frac{e}{-i\omega M} (E_x + v_y \frac{B_o}{c})$$

$$(5) \quad v_y = \frac{e}{-i\omega M} (E_y - v_x \frac{B_o}{c})$$

Substituting (5) → (4)

$$(6) \quad v_x = \frac{\frac{i\omega c i^c}{\omega B_o} E_x - \frac{\omega c i^2 c}{\omega^2 B_o} E_y}{1 - \frac{\omega c i^2}{\omega^2}}$$

$$(7) \quad v_y = \dots$$

and the expressions become overwhelmingly complicated as other species are added.

The problem can be greatly simplified by separating the components into left and right hand components. Thus, still being totally general, we assume

$$(8) \quad v_{\mp} = v_x \mp i\epsilon v_y \quad \text{and} \quad E_{\mp} = E_x \mp i\epsilon E_y$$

Equation (4) and (5) can now simply be added giving

$$(9) \quad v_{k\mp} = \frac{ic}{B_o} \frac{\epsilon_k \omega_{ck}}{\omega \pm \omega_{ck}} E_{\mp}$$

and just as before

$$(10) \quad v_{k,z} = \frac{ic}{B_o} \frac{\epsilon \omega_{ck}}{\omega} E_z$$

Defining the dielectric tensor

$$(11) \quad \vec{D} = \vec{K} \cdot \vec{E} = \vec{E} + \frac{4\pi i \vec{J}}{\omega}$$

we need now only to substitute (9) and (10) in (2), and then finally into (11) to get the full, general cold plasma dielectric tensor. An important property of tensors is that they can simply be added. The total dielectric tensor can be represented as the sum of the free space tensor, (an identity diagonal tensor), the ion current tensor, and the electron current tensor. Each of the charge particle tensors have on-diagonal polarization drift terms ( $K_{xx} = K_{yy}$ ), and off-diagonal  $\vec{E} \times \vec{B}$  drift terms ( $K_{xy} = -K_{yx}$ ), and of course, an inertia term along  $B_0$  ( $K_{zz}$ ).

When solving for a wave, we must always use the total dielectric tensor, but once the fields are found, we can look at each of the tensors individually to see what the velocities are. For example, two opposite cancelling currents do not affect the wave, but there is still kinetic energy stored in these velocities. Furthermore, each species tensor can be divided into a reactive part and a dissipative part. This formulation is particularly attractive in complicated situations near singularities, where the absorption mechanism is not well understood, and one wants to know which species are heated.

Furthermore, we will use the widely accepted notation introduced by Stix<sup>45</sup>.

$$(12) \quad \vec{K} \cdot \vec{E} = \begin{bmatrix} S & -iD & 0 \\ iD & S & 0 \\ 0 & 0 & P \end{bmatrix} \cdot \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix}$$

where

$$(13) \quad S = \frac{R + L}{2}$$

$$(14) \quad D = \frac{R - L}{2}$$

$$(15) \quad R = 1 - \sum_k \frac{\pi k^2}{\omega^2} \left( \frac{\omega}{\omega + \epsilon \omega_{ck}} \right)$$

$$(16) \quad L = 1 - \sum_k \frac{\pi k^2}{\omega^2} \left( \frac{\omega}{\omega - \epsilon \omega_{ck}} \right)$$

$$(17) \quad P = 1 - \sum_k \frac{\pi k^2}{\omega^2}$$

The R and L notations stand for right and left hand polarized. If R and L are the same as in the very low frequency regime, then D for difference is zero, and the wave is linearly polarized as we noted in Appendix 1. If R and L are not the same, we generally have an elliptically polarized wave.

The wave equation and dispersion relation follow easily as before from Maxwell's equations.

$$(18) \quad \nabla \times \vec{E} = -\frac{1}{c} \frac{\partial \vec{B}}{\partial t}$$

$$(19) \quad \nabla \times \vec{B} = \frac{4\pi \vec{J}}{c} + \frac{1}{c} \frac{\partial \vec{E}}{\partial t} = \frac{1}{c} \frac{\partial \vec{D}}{\partial t}$$

$$(20) \quad \vec{k} \times (\vec{k} \times \vec{E}) + \frac{\omega^2}{c^2} \vec{k} \cdot \vec{E} = 0$$

Defining a refractive index

$$(21) \quad \vec{n} = \frac{\vec{k}c}{\omega}$$

we have



$$(22) \quad \vec{n} \times (\vec{n} \times \vec{E}) + \vec{k} \cdot \vec{E} = 0$$

$$(23) \quad \begin{bmatrix} S - (n_y^2 + n_z^2) & -iD + n_x n_y & n_x n_z \\ iD + n_x n_y & S - (n_x^2 + n_z^2) & n_y n_z \\ n_x n_z & n_y n_z & P + (n_x^2 + n_y^2) \end{bmatrix} \cdot \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} = 0$$

We can usually choose our coordinate system so as to have  $n_y = 0$ , so that

(23) becomes

$$(24) \quad \begin{bmatrix} S - n_z^2 & -iD & n_x n_z \\ iD & S - n^2 & 0 \\ n_x n_z & 0 & P - n_x^2 \end{bmatrix} \cdot \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} = 0$$

Setting this determinant equal to zero gives the general dispersion relation.

#### IV-2.2. Zero electron mass dispersion relation and polarization

We now evaluate the dielectric tensor elements in the  $\omega \ll \omega_{ce}$  regime for a two component plasma.

$$(1) \quad R = 1 - \frac{\pi_e^2}{\omega(\omega - \omega_{ce})} - \frac{\pi_i^2}{\omega(\omega + \omega_{ci})}$$

We can make several important simplifications since

$$(2) \quad \begin{aligned} \pi_i^2 &\ll \pi_e^2 \\ \omega &\ll \omega_{ce} \end{aligned}$$

$$(3) \quad \frac{\pi_e^2}{\omega_{ce}} = \frac{\pi_i^2}{\omega_{ci}}$$

then

$$(4) \quad R \approx 1 + \frac{\pi_i^2}{\omega_{ci}^2(1 + \Omega)}$$

where we defined

$$(5) \quad \Omega = \frac{\omega}{\omega_{ci}}$$

For ICRF, the error in the tensor element will be of order  $\frac{\omega}{\omega_{ce}} \approx \frac{m}{M} \ll 1$ , a good approximation for any practical application.

Similarly,

$$(6) \quad L = 1 + \frac{\pi_i^2}{\omega_{ci}^2(1 - \Omega)}$$

$$(7) \quad S = \frac{R + L}{2} = 1 + \frac{\pi_i^2}{\omega_{ci}^2} \frac{1}{1 - \Omega^2}$$

$$(8) \quad D = \frac{R - L}{2} = \frac{\pi i^2}{\omega_{ci}^2} \frac{-\Omega}{1 - \Omega^2}$$

$$(9) \quad P = 1 - \frac{\pi e^2}{\omega^2} - \frac{\pi i^2}{\omega^2} \approx 1 - \frac{\pi e^2}{\omega^2}$$

Since  $\omega$  is of order  $\omega_{ci}$ , and at high density

$$\frac{\pi i^2}{\omega_{ci}^2} \text{ is of order } 10^3$$

$$\frac{\Omega}{1 - \Omega^2} \text{ is of order } 1$$

$$\frac{\pi e^2}{\omega^2} \text{ is of order } 10^6$$

we can, to a very good approximation, write

$$(10) \quad S = \frac{\pi i^2}{\omega_{ci}^2} \frac{1}{1 - \Omega^2}$$

$$(11) \quad D = -\Omega S$$

$$(12) \quad P = -\frac{\pi e^2}{\omega^2}$$

For the moment, let's assume that we are looking for waves of the Alfvén type with  $n_x \approx n_z \approx 40$  as we saw in Appendix 1. The third line of the wave tensor equation then reads

$$(13) \quad \begin{array}{l} n_x n_z E_x + (P - n_x^2) E_z = 0 \\ 40^2 E_x - 10^6 E_z = 0 \end{array}$$

and thus  $E_z \ll E_x$ , and  $E_z$  would have a negligible effect on the first two lines of the tensor. For practical purposes, we are now left with the 2 x 2 tensor equation

$$(14) \begin{bmatrix} S - n_{\parallel}^2 & -iD \\ iD & S - (n_{\perp}^2 + n_{\parallel}^2) \end{bmatrix} \cdot \begin{bmatrix} E_x \\ E_y \end{bmatrix} = 0$$

Since this amounts to assuming  $P \rightarrow -\infty$ , or simply  $m_e \rightarrow 0$ , this formulation is usually referred to as the zero electron mass approximation, and requires  $E_z = 0$ . Solving the determinant

$$(15) (S - n_{\parallel}^2)(S - (n_{\perp}^2 + n_{\parallel}^2)) - D^2 = 0$$

$$(16) n_{\perp}^2 = \frac{(S - n_{\parallel}^2)^2 - D^2}{S - n_{\parallel}^2}$$

Defining an Alfvén refractive index

$$(17) \vec{N} = \vec{n} \frac{\omega_{ci}}{\pi_i}$$

and multiplying both sides of (16) by  $\omega_{ci}^2/\pi_i^2$ , we have <sup>7</sup>

$$(18) N_{\perp}^2 = (A - N_{\parallel}^2) + \frac{A(1 - A)}{(A - N_{\parallel}^2)}$$

where we defined

$$(19) A = \frac{1}{1 - \Omega^2}$$

Figures 1-4 are plots of  $N_{\perp}^2$  vs  $N_{\parallel}^2$ , and  $N_{\perp}$  vs  $N_{\parallel}$  for different regimes. Several points are to be noted. First, one distinguishes clearly the fast wave from the slow wave, since the fast wave has a refractive index several times smaller than the slow wave. Second, the slow wave is evanescent for  $\Omega > 1$ , but the fast wave propagates both above and below  $\omega_{ci}$ . Third, for propagation across  $B_0$ ,  $k_{\parallel} = 0$ , then  $N_{\perp} = 1$  independently of the frequency regime. Fourth, for fast wave propagation along B,  $N_{\perp} = 0$ , and

$$(20) N_{\parallel} = \frac{1}{\sqrt{\Omega^2 + 1}}$$

so one can then write the approximate simple dispersion relation

$$(21) \quad k_{\perp}^2 + (\Omega + 1) k_{\parallel}^2 = k_A^2$$

From the first line of the dispersion relation (14),

$$(22) \quad E_x = \frac{D}{S - n_{\parallel}^2} iE_y$$

we can calculate the ratio of the left hand  $E_{+}$  to the right hand  $E_{-}$  polarized electric field components.

$$(23) \quad \frac{E_{+}}{E_{-}} = \frac{E_x + iE_y}{E_x - iE_y} = \frac{D + S - n_{\parallel}^2}{D - S + n_{\parallel}^2} = \frac{R - n_{\parallel}^2}{-L + n_{\parallel}^2} =$$

$$= \frac{\frac{1}{\Omega + 1} - N_{\parallel}^2}{\frac{1}{\Omega - 1} + N_{\parallel}^2}$$

For perpendicular propagation ( $N_{\parallel} = 0$ )

$$(24) \quad \frac{E_{+}}{E_{-}} = \frac{\Omega - 1}{\Omega + 1} = .2 \quad \text{for } \Omega = 1.5$$

.33	"	2
.6	"	4
.78	"	8

For parallel propagation (20),  $E_{+}/E_{-} = 0$  independently of  $\Omega$

If we allow  $k_y \neq 0$  then (IV-2.1(23))

$$(25) \quad E_x = \frac{D + i n_x n_y}{S - (n_y^2 + n_z^2)} iE_y$$

and

$$(26) \quad \frac{E_{+}}{E_{-}} = \frac{\frac{1}{\Omega + 1} + i N_x N_y - N_y^2 - N_z^2}{\frac{1}{\Omega - 1} + i N_x N_y + N_y^2 + N_z^2}$$

and the polarization is a function of all three components of  $\vec{k}$ .

# Fast and Slow Wave Dispersion Relations

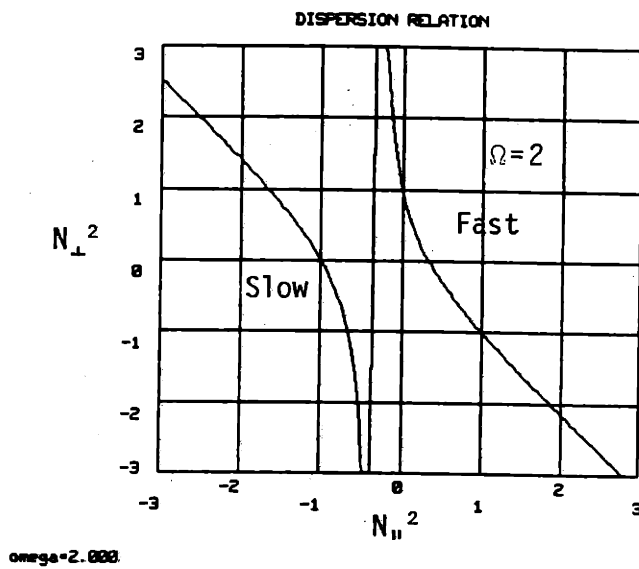


Figure 1

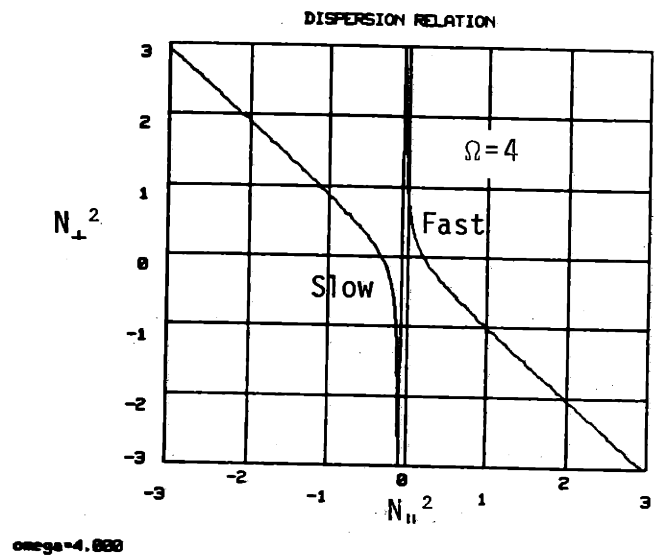


Figure 2

# Fast and Slow Wave Dispersion Relations

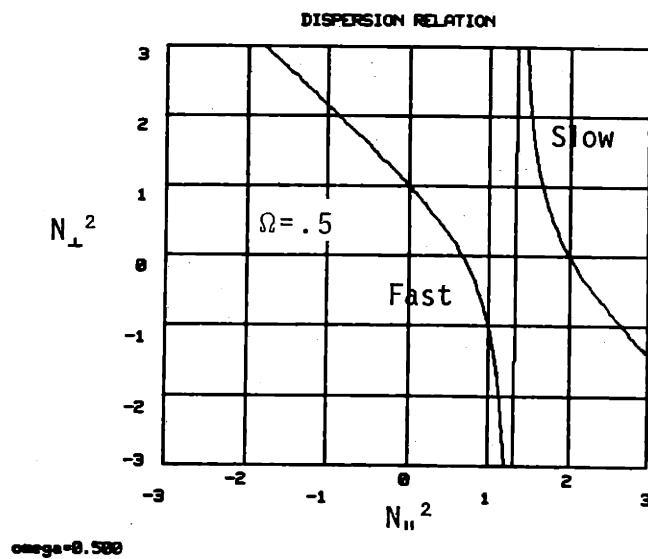


Figure 3

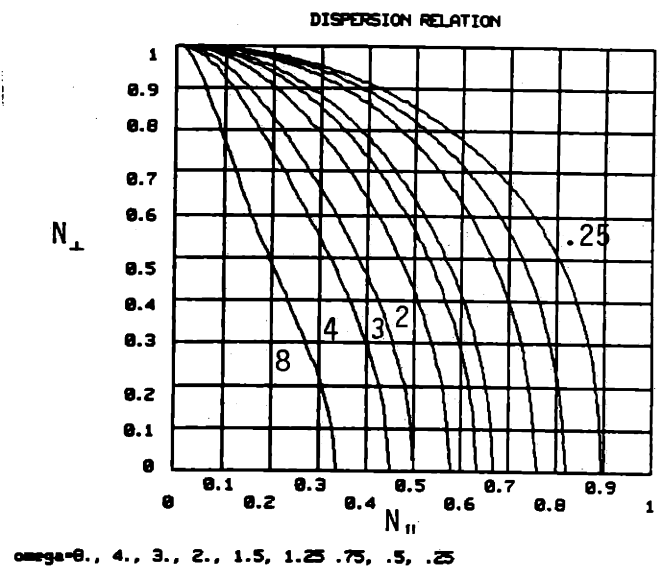


Figure 4

### IV-2.3. Inhomogeneous Cartesian waveguide and WKB

We now assume realistic Alcator density and temperature profiles of the form (Figures 1, 2, 3).<sup>28-29</sup>

$$(1) \quad n = n_0 \left[ 1 - \left( \frac{r}{9.7} \right)^2 \right] \quad 0 < r < 9 \text{ cm}$$

$$= n_0 \cdot .139 e^{-.12(r/9-1)} \quad 9 < r < 12.5 \text{ cm}$$

$$(2) \quad T = T_0 e^{-r^2/a_T^2} \quad 0 < r < 12.5 \text{ cm}$$

where

$$a_T^2 = \frac{3}{2} a_1^2 \frac{q_0}{q_1}, \quad a_1 = 9 \text{ cm}, \quad q_0 \approx .9, \quad q_1 \approx 5$$

Fixing  $k_{||}$  one can now calculate  $k_{\perp}$  as a function of  $r$  using eq. IV-2.2(16). Figure 5 shows  $k_{\perp}^2$  as a function of  $r$  for  $k_{||} = 0, .2, .4, .6, .8, 1.0, 1.2/\text{cm}$ , and for the standard condition

$$(3) \quad f_0 = 200 \text{ MHz}$$

$$n_0 = 5 \times 10^{14} \text{ Hydrogen}$$

$$\Omega = 2$$

One must note that, while in the center, the dispersion relation is of the type

$$(4) \quad k_{\perp}^2 + (1 + \Omega) k_{||}^2 = k_A^2$$

which allows  $k_{\perp}$  to be of order unity; the dispersion relation in the near vacuum edge is of the type

$$(5) \quad k_{\perp}^2 + k_{||}^2 = k_C^2 = 1.7 \times 10^{-3}/\text{cm}^2$$

Thus

$$(6) \quad k_{\perp} = ik_{||}$$

and we have an evanescent layer at the edge. Making  $k_{||}$  larger not only makes the layer ( $d_e$ ) thicker, but also decreases the decay length. A pessi-

mistic example can be estimated from Figure 5 with

$$(7) \quad k_{||} = .6/\text{cm}, d_e = 4 \text{ cm}$$

$$e^{-kx} = e^{-4 \times .6} = .09$$

Of course, one could also propagate.

$$(8) \quad k_{||} = .2, d_e \approx 1 \text{ cm}$$

$$e^{-.2} \approx .8$$

One can already see the difficulty in evaluating  $\beta_a = \frac{E_a}{E_0}$  in the radiation resistance, due to the evanescent edge effect alone. This effect is much more pronounced in Alcator, since our antenna is narrow and thus has a wide  $k_{||}$  spectrum, the central density is very high and thus can support a very large  $k_{||}$ , and the edge vacuum layer thickness is several centimeters. For low density tokamaks, typical parameters are

$$(9) \quad k_{||} \approx .05/\text{cm}, d_e \approx 10 \text{ cm}$$

$$e^{-.05 \times 10} \approx .6$$

and almost complete tunneling occurs.

To evaluate the eigenmode wave field shape, we must use a numerical integration technique.

Assuming  $k_y = 0$  and a wave field of the form

$$(10) \quad E(x, z, t) = E(x) e^{i(k_{||} z - \omega t)}$$

we have from Faraday's law

$$(11) \quad -k_{||} E_y = \frac{\omega}{c} B_x$$

$$(12) \quad k_{||} E_x = \frac{\omega}{c} B_y$$

$$(13) \quad \frac{\partial E_y}{\partial x} = \frac{i\omega}{c} B_z$$



and, from the x and y components of Ampère's law, we have

$$(14) \quad k_{||} B_y = \frac{\omega}{c} [S E_x - i D E_y]$$

$$(15) \quad i k_{||} B_x - \frac{\partial B_z}{\partial x} = \frac{-i\omega}{c} [i D E_x + S E_y]$$

Substituting (11, (12→14), 13→15), we have

$$(16) \quad \frac{\partial E_y^2}{\partial x^2} = \frac{-\omega^2}{c^2} \left[ \frac{-D^2}{S - n_{||}^2} + S - n_{||}^2 \right] E_y$$

which is recognized (from IV-2.2(16)) as

$$(17) \quad -k_{\perp}(x)^2 E(x) = \frac{\partial^2 E(x)}{\partial x^2}$$

Both sides of (17) can be integrated twice with respect to x

$$(18) \quad - \int_0^x \int_0^x k^2(x) E(x) d^2 x = \int_0^x \int_0^x \frac{\partial^2 E(x)}{\partial x^2} d^2 x = \int_0^x \left[ \frac{\partial E}{\partial x} - \frac{\partial E}{\partial x} \right]_{x=0} dx$$

$$= E(x) - E(x) \Big|_{x=0} - \left[ \frac{\partial E(x)}{\partial x} \right]_{x=0} x$$

and assuming a symmetric mode

$$(19) \quad E(x) = 1 - \int_0^x \int_0^x k^2(x) E(x) d^2 x$$

Figure 9 is a simplified block diagram of how one can easily calculate  $n(x)$ ,  $k^2(x)$ , and  $E_y(x)$  using a first order integration scheme. Although very

crude, it gives excellent qualitative and quantitative results. Figure 6 is a typical example with  $k_{||} = .63$ ,  $n = 5 \times 10^{14}$ . To arrive at a physically valid solution, one must adjust  $k_{||}$  so as to have  $E(x)|_{\text{wall}} = 0$ . This is a tedious trial and error process that will be circumvented in Section IV-5.4. One can find the asymmetrical solutions by changing the  $E$  and  $\frac{\partial E}{\partial r}$  initial boundary conditions. Different radial mode numbers ( $\mu$ ) can also be found by drastically changing  $k_{||}$ , and looking for a new solution.

Another way to solve equation (17) is to use WKB theory.

To zeroth order, we can write<sup>16</sup>

$$(20) \quad E(x) = E_0 e^{i\langle k \rangle \cdot x}$$

to first order

$$(21) \quad E(x) = E_0 e^{i \int k(x) \cdot dx}$$

and to second order

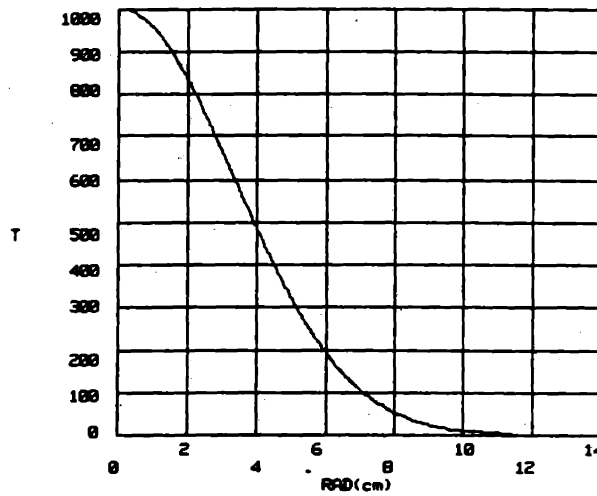
$$(22) \quad E(x) = E_0 \frac{e^{i \int k(x) \cdot dx}}{\sqrt{k}}$$

WKB theory, although analytically appealing, must be used carefully, especially since the wavelength is of the same order as the density scale length. Zero order WKB is obviously very crude, and is only useful either along  $B_0$  or with flat density profiles. First order WKB (Fig. 7) is in good agreement with the exact solution, as it treats correctly the evanescent layer. Second order WKB, which one might expect to be better, has a singularity when  $k_{\perp} = 0$ , and is of no use for our purposes (Figure 8).

In Section IV-4 we will see that collisionless damping power deposition is proportional to either the temperature, (Fig. 1) or to some exponential factor of the temperature. These profiles are extremely peaked, with half power width of only a few centimeters. We then should expect the

collisionless damping power to be deposited near the center of the plasma, almost independently of any reasonable electric field profile.

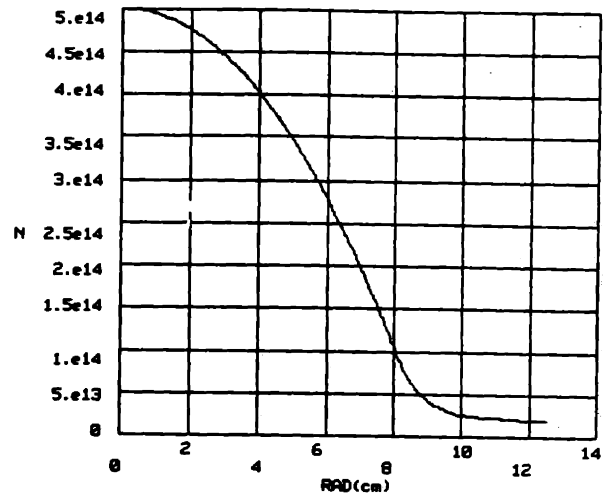
Temperature Profile



nprof= 1 t0= 1000.00 nhat=.50000E+15  
f=.20000E+09 kpar(1)= 0.63032 mu= 1.00 omega= 2.00

Figure 1

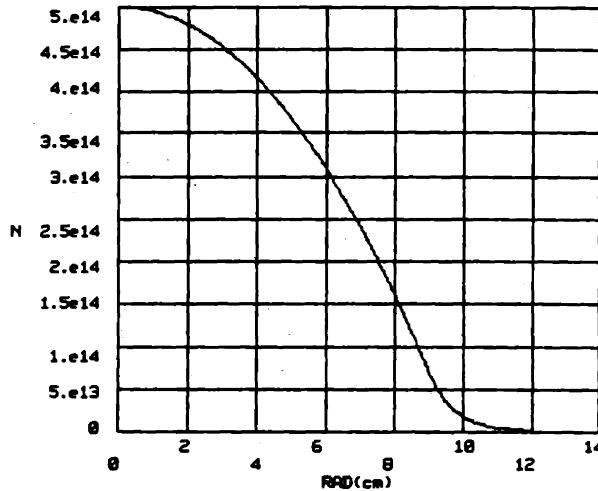
TFR Type Density Profile



nprof= 1 t0= 1000.00 nhat=.50000E+15  
f=.20000E+09 kpar(1)= 0.63032 mu= 1.00 omega= 2.00

Figure 2

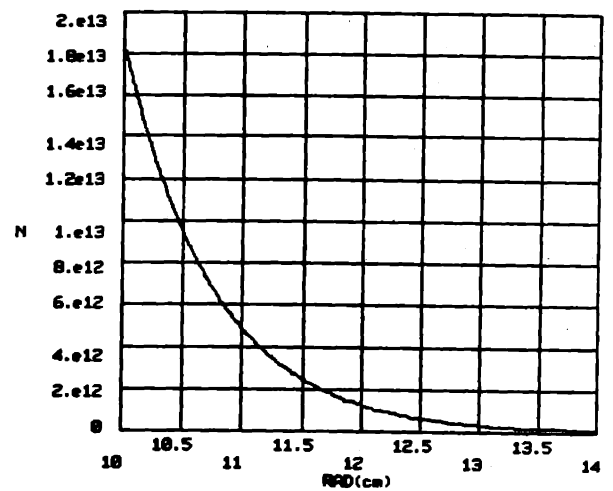
Alcator Type Density Profile



nprof= 2 t0= 1000.00 nhat=.50000E+15  
f=.20000E+09 kpar(1)= 0.67200 mu= 1.00 omega= 2.00

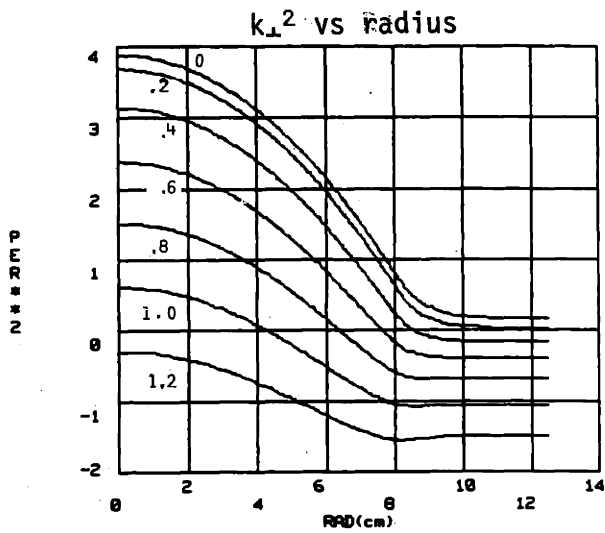
Figure 3

Edge Density Profile



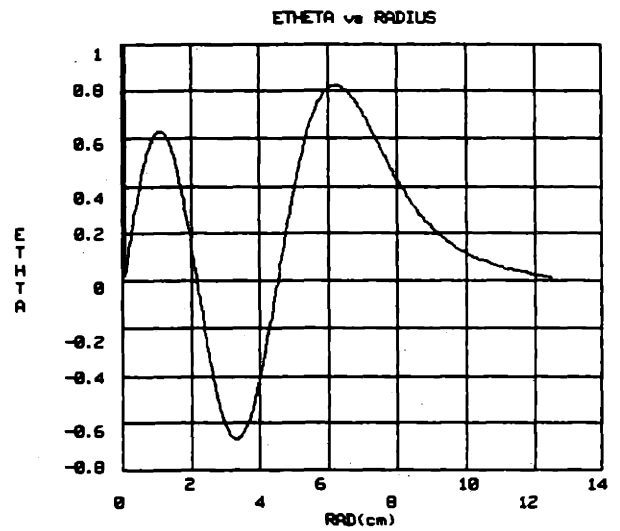
nprof= 2 t0= 1000.00 nhat=.50000E+15  
f=.20000E+09 kpar(1)= 0.67200 mu= 1.00 omega= 2.00

Figure 4



nprof= 1 t0= 1000.00 nhat=.50000E+15  
 f=.20000E+09 kpar(1)= 0.63032 mu= 1.00 omega= 2.00  
 Kpar= 0., .2, .4, .6, .8, 1.0, 1.2

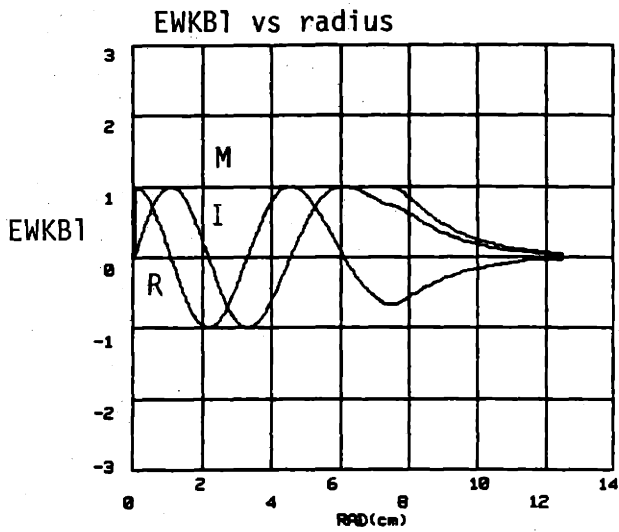
Figure 5



nprof= 1 t0= 1000.00 nhat=.50000E+15  
 f=.20000E+09 kpar(1)= 0.63032 mu= 1.00 omega= 2.00

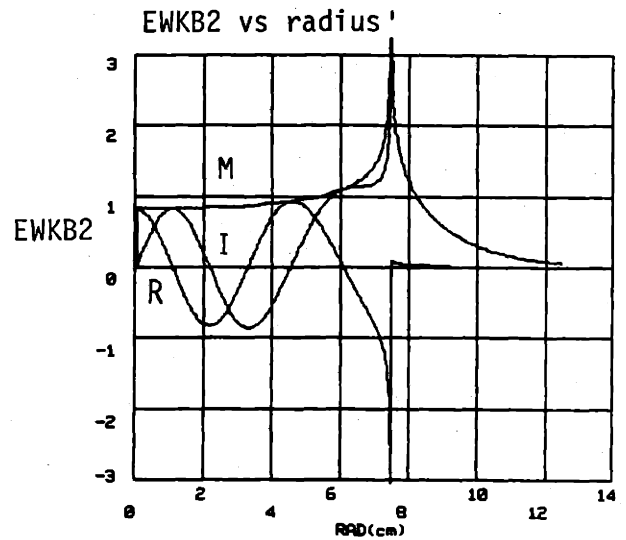
Figure 6

### First and Second Order WKB



nprof= 1 t0= 1000.00 nhat=.50000E+15  
 f=.20000E+09 kpar(1)= 0.63032 mu= 1.00 omega= 2.00

Figure 7



nprof= 1 t0= 1000.00 nhat=.50000E+15  
 f=.20000E+09 kpar(1)= 0.63032 mu= 1.00 omega= 2.00

Figure 8

# Block Diagram of Heuristic Code

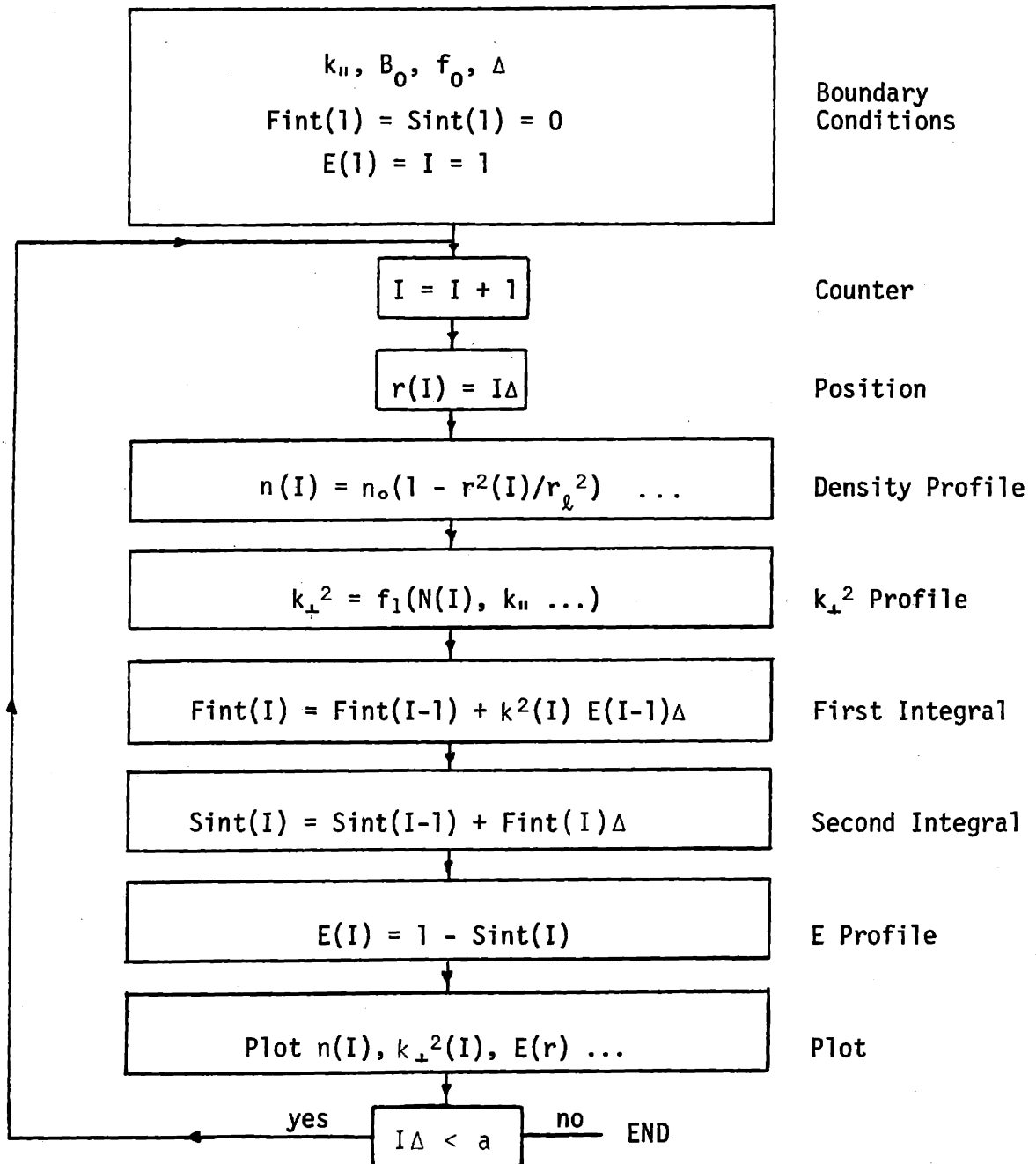


Figure 9

#### IV-2.4. Finite electron mass, fast and slow wave dispersion relations

If we now keep the full  $3 \times 3$  dispersion relation tensor with  $k_y = 0$  and solve for the determinant, we have

$$(1) \quad (S - n_{||}^2) [(S - (n_{\perp}^2 + n_{||}^2)) (P - n_{\perp}^2)] - D^2 (P - n_{\perp}^2) - n_{\perp} n_{||} (S - n_{\perp}^2 - n_{||}^2) n_{\perp} n_{||} = 0$$

which, after some simple algebraic manipulations, can be written in the form

$$(2) \quad a n_{\perp}^4 + b n_{\perp}^2 + c = 0$$

where  $a = 1$

$$b = - \left( \frac{P}{S} + 1 \right) (S - n_{||}^2) + \frac{D^2}{S}$$

$$(3) \quad c = \frac{P}{S} [(S - n_{||}^2)^2 - D^2]$$

Equation (2) has the fast wave root

$$(4) \quad n_{\perp+}^2 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \approx \frac{b}{2} \left( -1 + 1 - \frac{2c}{b^2} \right)$$

where we made the approximation  $\sqrt{1 - \epsilon} \approx 1 - \epsilon/2$ , since, in our regime  $b^2 \gg 4ac$

and thus

$$(5) \quad n_{\perp+}^2 \approx -\frac{c}{b} \approx \frac{(S - n_{||}^2)^2 - D^2}{S - n_{||}^2}$$

which is the same as we found by solving the  $2 \times 2$  determinant.

Similarly, we have the new slow wave root<sup>22</sup>

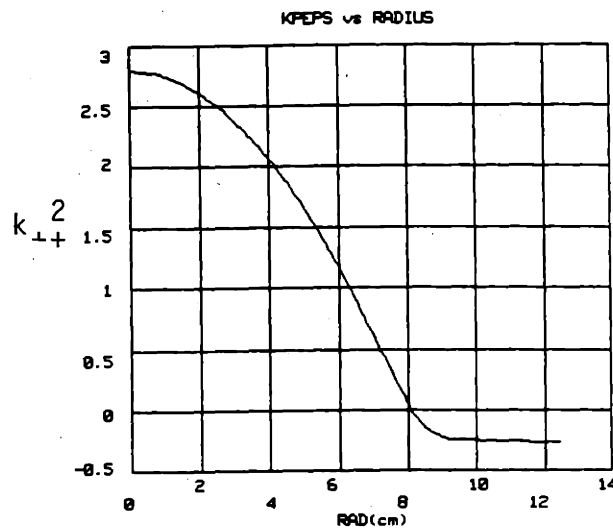
$$(7) \quad n_{\perp-}^2 \approx n_{\perp-}^2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a} \approx -b \approx \frac{P}{S} (S - n_{||}^2)$$

For  $\Omega > 1$  and reasonable densities,  $n_{\perp-}^2$  is of order  $P = \frac{-\pi^2 e^2}{\omega^2}$ , which means an extremely evanescent field. For this reason we are now confident that

only the fast wave can propagate in Alcator. Figures 1 and 2 are the fast and slow wave  $k_{\perp}^2$  profiles for standard condition  $B_0 = 66$  kG,  $f_0 = 200$  MHz,  $k_{\parallel} = .5/\text{cm}$  and  $5 \times 10^{14} / \text{cm}^3$  hydrogen central density.

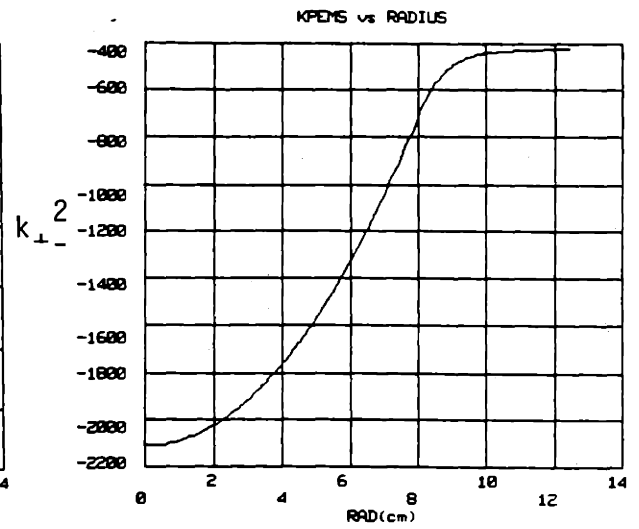


# Fast and Slow Waves at 200 MHz



H<sub>2</sub>, 66 kG, 200 MHz

Figure 1



f=.20000E+09 oma= 2.000 b0=.65587E+05 kpar=0.5000 mu=1.00  
 nhai=.5000E+15 pp= 9.000 rmax=12.500

Figure 2

#### IV-2.5. 1/R Magnetic field, two ion species and the two ion-ion hybrid resonance

We now introduce a 1/R magnetic field, so that

$$(1) \quad B = B_0 \frac{1}{1 + r/R_0}$$

and let  $\omega_{ci}$ ,  $\omega_{ce}$ ,  $\pi_i$ ,  $\pi_e$ , be functions of  $r$ . At this point, we also introduce several ion species, and numerically calculate  $R$ ,  $L$ ,  $S$ ,  $D$ , and  $P$  directly in their unsimplified general form (IV-2.1.(13)-(17)). Figures 1 and 2 show the fast and slow wave for one ion species at second harmonic with 1/R magnetic field. We notice a slight shift of the  $k_{\perp+}^2$  maximum towards the outside, which is simply explained by a lower magnetic field ( $k_A \propto 1/B$ ). The effect of this is to make the wave effective major radius some 3% larger than  $R_0$ .

For a mixed hydrogen and deuterium plasma with

$$(2) \quad \omega_0 \approx \omega_{cH} \approx 2 \omega_{cD}$$

and a smaller concentration of hydrogen than deuterium

$$(3) \quad n_{0H} + n_{0D} = n_e$$

$$(4) \quad \frac{n_H}{n_D} = \alpha < 1$$

we will encounter a singularity in the fast wave when

$$(5) \quad n_{\perp+}^2 = \frac{(S - n_{\parallel}^2)^2 - D^2}{S - n_{\parallel}^2} \rightarrow \infty$$

or simply

$$(6) \quad S - n_{\parallel}^2 = 0$$

a condition often called the Two Ion-Ion Hybrid (TIIH) resonance. Figures 3 and 4 show wave profiles for  $k_{\parallel} = .3/\text{cm}$ ,  $f_0 = 90 \text{ MHz}$ ,  $n_H = 2.5 \times 10^{13}/\text{cm}^3$ ,

$n_D = 5 \times 10^{14}/\text{cm}^3$ , and  $B_0 = 60 \text{ kG}$ . The  $k_{++}^2$  profile is now grossly non-symmetric since the resonance is critically dependent on the  $1/R$  magnetic field.

Neglecting the small electron contribution, equation (6) becomes

$$(7) \quad n_{ii}^2 - 1 + \frac{\pi_D^2}{\omega^2 - \omega_{cD}^2} + \frac{\pi_H^2}{\omega^2 - \omega_{cH}^2} = 0$$

and for  $n_{ii}^2 = 0$  and high density

$$(8) \quad \frac{\pi_H^2}{\pi_D^2} \approx - \frac{\omega^2 - \omega_{cH}^2}{\omega^2 - \omega_{cD}^2}$$

Combining (1), (2), (4) and (8), we have the simple formula

$$(9) \quad \Omega_H = \sqrt{\frac{1 + .5\alpha}{1 + 2\alpha}} = \begin{matrix} .97 & \text{for } \alpha = 5\% \\ .82 & \text{for } \alpha = 20\% \end{matrix}$$

and the hybrid layer is some 1.5 to 6 cm on the high field side of the  $\omega_{ci}$  resonance layer. Of course, for  $\alpha = 0$ , the resonance is right at the  $\omega = \omega_{cH}$  layer.

For very low densities, when  $\pi_i^2 \rightarrow 0$ , equation (7) can only be solved for  $\omega^2 \rightarrow \omega_{cH}^2$ . Thus, for a parabolic density profile, the two ion-ion hybrid resonance layer meets the minority cyclotron layer at the plasma edge independently of  $\alpha$ .

For the general case with finite  $n_{ii}$ , and  $\pi_i$ , we can write equation (7) in the form

$$(10) \quad n_{ii}^2 - 1 + n_e F(R, B_0, \alpha, \omega_0) = 0$$

$$(11) \quad n_e = \frac{1 - n_{ii}^2}{F(R, B_0, \alpha, \omega_0)}$$

Assuming a simple density profile

$$(12) \quad n_e(r) = n_{oe} \left(1 - \frac{r^2}{r_w^2}\right)$$

$$(13) \quad r = r_w \sqrt{1 - n_e(r)/n_{oe}}$$

and a two dimensional Cartesian coordinate system centered on the plasma,

$$(14) \quad r^2 = x^2 + y^2$$

$$(15) \quad y = \sqrt{r^2 - x^2}$$

$$(16) \quad R = R_o + x$$

Equations (11) and (12) can be numerically solved by slowly increasing  $x$  in (16)→(11) starting at  $x = -r_w$ , until the condition

$$(17) \quad 0 < n_e < n_{oe}$$

is met, at which point we have a resonance solution requiring a density  $n_e(r) = n_e$  and radius  $r$  from (13).

Thus, substituting (16) → (11) → (13) → (15),

we have a resonance surface of the form

$$(18) \quad (x,y) \equiv \left\{ x, \sqrt{r_w^2 \left[ 1 - \frac{1 - n_e^2}{n_{oe} F(R_o + x, B_o, \alpha, \omega_o)} \right]} - x^2 \right\}$$

and a vacuum vessel at

$$(19) \quad (x,y) = (x, \sqrt{r_w^2 - x^2})$$

Figures 5 and 6 are typical two ion-ion hybrid resonance surfaces for  $\alpha = .05, .1, .2, .3, .45, 1$  and  $k_{||} = .1, .5 / \text{cm}$ ,  $B_o = 70 \text{ kG}$ , and  $f_o = 97 \text{ MHz}$ . We see that at high density,  $k_{||}$  decreases the distance between the TIIH and cyclotron layers. For reasonable minority concentrations, the TIIH layer<sup>17</sup> will usually start near the dense plasma center and reach out to the cyclotron layer located in the low field side antenna.

# Fast and Slow Waves at 200 MHz

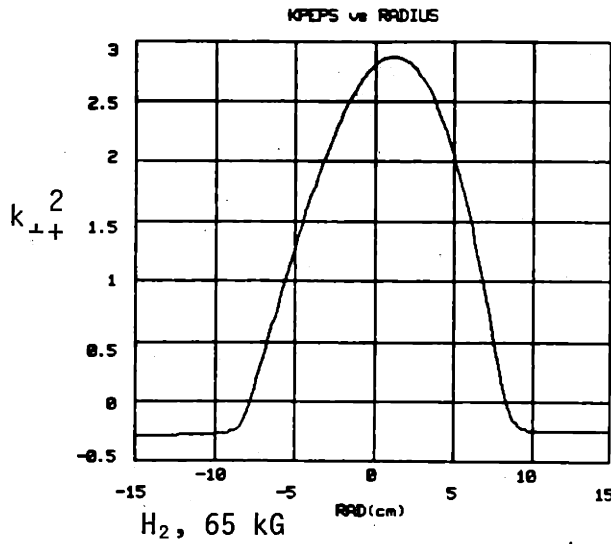
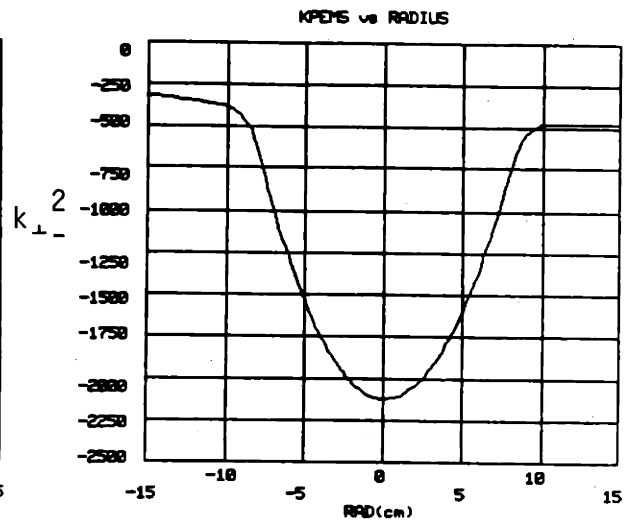


Figure 1



kpar= 0.580 f=.20000E+09 mu= 1.000 mu= 1.000 nprof= 1  
 nahat=.58000E+15 nbhat=.00000E+00 rmax=15.000 rmax=54.000  
 nstep= 200 b0=.65587E+05 pp= 9.00

Figure 2

# Fast and Slow Waves at 90 MHz

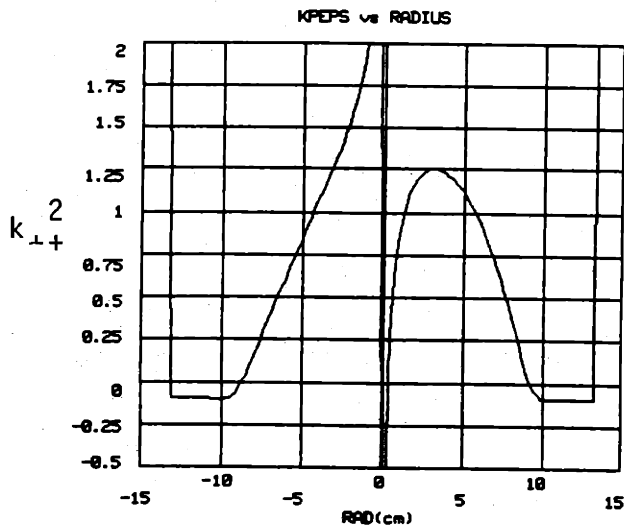
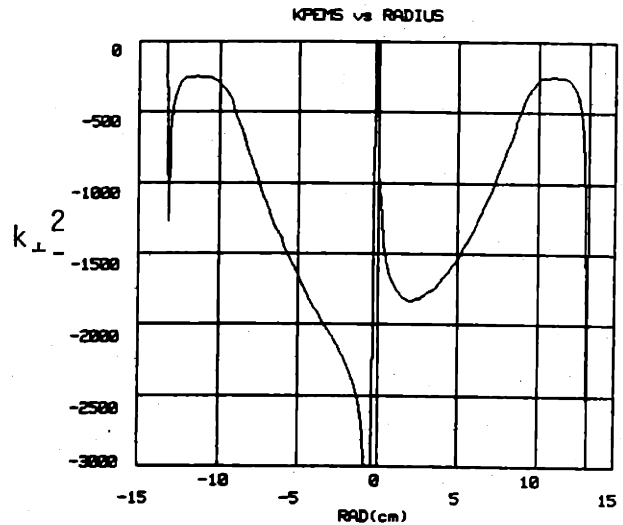


Figure 3



kpar= 0.380 f=.90000E+08 mu= 2.000 mu= 1.000 nprof= 2  
 nahat=.50000E+15 nbhat=.25000E+14 rmax=15.000 rmax=54.000  
 nstep= 200 b0=.60000E+05 pp= 1000.00

Figure 4

# TIH Layer Position for $k_{II} = .5/\text{cm}$

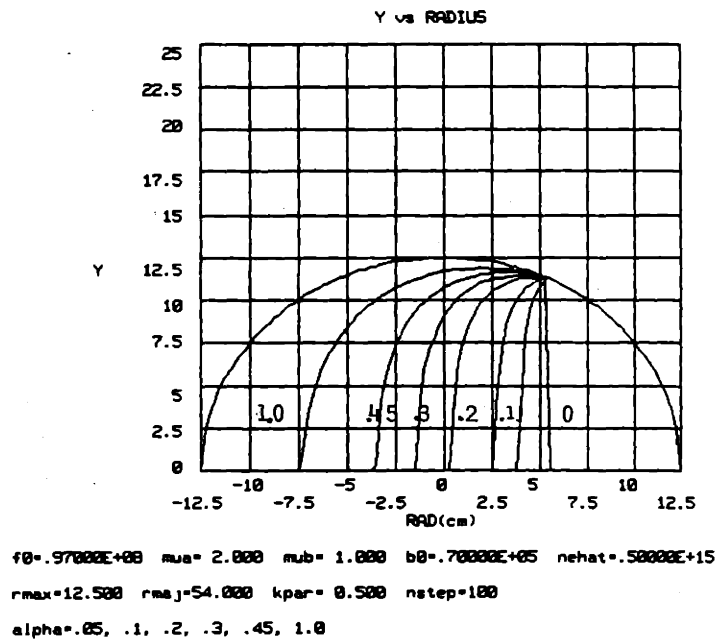


Figure 5

# TIH Layer Position for $k_{II} = .1/\text{cm}$

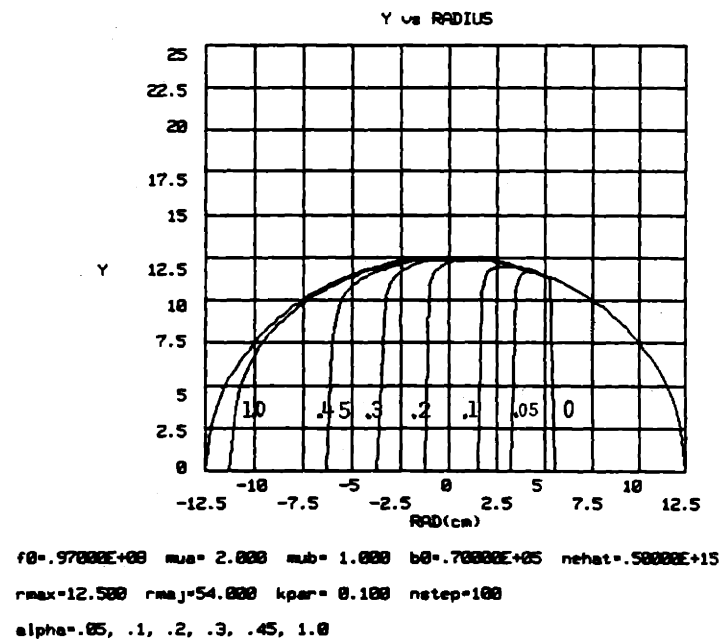


Figure 6

#### IV-2.6. Plasma edge lower hybrid resonance and $E_z$

In Figures IV-2.5 (3-4), we noticed a singularity in  $k_{\perp}^2$  when the density was decreased until the wave frequency was of the order of the ion plasma frequency. Specifically, close examination of Section IV-2.5 shows that this happens near

$$(1) \quad S = 1 + \frac{\pi_i^2}{\omega_{ci}^2} \frac{1}{1 - \Omega^2} \rightarrow 0^+$$

or simply

$$(2) \quad \omega^2 = \pi_i^2 + \omega_{ci}^2$$

From the usual definition of the lower hybrid wave<sup>45</sup>

$$(3) \quad \frac{1}{\omega_{LH}^2} = \frac{1}{\omega_{ci}^2 + \pi_i^2} + \frac{1}{\omega_{ce} \omega_{ci}} = \frac{1}{\omega_{ci}^2 + \pi_i^2} \quad \text{for } \pi_i^2 \ll \omega_{ci} \omega_{ce}$$

which is the same as (2), and we shall call this low density edge mode the lower hybrid resonance field. For the standard second harmonic condition in hydrogen at 200 MHz, this corresponds to  $n_e \approx 7 \times 10^{11}/\text{cm}^3$ . Although very low, this edge density could be obtained with normal density profiles and a central density less than  $10^{14}/\text{cm}^3$ .

In cold plasma theory, we can easily calculate  $E_z$  from the third line of the full (3 x 3) wave tensor

$$(4) \quad n_{\perp} n_{\parallel} E_x + (P - n_{\perp}^2) E_z = 0$$

under normal fast wave circumstances,  $n_{\perp}^2 \approx n_{\parallel}^2 \approx n_A^2 \approx 40^2$ , and  $P \approx 10^6$ , so that

$$(5) \quad \frac{E_z}{E_x} = \frac{-n_{\perp} n_{\parallel}}{P} \approx 10^{-3}$$

and  $E_z$  is only a small perturbation to the overall TE electric field.

(This is not true of the lower hybrid mode, since  $|P|$  is of the order of  $n^2$ ). Nevertheless, since  $S$ ,  $D$  and  $n^2$  are all of the same order, the parallel electron current

$$(6) \quad J_{||} = \frac{\omega}{4\pi i} P E_z \approx \frac{\omega}{4\pi i} n^2 E_{\perp}$$

is of the same order as the perpendicular currents

$$(7) \quad J_{\perp} = \frac{\omega}{4\pi i} (S E_x - iD E_y) \approx \frac{\omega}{4\pi i} n^2 E_{\perp}$$



#### IV-2.7. Fast Wave Energy Density

The total stored energy in the fast wave can be divided into magnetic, electric and kinetic energy components (MKS)

$$(1) \langle W_t \rangle = W_H + W_E + W_K = \frac{1}{2} \mu_0 H^2 + \frac{1}{2} \epsilon_0 E^2 + \sum_k \frac{1}{2} n_k m_k v_k^2$$

From Faraday's law, we can crudely write

$$(2) \quad E \approx \frac{\omega}{k} \mu_0 H \approx \sqrt{\frac{\mu_0}{\epsilon_A}} H$$

$$(3) \quad \frac{W_H}{W_E} \approx \frac{\epsilon_A}{\epsilon_0} \gg 1$$

and the electric field energy can be neglected.

In any wave, energy is transferred back and forth between two or more energy storage mechanisms. A closer look at the different components of Faraday's law

$$(4) \quad \begin{aligned} H_x &= -\frac{k_{||}}{\omega \mu_0} E_y = -N_{||} \sqrt{\epsilon_A / \mu_0} E_y \\ H_y &= \frac{k_{||}}{\omega \mu_0} E_x = N_{||} \sqrt{\epsilon_A / \mu_0} E_x \\ H_z &= \frac{k_{\perp}}{\omega \mu_0} E_y = N_{\perp} \sqrt{\epsilon_A / \mu_0} E_y \end{aligned}$$

shows that for  $k_{||} = 0$ , only  $H_z$  remains, and energy must be transferred between  $W_H$  and  $W_K$ , and

$$(5) \quad W_H = W_K$$

For  $k_{||} \neq 0$ , energy is shuffled between the different magnetic components and the particles velocities, so (5) is no longer valid.

Equation (1) may, in general, be rewritten for a lossless dielectric (CGS)<sup>7</sup>

$$\begin{aligned}
(6) \quad \langle W \rangle_t &= \frac{1}{16\pi} \left[ \vec{B}^* \cdot \vec{B} + \vec{E}^* \cdot \frac{\partial}{\partial \omega} (\omega \vec{K}) \cdot \vec{E} \right] \\
&= \frac{1}{16\pi} \epsilon_A E_y^2 \left\{ \left[ N_{||}^2 (F^2 + 1) + N_{\perp}^2 \right] + \frac{1}{2(1 - \Omega^2)^2} \left[ (1 + F)^2 (1 - \Omega)^2 \right. \right. \\
&\quad \left. \left. + (1 - F)^2 (1 + \Omega)^2 \right] \right\}
\end{aligned}$$

where the first bracket is the magnetic energy, and from the first line of the wave tensor

$$(7) \quad F = \frac{iE_x}{E_y} = \frac{-D}{S - n_{||}^2} = -\frac{1 - \Omega^2}{\Omega} \left[ N_{\perp}^2 + N_{||}^2 - \frac{1}{1 - \Omega^2} \right]$$

Finally, substituting (4) and (7) into (6), we have

$$\begin{aligned}
(8) \quad \langle W \rangle_t &= 2W_H = \frac{1}{8\pi} \epsilon_A E_y^2 && \text{for } k_{||} = 0 \\
&= \frac{1}{8\pi} \epsilon_A \frac{2 + \Omega}{(1 + \Omega)^2} E_y^2 && \text{for } k_{\perp} = 0 \\
&= \frac{1}{8\pi} \epsilon_A .44 E_y^2 && \text{for } k_{\perp} = 0 \text{ and } \Omega = 2 \\
&\approx \frac{1}{8\pi} \epsilon_A E_y^2 && \text{in general}
\end{aligned}$$

### II-3. Homogeneous Plasma Cylindrical Waveguide Field Solution

For simple analytical purposes, the most important model is the homogeneous cylindrical waveguide<sup>45,46,56,60</sup>. The zero electron mass cold plasma dispersion relation<sup>7</sup>, coupled with the electric field boundary conditions at the wall, gives the deterministic equation which in turn, uniquely defines  $k_z$  as a function of  $n_e$ , for the different values of radial and poloidal mode numbers<sup>22</sup>. This, furthermore, fully determines the cylindrical field solutions from which can be calculated the Poynting flux, antenna  $E_\theta$  field, probe signals and the radiation resistance<sup>21</sup>. An estimate of the number of possible propagating modes<sup>93</sup> and mode splitting<sup>22,52,55</sup>, due to the Ohmic current around the torus, is also calculated.

#### IV-3.1. Zero electron mass fast wave dispersion relation and mode cutoff

Proceeding exactly as in the previous chapter, we write Maxwell's equation and the equivalent dielectric tensor in cylindrical coordinates. From Faraday's law we have,

$$(1) \quad \nabla \times \vec{E} = -\frac{1}{c} \frac{\partial \vec{B}}{\partial t} = \frac{i \omega}{c} \vec{B}$$

from Ampère's law,

$$(2) \quad \nabla \times \vec{B} = \frac{1}{c} \frac{\partial \vec{D}}{\partial t} = \frac{-i \omega}{c} \vec{K} \cdot \vec{E}$$

and from the dielectric tensor

$$(3) \quad \vec{K} \cdot \vec{E} = \begin{bmatrix} S & -iD & 0 \\ iD & S & 0 \\ 0 & 0 & P \end{bmatrix} \cdot \begin{bmatrix} E_r \\ E_\theta \\ E_z \end{bmatrix}$$

where in cylindrical coordinates

$$(4) \quad \nabla \times \vec{A} = \begin{vmatrix} \hat{r}/r & \hat{\theta} & \hat{z}/r \\ \partial/\partial r & \partial/\partial \theta & \partial/\partial z \\ A_r & rA_\theta & A_z \end{vmatrix}$$

The basic methodology is to expand (1) and (2) in their  $r$ ,  $\theta$  and  $z$  components using (3) and (4). Then assuming <sup>22</sup>

$$(5) \quad A = A_0 e^{i(m\theta + k_z z - \omega t)}$$

and a TE eigenmode ( $E_z = 0$ ), so that

$$(6) \quad B_z \propto J_m(k_\perp r)$$

we can write an equation of the type

$$(7) \quad f(B_z) = g(E_\theta)$$

and using the boundary condition

$$(8) \quad E_\theta \Big|_{\text{wall}} = 0$$

we have the deterministic equation and the dispersion relation.

We thus proceed with Faraday's Law,

$$(1r) \quad \frac{-\partial E_\theta}{\partial z} = \frac{i \omega B_r}{c}$$

$$(1\theta) \quad \frac{\partial E_r}{\partial z} = \frac{i \omega B_\theta}{c}$$

$$(1z) \quad \frac{1}{r} \left[ \frac{\partial r E_\theta}{\partial r} - \frac{\partial E_r}{\partial \theta} \right] = \frac{i \omega B_z}{c}$$

and Ampere's law

$$(2r) \quad \frac{1}{r} \left[ \frac{\partial B_z}{\partial \theta} - \frac{\partial r B_\theta}{\partial z} \right] = \frac{-i\omega}{c} (S E_r - iD E_\theta)$$

$$(2\theta) \quad \frac{\partial B_z}{\partial r} + \frac{\partial B_r}{\partial z} = \frac{-i\omega}{c} (iD E_r + S E_\theta)$$

and substituting (1\theta) \rightarrow (2r)

$$(9) \quad \frac{i m B_z}{r} + \frac{D \omega}{c} E_\theta = E_r \left( \frac{-i\omega S}{c} + i c \frac{k_{||}^2}{\omega} \right)$$

and (1r) \rightarrow 2\theta

$$(10) \quad \frac{\partial B_z}{\partial r} + E_\theta \left[ \frac{i k_{||}^2 c}{\omega} - \frac{i \omega S}{c} \right] = - \frac{\omega D}{c} E_r$$

Finally, combining (9) and (10) we have

$$(11) \quad \frac{\partial B_z}{\partial r} + \frac{m D \omega^2}{r(c^2 k_{||}^2 - \omega^2 S)} B_z = \left[ \frac{i D^2 \omega^3}{c(c^2 k_{||}^2 - \omega^2 S)} + \frac{i \omega S}{c} - i \frac{k_{||}^2 c}{\omega} \right] E_\theta$$

Using the Boundary condition (8)

$$(12) \quad a \left. \frac{\partial B_z}{\partial r} \right|_{\text{wall}} = - \frac{m D \omega^2}{(c^2 k_{||}^2 - \omega^2 S)} B_z \Big|_{\text{wall}}$$

and substituting (6) into (12), we have the dispersion relation

$$(13) \quad k_{\perp} a J_m'(k_{\perp} a) = \frac{m D \omega^2}{\omega^2 S - c^2 k_{||}^2} J_m(k_{\perp} a)$$

The mode cutoff can be found by setting  $k_{||} = 0$ , and thus we have the simple deterministic equation

$$(14) \quad \frac{\chi J_m'(\chi)}{J_m(\chi)} = -m \Omega$$

where

$$(15) \quad \chi = k_{\perp} a = k_A a$$

For  $m = 0$ ,  $x$  will be simply the zeroes of  $J_0'$ . Using the Bessel function identities<sup>137</sup>

$$(16) \quad J_n'(x) = J_{n-1}(x) - \frac{n J_n(x)}{x}$$

$$(17) \quad J_{-n}(x) = (-1)^n J_n(x)$$

we can calculate the  $\pm 1$  mode cutoffs (Table 1) for hydrogen and  $\Omega = 2$ .

We note that  $m = +1$  has the lowest density cutoff, and is therefore the fundamental mode of the system.

We can now easily calculate the electric and magnetic field components. Of course from (6)

$$(18) \quad B_z = A J_m(k_\perp r) e^{i(m\theta + k_{\parallel} z - \omega t)}$$

and substituting (17)  $\rightarrow$  (11)

$$(19) \quad E_\theta = B J_m'(k_\perp r) + C J_m(k_\perp r)$$

and (19)  $\rightarrow$  (1r)

$$(20) \quad B_r = \frac{-k_{\parallel} C}{\omega} E_\theta$$

and (18, 19)  $\rightarrow$  (9)

$$(21) \quad E_r = D J_m'(k_\perp r) + E J_m(k_\perp r)$$

and (21)  $\rightarrow$  (10)

$$(22) \quad B_\theta = \frac{k_{\parallel} C}{\omega} E_r$$

where  $A, B, C, D, E$  are constants proportional to the wave field, and functions of the plasma and wave parameters.

Table 1

$\mu$	$m$	-1	0	+1
1		.32	.19	$.10 \times 10^{14}/\text{cm}^3$
2		.89	.63	.42
3		1.71	1.34	.98
4		2.79	2.28	1.83
5		4.09	3.52	2.90

Eigenmode cutoff density for  $m = 0 \pm 1$ ,  $\mu = 1-5$ ,  $\Omega = 2$ , hydrogen and  
 $a = 10 \text{ cm}$

### IV-3.2. Number of propagating eigenmodes at high density

Using equation IV-3.1(16), we can transform IV-3.1(14) to

$$(1) \quad J_{m-1} = \frac{m(1 - \Omega)}{\chi}$$

For reasonable values of  $m$ ,  $\Omega$ , and  $\chi$ , i.e.

$$m = 3$$

$$(2) \quad \Omega = 2$$

$$\chi = k_A a = 10$$

and, since  $J_{m-1}$  has roughly the same amplitude as  $J_m$ , we can approximate equation (1) as

$$(3) \quad J_{m-1}(\chi) = 0$$

From Figure 1 and IV-3.1(17), we can write the very crude but simple cutoff dispersion relation

$$(4) \quad k_{\perp} a = \mu(\pi + \frac{m}{2})$$

In a typical Alcator high density plasma with  $\lambda_A = 3.8$  cm and  $a = 10$  cm,

$$(5) \quad \frac{k_{\perp} a}{\pi} = \nu \approx 5$$

and we can propagate as many as 45 perpendicular modes, as shown in Table 1.

Physically, we could have arrived at the same result by crudely assuming

$$(6) \quad \begin{aligned} k_{\perp} &= k_r + k_{\theta} \\ k_r a &= \mu \pi \\ k_{\theta} 2\pi \langle r \rangle &= m 2\pi \\ \langle r \rangle &= \ll a \end{aligned}$$



Of course, at large  $r$ , where  $1/r$  effects can be neglected, we must have

$$(7) \quad k_{\perp}^2 = k_r^2 + \left(\frac{m}{r}\right)^2$$

At this point, we must use these simple relations for statistical purposes only. For example, in a normal dielectric, the lowest TE mode of a rectangular guide is  $TE_{01}$ . In a circular guide, the lowest mode is  $TE_{11}$ , not  $TE_{01}$ . Accordingly, a careful look at the electric field would have shown that the circular  $TE_{11}$  is only a slightly perturbed Cartesian  $TE_{01}$ , and would indeed require less  $k_{\perp}$  than the more distorted circular  $TE_{01}$ . With an only slightly more sophisticated version of equation (4),<sup>7</sup> which includes a phase factor dependent on the sign of  $m$ , the mode spectrum would then be correctly symmetric about  $m = -1$  instead of  $m = 0$ .

Cartesian fields ( $\sin$ ,  $\cos$ ) can be mapped into cylindrical coordinate fields ( $J_m$ ,  $J_m'$ ) by using the crude approximation

$$(8) \quad J_1(x) \cong \frac{\sin(x)}{\sqrt{1 + \pi x/2}}$$

as shown in Figure 2. Our corrected inhomogeneous Cartesian solution (IV-2.3) then becomes a very good approximation to the exact solution, as we will see in Section IV-5.3.

Table 1

Possible perpendicular eigenmodes for  $k_{\perp}a = 5$ .

$\mu$	$m$
1	$0 \pm 1 \pm 2 \pm 3 \pm 4 \pm 5 \pm 6 \pm 7 \pm 8$
2	$0 \pm 1 \pm 2 \pm 3 \pm 4 \pm 5 \pm 6$
3	$0 \pm 1 \pm 2 \pm 3 \pm 4$
4	$0 \pm 1 \pm 2$
5	0

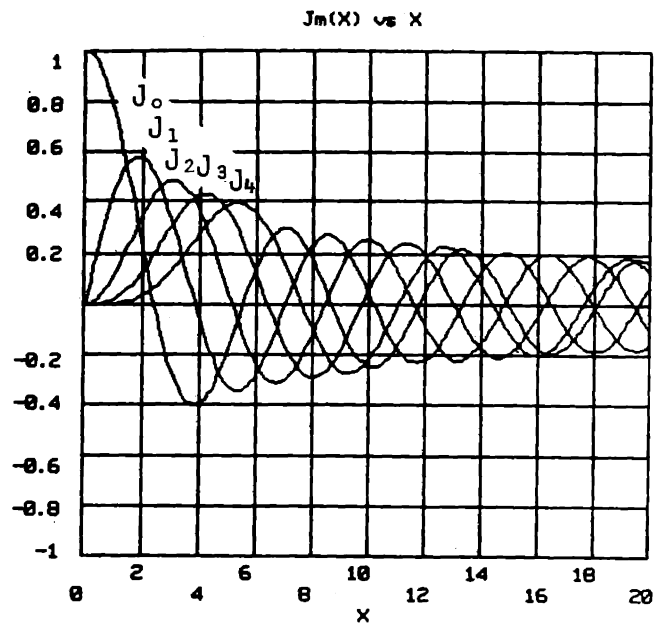


Figure 1

$J_1(x)$  and  $\frac{\sin x}{\sqrt{1 + \pi x/2}}$  vs.  $x$

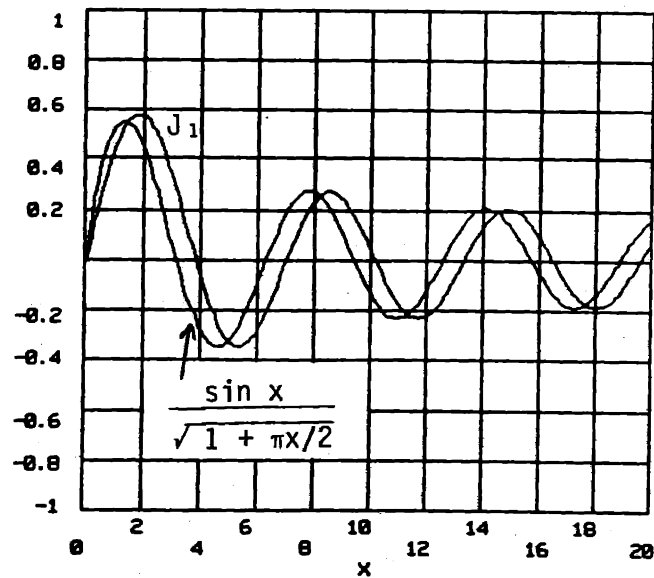


Figure 2

#### IV-3.3. Mode splitting

For a wave field of the form

$$(1) \quad e^{i(m\theta + k_{||}z - \omega t)}$$

we have a constant phase point at

$$(2) \quad \theta_w = \frac{-k_{||}z}{m}$$

So far, our wave field structure was always "tied" to a stationary magnetic field. If the magnetic field frame of reference is rotated, our field solutions are still valid in that reference frame. Unfortunately, we would like a solution in the laboratory frame of reference. In a tokamak, the confining field has a similar constant phase point (Figure 1)

$$(3) \quad \theta_B = \frac{B_{\theta} z}{B_z r}$$

Assuming  $k_{||}$  positive and  $m < 0$ , the wave field rotates with a typical parallel wavelength of a few centimeters. When positive  $B_{\theta}$  is added, the wave field wraps around faster, and thus, as seen in the laboratory, has a shorter parallel wavelength.

$$(4) \quad y \approx \lambda \quad \theta_B \approx \Delta\lambda\theta_w$$

and

$$(5) \quad \frac{\Delta\lambda}{\lambda} \approx \frac{\theta_B}{\theta_w} \ll 1$$

Substituting (2) and (3) in (5)

$$(6) \quad \Delta k_{||} = \frac{B_{\theta}}{B_z} \frac{m}{r} = \frac{m}{qR}$$

where  $q$  is the usual safety factor.<sup>1,22</sup> If we now simulate an identical wave going against  $B_{\theta}$  by simply reversing  $B_{\theta}$ , the wave wraps around

more slowly, and thus has a longer parallel length. For typical Alcator experiments

$$(7) \quad \begin{array}{ll} k_{||} = .5/\text{cm} & m = 2 \\ q = 3 & R = 54 \text{ cm} \end{array}$$

and

$$(8) \quad \frac{\Delta k_{||}}{k_{||}} \cong 2.5\%$$

# Mode Splitting

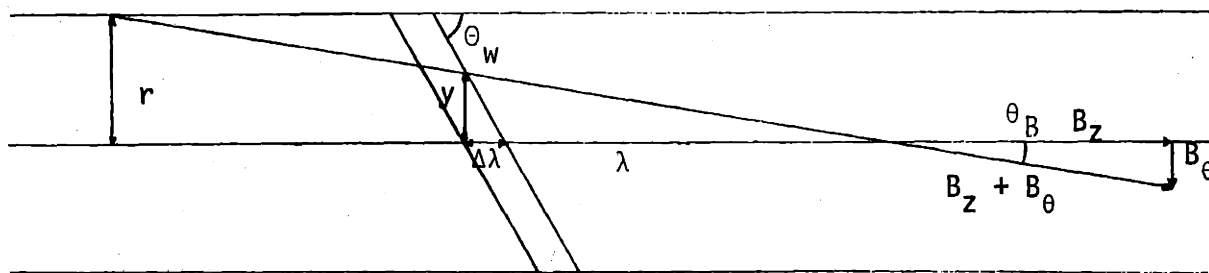


Figure 1

#### IV-4. Hot Plasma Model and Damping Mechanisms

Five basic damping mechanisms<sup>32</sup> of importance can be found in the Alcator experiment<sup>20,26</sup>. Each of the damping decrements depend heavily on the wave field structures and plasma parameter profiles. The first and simplest are wall losses, which are always present and give an upper bound on  $Q$ <sup>32</sup>. Wall losses are dependent on parallel and perpendicular mode numbers and, in particular, the edge H field. Second harmonic of the ion gyrofrequency damping can also be important if  $k_{\perp} \rho_{ci}$  is large enough, which for practical purposes, means large radial mode numbers in the hot plasma center<sup>30-32</sup>. Fundamental and two ion-ion hybrid damping mechanisms have been discussed the most in the literature, and have the largest damping decrements<sup>10,11,12,16,17,27,48,49</sup>.

Electron Landau Damping (ELD) and Transient Time Magnetic Pumping (TTMP) can be important in the high temperature center with large enough  $k_{\parallel}$  and  $E_z$  in the absence of other strong damping mechanisms<sup>30-32</sup>.

Finally, collisional and near-field damping may contribute to the<sup>90,96</sup> loading resistance, and remove power from the antenna. Also, a whole generation of non-linear effects and surface waves may, unfortunately, heat the plasma edge<sup>17,32,74</sup>.

In this and the next Sections, all five types of damping mechanisms will be qualitatively evaluated in simple geometry, and then used in the weak damping approximation to numerically calculate wave damping length, radiation resistance and power deposition profiles for realistic experimental mode numbers, field strengths, plasma parameters and profiles.

#### IV-4.1. Wall damping

We now allow  $k_{ii}$  to be complex, so the power flowing down our simple waveguide is

$$(1) \quad \begin{aligned} P_f &\propto [E_o e^{i((k_r + ik_i) z - \omega t)}]^2 \\ &\propto P_o e^{-2k_i z} \end{aligned}$$

The power dissipated per unit length is

$$(2) \quad -\frac{\partial P_f}{\partial z} = 2 k_i P_f$$

and the wave damping length is

$$(3) \quad \frac{1}{k_i} = \frac{-2P_f}{\partial P_f / \partial z} = \frac{2 \text{ Power Flowing}}{\text{Power Dissipated}}$$

In Section IV-1.3, we calculated the power flowing down a simple waveguide as (MKS)

$$(4) \quad P_f = \frac{a^2 k_z}{\omega \mu_o} E_o^2$$

For wall losses, the power dissipated per unit length can be calculated from the surface current resistive loss on the vacuum vessel

$$(5) \quad J_s = \frac{1}{\omega \mu_o} \frac{\partial E}{\partial x} \Big|_{\text{wall}}$$

The power deposited per unit area and unit length are then<sup>134</sup>

$$(6) \quad P/m^2 = \frac{J_s^2}{2 \sigma \delta}$$

$$(7) \quad P/m = 2\pi a P/m^2$$

where the skin depth is

$$(8) \quad \delta = \sqrt{\frac{2}{\omega \mu \sigma}}$$

Finally, substituting (4) - (8) into (3), we have

$$(9) \quad \frac{1}{k_i} = \frac{2\sqrt{2}}{\pi} \left[ \frac{E_0}{\left. \frac{\partial E}{\partial x} \right|_{\text{wall}}} \right]^2 a k_{ii} \sqrt{\omega \mu_0 \sigma} \cong 740 \text{ meters}$$

for high density (IV-1.3.(7)) and a stainless steel ( $\sigma = 1.1 \times 10^6$  mho/m) vacuum vessel.

We again see the critical importance of knowing the edge field. Physically, for a given central field, and thus flowing power, a smaller edge field will dissipate less power and have a longer damping length.



#### IV-4.2. Cyclotron damping

To first order, we can write the left handed particle trajectories and velocities (positive ions), as

$$(1) \quad x = r_o \sin \omega_o t$$

$$(2) \quad y = r_o \cos \omega_o t$$

$$(3) \quad v_x = r_o \omega_o \cos \omega_o t$$

$$(4) \quad v_y = -r_o \omega_o \sin \omega_o t$$

For a right handed electric field ( $E_-$ )

$$(5) \quad E_x = E_o \cos \omega_o t$$

$$(6) \quad E_y = E_o \sin \omega_o t$$

we can write the average power to the ion as

$$(7) \quad P_- = \frac{\int_0^t q(v_x E_x + v_y E_y) dt'}{t} = r_o \omega_o q E_o [\langle \cos^2 \omega_o t \rangle - \langle \sin^2 \omega_o t \rangle] = 0$$

and, on the average, no power is coupled.

For a left handed electric field ( $E_+$ ),

$$(8) \quad E_x = E_o \cos \omega_o t$$

$$(9) \quad E_y = -E_o \sin \omega_o t$$

and the power coupled to the ion is simply,

$$(10) \quad P_+ = r_o \omega_o q E_o [\langle \cos^2 \omega_o t \rangle + \langle \sin^2 \omega_o t \rangle] = r_o \omega_o q E_o$$

Of course, negative power can be obtained by adjusting the phase of the electric field with respect to the ion velocity, but the right hand electric field component cannot couple power independently of phase, and can be ignored in the following power absorption calculations.

To understand this initial phase effect, we proceed directly to the tokamak

geometry, where particles are streaming along the rotationally transformed field lines in a  $1/R$  magnetic field. For ICRF, the electric field is a wave propagating along  $z$ , and the resonant condition is

$$(11) \quad \omega_o = \omega_{ci} + k_{\parallel} V_{\parallel}$$

or simply, the cyclotron frequency in the ion's moving reference frame.

Since the ions are moving along  $z$  in a slowly varying magnetic field, we can write.<sup>7</sup>

$$(12) \quad \omega_{ci}(t) = \omega_{ci} + t \omega'_{ci}$$

and the equations of motion become

$$(13) \quad v_x - \omega_{ci}(t) v_y = q/m E_x \cos \omega_o t$$

$$(14) \quad v_y + \omega_{ci}(t) v_x = -q/m E_y \sin \omega_o t$$

Combining (13) and (14) in a rotating coordinate system

$$(15) \quad E_{\pm} = \frac{E_x \pm E_y}{2}$$

$$(16) \quad u = v_x + i v_y$$

and neglecting the non-resonant right hand electric field, we have

$$(17) \quad \frac{du}{dt} + i \omega_{ci}(t) u = \frac{q}{m} E_+ e^{-i\omega t}$$

Equation (17) is linear, of first order, may be integrated using the integrating factor<sup>132</sup>

$$(18) \quad p = e^{i\omega_{ci}(t)}$$

and has the solution<sup>7</sup>

$$(19) \quad u(t) = \exp\left(-\int_{-\infty}^t i\omega_{ci}(t)dt\right) \left[ u(-\infty) + \frac{q}{m} E_+ \sqrt{\frac{2\pi i}{\omega_{ci}}} \right]$$

The first factor in (19) is simply the phase angle of the velocity, and is analogous to WKB solutions, with  $k(x)$  substituted by  $\omega_{ci}(t)$ , and  $x$  by  $t$ . Assuming that  $u(-\infty)$  is randomly phased with respect to  $E_+$ , the average energy kick per pass through the resonant layer is

$$(20) \quad W = \frac{m}{2} \langle u(t) u(t)^* - u(-\infty) u(-\infty)^* \rangle = \frac{e^2}{2m} E_+^2 \left| \frac{2\pi}{\omega_{ci}} \right|$$

and is independent of initial phase and perpendicular energy. We can now integrate this energy kick over a distribution of resonant particles to find the power absorbed,

$$(21) \quad P/\text{cm}^3 = \frac{\pi q^2 E_+^2}{m k_{||}} \iint dv_x dv_y f(v_{\perp}, v_{\text{res}})$$

where from (11)

$$(22) \quad v_{\text{res}} = \frac{\omega_o - \omega_{ci}}{k_{||}}$$

Physically, the  $1/k_{||}$  factor is due to the fact that, for a given layer thickness  $\Delta x$ , perpendicular distribution function, etc., the integration interval width along  $v_{||}$  is proportional to  $1/k_{||}$ .

Substituting a Maxwellian distribution function<sup>1</sup>

$$(23) \quad f_m = \frac{1}{\sqrt{\pi} v_{th}} e^{-\left[\frac{v}{v_{th}}\right]^2}$$

into (21), and making some changes in variables that cancel out the  $1/R$  magnetic field dependence, gives the usual cyclotron power absorption formula<sup>21,31,32</sup>

$$(24) \quad P/\text{cm}^3 = \frac{\pi_i^2}{8 \sqrt{\pi}} \frac{E_+^2}{k_{||} v_{thi}} e^{-\left[\frac{\omega - \omega_{ci}}{k_{||} v_{thi}}\right]^2}$$

We could have arrived at the same result in uniform  $B_0$  theory by an almost identical method, as in Section IV-2.1. First, we write the momentum equation including the parallel velocity,  $v_{\parallel}$ .<sup>45</sup>

$$(25) \quad m \left[ \frac{\partial \vec{v}}{\partial t} + \vec{v} \frac{\partial \vec{v}}{\partial z} \right] = q \left[ \vec{E}_1 + \frac{\vec{v} \times \vec{B}_0}{c} + \frac{\vec{v}}{c} \times \vec{B}_1 \right]$$

Again we assume

$$(26) \quad \vec{v} = v_x \hat{x} + i v_y \hat{y}$$

$$\vec{E} = E_x \hat{x} + i E_y \hat{y}$$

and solve for  $\vec{v}$  adding the restriction  $\vec{v} = 0$  at  $t = 0$

$$(27) \quad v_{\pm} = \frac{i e E_{\pm} (\omega - k_{\parallel} V) e^{i(k_{\parallel} z - \omega t)} 1 - e^{i(\omega - k_{\parallel} V \mp \omega_{ci})t}}{m \omega (\omega - k_{\parallel} V \mp \omega_{ci})}$$

Averaging (27) over a Maxwellian distribution and random initial phase, we can then write a dielectric tensor with complex elements similar to  $S$  and  $D$  which, themselves, are again functions of complex  $R$  (right hand) and  $L$  (left hand) components. The resistive parts of  $S$  and  $D$  then represent dissipative effects, since the current is now in phase with the electric field.

We can thus write<sup>21</sup>

$$(28) \quad K_{xx} = K_{yy} = S + i\gamma$$

$$K_{yx} = -K_{xy} = iD + \gamma$$

and

$$(29) \quad P/\text{cm}^3 = \frac{1}{2} \text{Re} (\vec{E}^* \cdot \vec{J}) = \frac{1}{4} (\vec{E}^* \cdot \vec{J}) + \text{cc} = \frac{-i\omega}{16\pi} \vec{E}^* \cdot (\vec{K} - 1) \cdot \vec{E} + \text{cc}$$

In equation (7) we assumed the ions were "free falling" in the resonant electric field, so the power delivered was proportional to  $E_0 t$ . In practice, ions are not allowed to free fall, since the interaction time with

the resonant field is finite, and phase incoherent from one pass to another through the resonant layer. This finite interaction leads to a collision type resistivity, and thus to an absorbed power proportional to  $E_o^2$ . The interaction force is, nevertheless, simply  $qE$  in the ion reference frame.

If we now assume an electric field at the second harmonic of the ion gyrofrequency, equation (7) becomes

$$(30) \quad P = r_o \omega E_o [\langle \cos \omega_o t \cos 2 \omega_o t \rangle + \langle \sin \omega_o t \sin 2 \omega_o t \rangle] = 0$$

and no power can be coupled independently of polarization or initial phase.

On the other hand, if we allow  $E$  to have a gradient,

$$(31) \quad E_x = E_o + \frac{\partial E}{\partial x} x = E_o (1 + k_{\perp} r_o \cos \omega_o t)$$

power can be coupled through the non-linear term

$$(32) \quad \langle \cos \omega_o t \cos \omega_o t \cos 2 \omega_o t \rangle \neq 0$$

Our effective interaction force is now

$$(33) \quad \frac{k_{\perp} r_o}{\sqrt{2}} q E_o = \sqrt{\lambda} q E_o$$

instead of simply  $qE_o$ , and all of our previous results can be upgraded for the second harmonic regime by simply replacing  $E_o$  by  $\sqrt{\lambda} E_o$  and  $\omega_{ci}$  by  $2 \omega_{ci}$ . Similarly, we could go to the  $n$ th harmonic by inserting the factors

$$(34) \quad \sqrt{\lambda}^{n-1} \text{ and } n \omega_{ci}$$

From now on, we shall consider the second harmonic regime at high density in the tokamak geometry <sup>21,31,32</sup>

$$(35) \quad P/\text{cm}^3 = \frac{\pi_i^2}{8\sqrt{\pi}} \frac{\lambda}{k_{\parallel} V_{thi}} |E_+|^2 e^{-\left(\frac{\omega - 2\omega_{ci}}{k_{\parallel} V_{thi}}\right)^2}$$

In this case, the wave energy is deposited in the perpendicular component of the ion energy for predominantly large Larmor radii. At high power, this will lead to the formation of energetic perpendicular ion tails. If the resonant layer is at the center of the plasma, the resonant region will be a cylinder of radius  $R_0$ , height  $2a$  and effective thickness  $\Delta R$  (such that the exponent is negative one on each side of the resonance )<sup>21</sup>

$$(36) \quad \Delta R = \frac{k_{\perp} V_{thi}}{\omega} \approx .6 \text{ cm in the hot plasma center.}$$

The heat transfer along  $\theta$  is much greater than along  $r$  due to the rotational transform, and so we can write an average power per volume as a function of minor radius by integrating around  $\theta$  at  $r$  and dividing by  $2\pi r$ .<sup>21</sup>

$$(37) \quad P/\text{cm}^3 = \frac{\pi i^2}{16\pi\omega} \frac{R_0 k_{\perp}^2 \rho_i^2}{r} |E_+|^2 = \frac{\pi i^2}{16\pi\omega} \frac{R_0}{r} \rho_i^2 |E'_+|^2$$

This result is independent of  $k_{\parallel}$  and peaked at the center of the plasma (small  $r$ ). The apparent singularity can be removed by displacing the resonant layer by  $\Delta R$  ( $r \rightarrow r + \Delta R$  in denominator). In any case, we will either want the power deposition profile or the power deposited per unit length, which removes the singularity altogether

$$(38) \quad P/\text{cm} = \int_0^{a_{\text{hot}}} 2\pi r P/\text{cm}^3 dr \approx 1.7 \times 10^9 \frac{\text{ergs}}{\text{sec-cm}} = 170 \text{ W/cm}$$

where we assumed  $a_{\text{hot}} = 5 \text{ cm}$ ,  $n_e = 5 \times 10^{14}/\text{cm}^3$ ,  $E_+ = .16 \text{ statvolt/cm}$  and  $k_{\perp} = 1/\text{cm}$ . Also assuming  $E \approx .5 \text{ statvolt/cm}$  and  $k_{\parallel} = .5/\text{cm}$ , the power flowing is approximately

$$(39) \quad P_f = \int_S \frac{cEXH}{8} ds \approx \pi a^2 \frac{c^2 k_{\parallel}}{\omega 8 \pi} \cong 1.1 \times 10^{12} \text{ ergs/sec} = 110 \text{ kW}$$

The damping length is then

$$(40) \quad \frac{1}{k_{\parallel i}} = \frac{2 P_f}{P_{\text{dis}}} = 13 \text{ meters}$$

which is several times the circumference of the torus, but is nevertheless much shorter than the wall damping length.

#### IV-4.3. Electron Landau damping and transit time magnetic pumping

The collisionless resonant damping condition  $\omega - n\omega_c - k_{||} V = 0$  can also be significant for  $n = 0$  if the phase velocity is of the order of the parallel thermal velocity.<sup>1</sup> For electrons at high density in Alcator

$$(1) \quad \alpha = \frac{v_{p||}}{v_{the||}} = \frac{2.5 \times 10^9 \text{ cm/sec}}{1.3 \times 10^9 \text{ cm/sec}} = 1.9$$

and we thus proceed to calculate what is usually called electron Landau damping. Considering only motions and fields along  $z$ , and without using complex variables, we can write the momentum equation and its solution just as in the previous Section.<sup>1,45</sup>

$$(2) \quad \frac{m d\vec{v}_I}{dt} = e\vec{E} \cos [k(z_0 + v_0 t) - \omega t]$$

$$(3) \quad \vec{v}_I = \frac{e\vec{E}}{m} \frac{\sin [k(z_0 + v_0 t) - \omega t] - \sin kz_0}{kv_0 - \omega}$$

The power absorption is then found by averaging the change in kinetic energy over the initial condition  $z_0$  and distribution function  $f(v_0)$

$$(4) \quad P/\text{cm}^3 = n_0 \left\langle \frac{d}{dt} \frac{mv^2}{2} \right\rangle_{z_0, v_0}$$

$$= \frac{-\pi n_0 \omega e^2 E_{||}^2}{2 m k_{||}^2} \left. \frac{\partial f(v_0)}{\partial v_0} \right|_{v_0 = \frac{\omega}{k_{||}}}$$



Substituting a Maxwellian distribution function (IV-4.2(23)) and (1), we have

$$(5) \quad P/\text{cm}^3 = \frac{\sqrt{\pi} n_0 e^2 E_z^2}{\omega m} \alpha^2 e^{-\alpha^2}$$

The difficulty is now in calculating  $E_z$  as a function of  $E_y$ . It is tempting to use the third line of the dielectric tensor, but as we shall see later, for the hot dielectric tensor when  $1/\alpha$  is not small, the  $k_{zy}$  element can be of the order of  $n_A^2$  instead of zero, and  $k_{zz}$  is  $1/k_{||}^2 \lambda_D^2$  instead of  $-\pi^2 e/\omega^2$ .<sup>7</sup> To circumvent these problems, we shall assume the wave is more or less compressional ( $k_{\perp} > k_{||}$ ), and the electrons can keep quasi-neutrality through Debye shielding ( $V_{te} \geq V_p$ ).<sup>7</sup> Both approximations are quite crude for Alcator, but will nevertheless give a reasonable answer. We can thus write from compressibility

$$(6) \quad \frac{n_{i1}}{n_0} = \frac{B_1}{B_0}$$

from quasi-neutrality

$$(7) \quad n_{i1} \approx n_{e1}$$

from hot electron shielding

$$(8) \quad n_{e1} = n_0 e^{\frac{e\phi}{KT_e}} = n_0 \left( 1 + \frac{e\phi}{KT_e} \right)$$

and from Ampère's law

$$(9) \quad \nabla \times \vec{E} = - \frac{\partial \vec{B}_1}{c \partial t}$$

Substituting (7)→(6)→(8)→(9), we have

$$(10) \quad i k_x E_y = \frac{i\omega}{c} \frac{e\phi}{KT_e} B_0$$

and noting that

$$(11) \quad E_z = -\frac{\partial\phi}{\partial z} = -i k_{||} \phi$$

then

$$(12) \quad E_z = -\frac{i k_x k_{||} KT_e c}{\omega e B} E_y$$

Substituting (12) into (5) finally gives the compact form<sup>130</sup>

$$(13) \quad P/\text{cm}^3 = \frac{\omega \beta_e}{16 \sqrt{\pi}} \left[ \frac{k_{\perp} c}{\omega} \right]^2 \alpha e^{-\alpha^2} |E_y|^2$$

Note that, for Alcator, we could have achieved a similar result by blindly using the cold dielectric tensor, since

$$(14) \quad \frac{1}{k_{||}^2 \lambda_D^2} = \frac{\pi^2 e}{\omega^2} \frac{v_p^2}{v_{the}^2} \approx |P|$$

except that the phase of  $E_z$  would have been wrong (which is of no importance here).

Until now, we have only allowed  $qE$  forces on the particles, but we could also include  $-\mu \nabla B$  forces, which would give rise to transit time magnetic pumping (TTMP). Careful analysis of the hot dielectric tensor<sup>7</sup> shows that ELD and TTMP are coherent, and cross terms cancel in the power deposition calculation (IV-4.2.(26)), thus leaving ELD ( $E_z$ ) alone in the form

of (13), which is half the TTMP power loss. We note also that the wave energy is deposited in the electron parallel velocity near the thermal velocity, unlike  $2 \omega_{ci}$  damping, which favors large Larmor radius ions. Using the same numbers as in Section II-4.2. ( $E_{\perp} = 150$  V/cm,  $a_{hot} = 5$  cm,  $k_{\parallel} = .5/\text{cm}$ ,  $k_{\perp} = 1/\text{cm}$ ,  $\alpha = 1.9$ ,  $\beta_e = 5.6 \times 10^{-3}$  and  $P_f = 110$  kW)

$$(15) \quad P/\text{cm} = \pi a_{hot}^2 P/\text{cm}^3 = 1.4 \times 10^8 \frac{\text{erg}}{\text{sec} \cdot \text{cm}} = 14 \text{ W/cm}$$

$$(16) \quad \frac{1}{k_{i\parallel}} = \frac{2P_f}{P_{dis}} = 160 \text{ meters}$$

The damping length is then many times the circumference of the torus, but is, nevertheless, shorter than the wall damping length, and much longer than the  $2 \omega_{ci}$  damping length.

#### IV-4.4. The hot dielectric tensor and approximations

So far, we have looked into selected topics of hot plasma effects, just as in Appendix 1 we investigated selected simple wave regimes. Now we introduce a totally general wave propagation and damping formulation, the hot dielectric tensor and wave equation.<sup>45,20</sup> The basic constitution of the hot tensor is the same as the cold one, i.e., a sum of a vacuum, electron and ion terms, except that the elements can also be resistive. Unfortunately, as we saw in Section IV-4.2., the resistive and even reactive particle currents are much more difficult to calculate than in cold plasma theory. The basic trick is to calculate the field-particle interaction in the particle zeroth order trajectory reference frame, and then average over the particle velocity distribution function. The product of this formidable analytic computation is, even in its most compact form for Maxwellian distribution functions, a somewhat overwhelming series of infinite sums involving just about every plasma variable ( $T_e$ ,  $k_x$ ,  $\lambda$  ...  $V_d$ ) and a particularly nasty integral, the dispersion function. Nevertheless, with a number of not too restrictive and quite accurate approximations at high density,<sup>20,26</sup>

$$(1) \quad V_{\text{drift}} = 0$$

$$(2) \quad T_i = T_e$$

$$(3) \quad k_y = 0$$

$$(4) \quad j = e, i$$

$$(5) \quad -5 < n < +5$$

the formulation becomes manageable, and general trends become more apparent.

The basic expansion parameters are as one would expect from our simple

treatment of collisionless damping:<sup>20</sup>

$$(6) \quad \lambda_j = -\frac{k_\perp^2 \rho_j^2}{2}$$

$$(7) \quad \zeta_{nj} = \frac{\omega + n\omega_{cj}}{k_\parallel v_{thj}}$$

$k_y = 0$  is the most restricting approximation, since  $k_y$  affects the wave polarization, and thus the reactive and resistive components of the tensor elements. Although the dispersion relation is still found by simply setting the wave equation determinant to zero, we cannot write an equivalent biquadratic equation, since the tensor elements are infinite sums already involving  $k_x$ ,  $k_y$ ,  $k_\perp$ ,  $k_\parallel$ . Without the further, major assumption that  $\lambda_j \ll 1$ , the many dispersion relation solutions can only be found by using sophisticated numerical methods. Restricting  $\lambda \ll 1$  basically reproduces the cold (2 x 2) tensor with very small<sup>26</sup> changes in the reactive components due to ELD, TTMP and cyclotron damping, and does not allow any new wave solutions.

A realistic, comprehensive study of the effect of our approximations (equations (1) - (5)) can only be done on advanced algebraic manipulators such as MAXIMA. In particular, at low density in the runaway regime, the streaming parameter becomes appreciable ( $V_D \neq 0$ ),  $T_i \neq T_e$ , discrete positive and negative  $m$  modes dominate ( $k_y = 0$ ),  $Z_{eff} > 1$  ( $j = e, i$ , impurities), and all approximations break down lamentably.

Fortunately, in Alcator, nearly pure hydrogen high density plasmas can be produced with very small drift velocities and nearly isotropic Maxwellian distribution functions. For simplicity and not necessity, we

will also assume for now, that  $k_y = 0$ . The results of solving the cold and hot wave determinants for a hot, dense plasma center and cold edge, for the same  $k_{||}$  and roughly the same magnetic field are shown in Tables 1 and 2. As we suspected and assumed until now, the  $2 \times 2$  part of the cold tensor is almost the same as its hot plasma counterpart for the fast wave branch, since indeed  $\lambda \ll 1$ , and so all our previous work is well founded. Also, as one would expect, at the cold edge,  $k_{xz}$  and  $k_{yz}$  are very small, and  $k_{zz} = P$ . Unfortunately, this is not the case with the hot plasma center, which led to our earlier difficulty in calculating  $E_z$  from the third line of the wave tensor. It is important to note here that, although the reactive components of the fast wave tensor elements are insensitive to the major radius position, that is, the magnetic field, the ion resistive components are dependent on the narrow harmonic resonant condition

$$(8) \quad \omega - n\omega_c - k_{||} V = 0$$

and the power deposition profile is very peaked at the resonant layer in a one dimensional plasma formulation.

It is also interesting to note that, for  $k_y = 0$  ( $m = 0$ ), from the first line of the wave tensor, we have

$$(9) \quad \frac{E_y}{E_x} = \frac{E_\theta}{E_r} = \frac{S - n_{||}^2}{-iD} \quad \begin{aligned} &= 5.46 \text{ at the edge,} \\ &= .60 \text{ at the center,} \\ &= 1 \text{ at the cutoff layer}(k_{\perp}^2 = 0) . \end{aligned}$$

which means that, from a global point of view, the antenna couples to an  $E_\theta$  edge wave that gradually transforms into an  $E_r$  wave in the center of the plasma.

The mere fact that the wave equation is of higher order than biquadratic means that other wave solutions exist. The most important solution (besides the slow wave which did not even depend on hot plasma effects, but was found evanescent in our regime and could be neglected) is the ion Bernstein wave. This new wave is critically dependent on  $\lambda$  and  $\zeta$ , and thus cannot be coupled from the cold edge which does not allow such a mode, as we saw in cold plasma theory. Nevertheless, the wave can couple power from the fast wave near the center if the Bernstein wave  $k_{\perp}$  becomes equal to the fast wave  $k_{\perp}$ . The two modes are then locally non-orthogonal, and power can be coupled between the waves giving rise to a telescopingly complicated problem. In the two ion-ion hybrid regime, even with cold plasma theory, we encountered singularities in  $k_{\perp}$  which would have made  $\lambda \gg 1$ , and new waves would have immediately appeared. In fact, hot plasma treatment of the two ion-ion hybrid regime would have removed the singularity, but the general location of the resonance would still be governed by the cold plasma  $k_{\perp}$ , which "controls" the hot plasma expansion parameter  $\lambda$ .

Table 1

Comparison of tensor elements in hot plasma center

Tensor Elements	Cold Plasma Fast Wave	Hot Plasma Fast Wave		Hot Ion Bernstein Wave	
$K_{xx}, S$	-735	+i184	-736	-i44.7	+151
$K_{xy}, -iD$	-i1471	-i1468	-184	-i582	+39.5
$K_{zz}, P$	$-10^6$	$+i1.4 \times 10^6$	$-9.7 \times 10^5$	$+i1.4 \times 10^6$	$-9.7 \times 10^5$
$K_{xz}$	0	+i.44	+4.99	-i39.9	-48.2
$K_{yz}$	0	+i1003	+1406	-i775	+3637
$k_{\perp}$	1.67	+i.04	+1.65	-i2.62	+2.47

## Basic Parameters

200 MHz  
 65.8 kG  
 $k_{\parallel} = .5/\text{cm}$   
 $n_e = 5 \times 10^{14}/\text{cm}^3$   
 Hydrogen

$k_y = 0$   
 $T_i = T_e = 1 \text{ keV}$   
 $\Omega = 2$   
 $\omega_{pi}^2 / \omega_{ci}^2 = 2207$



Table 2

Comparison of tensor elements in cold plasma edge

Tensor Elements	Cold Plasma Fast Wave	Hot Plasma Fast Wave		Hot Ion Bernstein Wave
$K_{xx}$ , S	-13.7	-i.036	-13.6	
$K_{xy}$ , -iD	-i29.4	-i29.3	+i.036	No
$K_{zz}$ , P	$-2.0 \times 10^4$	$-i7.5 \times 10^{-9}$	$-2.03 \times 10^4$	
$K_{xz}$	0	$-9.7 \times 10^{-5}$	$+1.38 \times 10^{-8}$	Solution
$K_{yz}$	0	$-5.0 \times 10^{-6}$	-.063	
$k_{\perp}$	i.511	+i.514	$-4.05 \times 10^{-5}$	

## Basic Parameters

200 MHz  
 65.8 kG  
 $k_{\parallel} = .5/\text{cm}$   
 $n_e = 10^{13}/\text{cm}^3$   
 Hydrogen

$k_y = 0$   
 $T_i = T_e = 10 \text{ eV}$   
 $\Omega = 2$   
 $\omega_{pi}^2/\omega_{ci}^2 = 44.1$

#### IV-4.5. Collisional Damping

To take into account collisional damping, we return again to the basic momentum equation, but including collisions <sup>1</sup>

$$(1) \quad m_k n_k \frac{d\vec{v}_k}{dt} = q_k n_k \left( \vec{E} \times \frac{\vec{v}_k \times \vec{B}}{c} \right) - \gamma_k n_k m_k \vec{v}_k$$

Equation (1) can be cast in the form of equation IV-2.1.(3) with the substitution

$$(2) \quad m_k \rightarrow m_k^+ = m_k (1 + i \gamma_k / \omega)$$

and

$$(3) \quad \Omega \rightarrow \Omega^+$$

$$(4) \quad \frac{\pi_e^{+2}}{\omega_c^+} = \frac{\pi_e^2}{\omega_c}$$

and our cold dielectric tensor is easily upgraded to include collisions.

We will confine this derivation to ICRF, and assume

$$(5) \quad \Omega_e \ll \Omega_i < \sqrt{\frac{m_i}{m_e}} \approx 6.5$$

$$(6) \quad \gamma_i = \sqrt{\frac{m_e}{m_i}} \gamma_e, \quad (T_i = T_e)$$

$$(7) \quad n_s = \frac{E}{J} = \frac{m \gamma_{ei}}{n_e e^2}$$

$$(8) \quad \frac{\gamma_k}{\omega} = \tau_k \ll 1$$

The new dielectric tensor elements are then calculated from IV-2.1 as

$$(9) \quad P^{\dagger} = 1 - \frac{\pi_e^{+2}}{\omega^2} - \frac{\pi_i^{+2}}{\omega^2} \\ \approx - \frac{\pi_e^2 (1 - i \tau_e) + \pi_i^2 (1 - i \tau_i)}{\omega^2} \\ \approx P (1 - i \tau_e)$$

$$(10) \quad R^\dagger = -\frac{\pi_e^{\dagger 2}}{\omega (\omega - \omega_{ce}^\dagger)} - \frac{\pi_i^{\dagger 2}}{\omega (\omega + \omega_{ci}^\dagger)}$$

$$= -\frac{\pi_i^2}{\omega \omega_{ci}} \left[ \frac{1}{\Omega_e^\dagger - 1} + \frac{1}{\Omega_i^\dagger + 1} \right]$$

$$(11) \quad L^\dagger = -\frac{\pi_i^2}{\omega \omega_{ci}} \left[ \frac{1}{\Omega_e^\dagger + 1} + \frac{1}{\Omega_i^\dagger - 1} \right]$$

$$(12) \quad \frac{1}{\Omega^{\dagger 2} - 1} \approx \frac{1}{\Omega^2 - 1} (1 - i 2\tau \frac{\Omega^2}{\Omega^2 - 1})$$

$$(13) \quad \frac{\Omega^\dagger}{\Omega^{\dagger 2} - 1} \approx \frac{\Omega}{\Omega^2 - 1} (1 - i\tau \frac{\Omega^2 + 1}{\Omega^2 - 1})$$

and using (12), (13) and (5),

$$(14) \quad S^\dagger = \frac{R^\dagger + L^\dagger}{2} \approx S (1 + i \tau_i \frac{1 + \Omega_i^2}{1 - \Omega_i^2})$$

$$(15) \quad D^\dagger = \frac{R^\dagger - L^\dagger}{2} \approx D (1 + i \tau_i \frac{2}{1 - \Omega_i^2})$$

Power absorption is then calculated as

$$(16) \quad P_{\parallel} = \text{Re} \frac{\mathbf{J}_{\parallel} \cdot \mathbf{E}_{\parallel}^*}{2} = \text{Re} \frac{\omega}{i8\pi} P^\dagger E_z E_z^*$$

$$= \frac{\pi_e^2}{\omega^2} \frac{\gamma_e}{8\pi} E_z^2 \approx \frac{\pi_i^2}{\omega_{ci}^2} \frac{\gamma_i}{8\pi} \sqrt{\frac{m_e}{m_i}} E_y^2$$

where we used (6), and assumed  $E_z \approx m_e/m_i E_y$ , and

$$(17) \quad P_{\perp} = \text{Re} \frac{\omega}{i8\pi} \left[ (S^\dagger E_x - iD^\dagger E_y) E_x^* + (iD^\dagger E_x + S^\dagger E_y) E_y^* \right]$$

$$= \text{Re} \frac{\omega E_y}{i8\pi} \left[ S^\dagger \left[ \frac{D^2}{(S - n_{\parallel}^2)^2} + 1 \right] - D^\dagger \frac{2D}{S - n_{\parallel}^2} \right]$$

Further assuming  $n_{ii} = 0$  and  $D = -\Omega S$ , we have

$$(18) \quad P_{\perp} = \frac{\pi i^2}{\omega^2 c i} \frac{\gamma_i}{8\pi} E_y^2 \approx \sqrt{\frac{m_i}{m_e}} P_{ii}$$

From Spitzer resistivity (7), and the ion-electron collision frequency (6)<sup>75,82</sup>

$$(19) \quad \gamma_{ie} = 4.8 \times 10^{-8} n_e \lambda_c T^{-3/2} \mu^{-1/2} \approx 2.3 \times 10^6 / \text{sec}$$

and the ion-neutral collision frequency<sup>82</sup>

$$(20) \quad \gamma_{in} = n_n \sigma_n v_{thi} \approx 1.5 \times 10^6 / \text{sec}$$

$$(21) \quad \gamma_i = \gamma_{in} + \gamma_{ie} \approx 3.8 \times 10^6 / \text{sec}$$

where we assumed  $T_i = T_e = 10 \text{ ev}$ ,  $\sigma_n = 5 \times 10^{-15} \text{ cm}^2$ ,  $n_n = n_e = 10^{14} / \text{cm}^3$ .

The cold neutrals from the edge will have a short mean free path, due to ionization by electron impact<sup>78</sup>

$$(22) \quad \gamma_{ion} \approx \langle \sigma v_e \rangle n_e \approx 1.9 \times 10^6 / \text{sec}$$

$$(23) \quad \lambda_{mfp} = \frac{v_{thi}}{\gamma_{ion}} = 1.1 \text{ cm}$$

where we assumed  $T_n = 5 \text{ ev}$ ,  $T_e = 30 \text{ ev}$  and  $n_e = 10^{14} / \text{cm}^3$ .

Finally, the power deposition and damping length can be approximately calculated as

$$(24) \quad P / \text{cm}^3 \approx P_{\perp} \approx 3.1 \times 10^6 \frac{\text{erg}}{\text{sec cm}^3} \approx .31 \text{ W/cm}^3$$

$$(25) \quad P / \text{cm} \approx 2\pi a \lambda_{mfp} P / \text{cm}^3 \approx 19.5 \text{ W/cm}$$

$$(26) \quad \frac{1}{k_i} = \frac{2 P_f}{P / \text{cm}} \approx 110 \text{ meters}$$

where we assumed from IV-4.2,  $P_f = 110 \text{ kW}$  and  $E_{\perp} \approx 60 \text{ V/cm}$ .

From (18) and (20) it is clear that

$$(27) \quad P \propto n E_{\perp}^2 T^{-3/2}$$

and

$$(28) \quad P_{\text{center}} < P_{\text{edge}}$$

Even if the collisional damping length is much larger than the second harmonic damping length, it may have very detrimental effects since  $nT$  is much smaller at the plasma edge. Antenna near-fields can be several times larger than the wave fields (and  $E_z$  of the order of  $E_\perp$  for unshielded antenna) giving rise to large power absorption, and even breakdown.

Collisional power absorption could also have been crudely estimated by simply writing <sup>1,75,90,96,144</sup>

$$(29) \quad P/\text{cm}^3 = \frac{J^2}{2} \eta_S$$

where

$$(30) \quad J \approx \frac{\omega}{4\pi} \epsilon_A E_\perp \approx J_\perp \approx J_\parallel$$

and  $\eta$  is the Spitzer (or any other) resistivity. Note that this is very different from writing  $P/\text{cm}^3 = E^2 \eta/2$  as in ohmic heating, since the resistivity is in "series" with a much larger wave reactance.<sup>1</sup>

$$(31) \quad \chi_w = \frac{E}{J} = \frac{1}{\omega \epsilon_0 \epsilon_A} \approx 100 \, \Omega \cdot \text{cm}$$

$$>> \eta_S \approx .08 T^{-1.5} \approx 10^{-4} \, \Omega \cdot \text{cm}$$

#### IV-5. Inhomogeneous Cylindrical Plasma Numerical Model

We now turn to a somewhat more sophisticated model, the inhomogeneous plasma-filled circular waveguide. The procedure for obtaining the field solutions is a combination of all our previous models, where we will sacrifice simplicity for more precise results, especially at the low density edge near the antenna. A numerical code<sup>21</sup> written by J. Adams (TFR, Fontenay - aux - Roses, France) and further extended to incorporate  $E_z$  and hot plasma effects, is used extensively. A similar code is also used to generate the full inhomogeneous plasma eigenmode dispersion relations. Using these codes, radiation resistance and probe signal components are calculated much more precisely.

##### IV-5.1. Inhomogeneous plasma eigenmode differential equations and large $r$ approximation

In this Section, we will again start with Maxwell's equations and the dielectric tensor in cylindrical coordinates, as in Section IV-3.1., and assume a wave of poloidal, toroidal and temporal form

$$(1) \quad A = A_0 e^{i(m\theta + k_z z - \omega t)}$$

but leave the radial behavior unspecified. We will also solve for a second order differential equation in  $E_\theta$ , instead of  $B_z$ , so that the physical solution with  $E_\theta = 0$  at the wall can easily be recognized, just as in Section IV-2.3.

Starting with equation(2r) of Section IV-3.1. and substituting (1), we have

$$(2) \quad \frac{im}{r} B_z - ik_z B_\theta = \frac{-i\omega}{c} (SE_r - iDE_\theta).$$

Then substituting (1 $\theta$ ) and (1 $z$ ) into (2) and collecting terms, we have a first order differential equation in terms of  $E_\theta$  and  $E_r$  alone<sup>21</sup>

$$(3) \quad i E_r \left[ k_{||}^2 + \frac{m^2}{r^2} - \frac{\omega^2 S}{c^2} \right] = \frac{m}{r} E_\theta' + E_\theta \left[ \frac{m}{r^2} + \frac{\omega^2}{c^2} D \right]$$

where the prime denotes differentiation with respect to  $r$ . Similarly, we can also start with equation (2 $\theta$ ), and substituting (1),

$$(4) \quad -B_z' + i k_{||} B_r = \frac{-i\omega}{c} (i D E_r + S E_\theta)$$

and again, substituting (1 $r$ ) and (1 $z$ ) and collecting terms, we have

$$(5) \quad E_\theta'' + \frac{E_\theta'}{r} + E_\theta \left[ \frac{\omega^2 S}{c^2} - k_{||}^2 - \frac{1}{r^2} \right] = \frac{m}{r} i E_r' - i E_r \left[ \frac{m}{r^2} + \frac{\omega^2 D}{c^2} \right]$$

Let's pause now from our general derivation and assume that  $r \rightarrow \infty$  but  $m$  stays finite, which simulates a Cartesian coordinate system with  $k_y = 0$ . Equation (3) then becomes

$$(6) \quad E_r \left[ \frac{\omega^2 S}{c^2} - k_{||}^2 \right] - \frac{i D \omega^2}{c^2} E_\theta = 0$$

which we recognize as the first line of the wave tensor equation.

Similarly, equation (5) becomes

$$(7) \quad E_\theta'' + E_\theta \left[ \frac{\omega^2 S}{c^2} - k_{||}^2 - \frac{\omega^4 D^2}{c^4} \frac{1}{\frac{\omega^2 S}{c^2} - k_{||}^2} \right] = 0$$

where the factor multiplying  $E_\theta$  is easily recognized as the fast wave  $k_{\perp}^2$  in agreement with our inhomogeneous plasma Cartesian waveguide treatment of Section IV-2.3.

Returning to our general derivation, we can rewrite (3) and (5) as

$$(8) \quad A i E_r = B E_\theta' + C E_\theta$$

$$(9) \quad E_\theta'' + D E_\theta' + F E_\theta = B i E_r' - C i E_r$$

where  $A$ ,  $B$ ,  $C$ ,  $D$ , and  $F$  are all functions of  $r$ .

By substituting (8) and the derivative of (8),

$$(10) \quad i E_r' = \frac{B}{A} E_\theta'' + \left[ \left( \frac{B}{A} \right)' + \frac{C}{A} \right] E_\theta' + \left( \frac{C}{A} \right)' E_\theta$$

into (9) and collecting terms, we have a second order differential equation in  $E_\theta$  alone,

$$(11) \quad E_\theta'' = \frac{A}{A - B^2} \left[ [-D + B \left( \frac{B}{A} \right)'] E_\theta' - [F - B \left( \frac{C}{A} \right)' + \frac{C^2}{A}] E_\theta \right]$$

where we note the possibility of a singularity at

$$(12) \quad A - B^2 = 0$$

which is simply our two ion-ion hybrid resonance condition,  $S - n_{ii}^2 = 0$ .

Also, unlike equation (7), we must take derivatives of the plasma parameter functions,  $A'$ ,  $B'$  and  $C'$ , which are trivial except for the dielectric tensor element factors  $S'$  and  $D'$ , which can be written as

$$(13) \quad S, D \propto n(r)$$

and these derivatives are easily calculated from the density profile  $n(r)$  and  $\frac{\partial n(r)}{\partial r}$ .

Cold plasma  $E_z$  can be calculated from IV-3.1-(2z).

$$(14) \quad \frac{1}{r} \left[ \frac{\partial r B_\theta}{\partial r} - \frac{\partial B_r}{\partial \theta} \right] = -\frac{i\omega}{c} P E_z$$

and substituting (1r) and (1θ), we have

$$(15) \quad E_z = \frac{k_{ii} c^2}{p \omega^2} \left[ \frac{i E_r}{r} + i E_r' - \frac{m}{r} E_\theta \right]$$



#### IV-5.2. TFR-EZ code structure

We now wish to solve numerically equation IV-5. 1.(11), which can be rewritten as

$$(1) \quad E_{\theta} = F(r, E_{\theta}, E'_{\theta})$$

and where we will use a density profile,

$$(2) \quad \begin{aligned} n(r) &= n \left(1 - \frac{r^2}{a^2}\right) & 0 < r < .9a \\ &= n [ .04 + .15e^{-12(r/a - .9)} ] & .9a < r < \text{wall} \end{aligned}$$

and  $n_a$ ,  $n_b$ ,  $B_o$ ,  $\omega_o$  etc., to calculate  $F$ .

In Section IV-2.3, we used a first order integration method to solve

$$(3) \quad y' = G(r, y)$$

$$(4) \quad y_{k+1} = y_k + \frac{1}{6} (6a_1)$$

with  $a_1 = h G(r_k, y_k)$

$h$  = step size

Quite similarly, we could have written to fourth order, using the Runge-Kutta method<sup>132</sup>

$$(5) \quad y_{k+1} = y_k + \frac{1}{6} (b_1 + 2b_2 + 2b_3 + b_4)$$

with

$$b_1 = h G(r_k, y_k)$$

$$b_2 = h G\left(r_k + \frac{h}{2}, y_k + \frac{b_1}{2}\right)$$

$$b_3 = h G\left(r_k + \frac{h}{2}, y_k + \frac{b_2}{2}\right)$$

$$b_4 = h G(r_k + h, y_k + b_3)$$

Now solving equation (1) to fourth order, we similarly write

$$(6) \quad \begin{aligned} E_{k+1} &= E_k + \frac{1}{6} (b_1 + 2b_2 + 2b_3 + b_4) \\ E'_{k+1} &= E'_k + \frac{1}{6} (b'_1 + 2b'_2 + 2b'_3 + b'_4) \end{aligned}$$

where we have successively calculated

$$(7) \quad \begin{aligned} b_1 &= h E'_k \\ b'_1 &= h F(r_k, E_k, E'_k) \\ b_2 &= h (E'_k + \frac{b'_1}{2}) \\ b'_2 &= h F(r_k + \frac{h}{2}, E_k + \frac{b_1}{2}, E'_k + \frac{b'_1}{2}) \end{aligned}$$

and so on for  $b_3$ ,  $b'_3$ ,  $b_4$ , and  $b'_4$ .

To initialize the program, we need to specify  $E_\theta$  and  $E'_\theta$  at  $r = 0$  for each poloidal mode number  $m$ , just as we needed to initialize FINT (first integral) and SINT (second integral) in Section IV-2.3. By judiciously varying  $k_\parallel$ , we can then solve for the desired radial mode number  $\mu$ , while keeping  $E_\theta(w) = 0$ . Since we now have  $E_\theta$ ,  $E'_\theta$ ,  $S$ ,  $D$ , etc. for all  $r$ , we can successively calculate  $iE_r$  from Section IV-5.1.(3),  $E_z$  from IV-5.1.(15),  $B_r$  from IV-3.1.(1r),  $B_\theta$  from IV-3.1.(1\theta) and  $B_z$  from IV-3.1.(1z). We can calculate  $|E_+|$ ,  $|E'_+|$ ,  $|E'_+|^2$  from

$$(8) \quad E_\pm = \frac{E_\theta \mp iE_r}{2} \quad (\text{statvolts/cm})$$

and the Poynting flux along  $+z$  and  $-z$ , as

$$(9) \quad \Phi(r) = \frac{k_\parallel c^2}{4\omega} \int_0^r [|E_r|^2 + |E_\theta|^2] r dr \quad (\text{ergs/sec})$$

Similarly, power deposition per cubic centimeter and per unit length for ELD and second harmonic are calculated from IV-4.3. (13) and IV-4.2.(37) with a Gaussian temperature profile defined by IV-2.3.(2). Finally, the single pass radiation resistance can be found from (6),  $E_\theta$  ( $r =$  antenna) and equation IV-1.2 (5).

Figure 1 is a simplified operational block diagram of TFR-EZ. Data can either be written directly in the beginning of the main program or through more convenient files. The initial data is then normalized and reduced to more compact forms, and proper initial conditions are set for  $E_\theta$  and  $E'_\theta$  as a function of  $m$ . ADIRKG is a general fourth order subroutine that sets up (6) and (7). SYDELC is a specific subroutine that calculates (7) from the initial reduced data. CALPI calculates  $n(r)$  and  $\partial n(r)/\partial r$  from (2). The field components can then be calculated and printed as the main loop increases  $r$  to a maximum radius, where all the field components can be plotted as a function of minor radius.

# Block Diagram of TFR-EZ

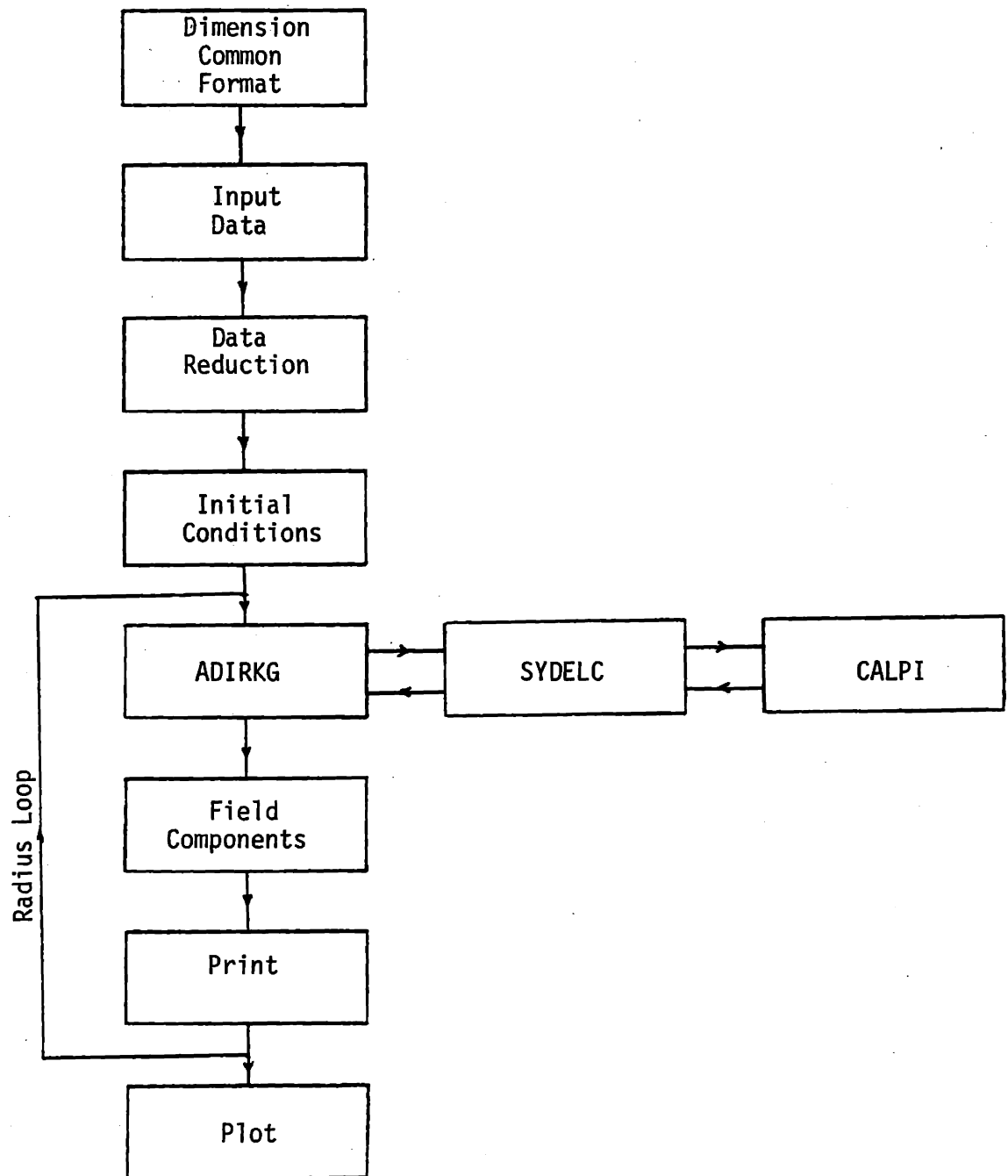


Figure 1

Similarly, power deposition per cubic centimeter and per unit length for ELD and second harmonic are calculated from IV-4.3. (13) and IV-4.2.(37) with a Gaussian temperature profile defined by IV-2.3.(2). Finally, the single pass radiation resistance can be found from (6),  $E_\theta$  ( $r =$  antenna) and equation IV-1.2 (5).

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# Block Diagram of TFR-EZ

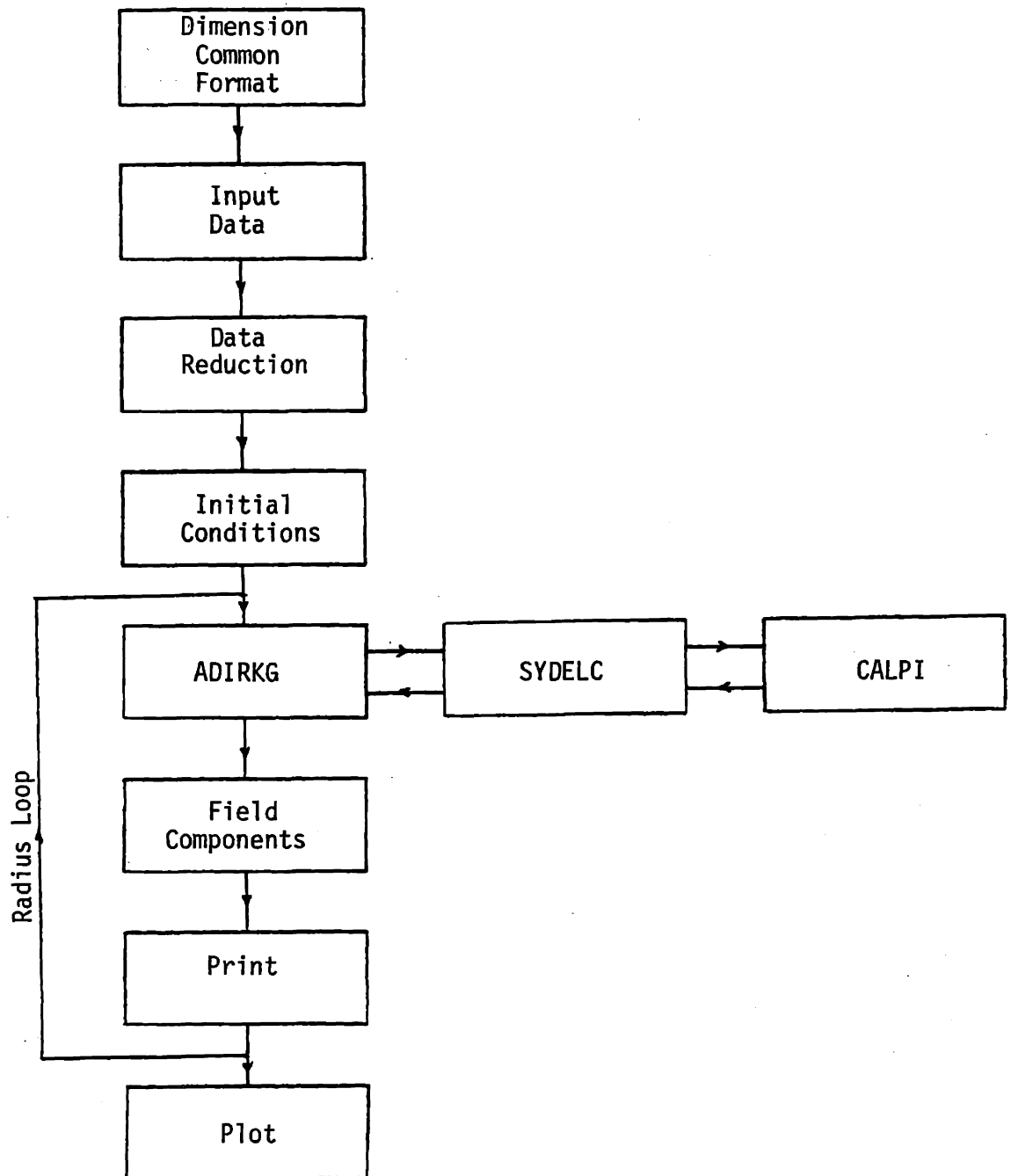


Figure 1

### IV-5.3. Field profiles at high density

In this Section, we will discuss the Alcator ICRF field profiles for a hot ( $T_e = T_i = 1\text{keV}$ ) high density ( $\hat{n}_e = 5 \times 10^{14}/\text{cm}^3$ ) hydrogen plasma in the second harmonic regime ( $f_0 = 200\text{ MHz}$ ).

Figures 1-4 show  $E_\theta$  vs  $r$  for  $m = 0$  and radial mode numbers  $\mu = 1-4$ . Note how Figure 3 is in excellent agreement with our simple Cartesian model of Figure IV-2.3.(6) (except for the Bessel function effect discussed in Section IV-3.2.). Note also the extreme importance of the evanescent layer for the smaller  $\mu$  ( $R_R \approx 10^{-3}\Omega$ ). The evanescent layer is, on the other hand, almost negligible for TFR, as can be seen from Figures 5 and 6, since  $k_{||}$  is of order .05/cm instead of .5/cm for Alcator.

Figures 7, 8, and 9 are the complete set of electric and magnetic fields and power absorption profiles,  $E_\theta$ ,  $E_r$ ,  $E_z$ ,  $B_\theta$ ,  $B_r$ ,  $B_z$ ,  $S_{\pm z}$ ,  $E_+^2$ ,  $E_+'^2$ ,  $P_{\text{eld}}/\text{cm}^3$ ,  $P_{2\omega_{ci}}/\text{cm}^3$ ,  $P_{2\omega_{ci}}/\text{cm}$ , as well as the radiation resistance calculated from  $E_\theta$  (11.5 cm), for  $\mu = 3$ ,  $m = 0, -1, +1$ . Many particularities need to be noted as follows.

The simplest mode to visualize is the asymmetric ( $E_\theta$ , about  $\pm r$ )  $m = 0$  mode shown in Figure 7.  $m = \pm 1$  are symmetric as shown in Figures 8 and 9. We must be careful with the definition of  $\mu$ , since only  $B_z$  (in homogeneous plasma) is a pure Bessel function ( $E_\theta$  is the sum of two Bessel functions, IV-3.1.(19)), so that the number of zero crossings is different for  $E_\theta$  and  $B_z$ . We will use  $E_\theta$  in defining  $\mu$  because of the wall boundary condition. Since the  $m = +1$  has the lowest density cutoff, it has the largest  $k_{||}$  for a given  $\mu$ , and thus has the largest evanescent layer and smallest radiation resistance ( $R_{+1} < R_0 < R_{-1}$ ).

In the plasma center, the magnitudes of  $E_r$  and  $E_\theta$  are comparable for all modes, and about three orders of magnitude larger than  $E_z$  (also  $B_\theta \approx B_r \approx B_z$ ). The phase between  $E_r$  and  $E_\theta$ , on the other hand, is very different

for the different poloidal mode numbers, so that the  $m < 0$  are essentially left-handed ( $E_\theta \approx -iE_r$ ),  $m > 0$  are right-handed ( $E_\theta \approx iE_r$ ), and  $m = 0$  is about half and half.<sup>21</sup> Second harmonic damping is dependent on the gradient of the left-hand component, so that the  $m < 0$  will have shorter parallel damping length ( $\approx 5$  meters) than the  $m > 0$  modes ( $\approx 60$  meters), as shown in Figure 10.

$$(1) \quad \frac{1}{k_i} = \frac{2P_f}{P_{2\omega_{ci}}/\text{cm}} = \frac{\text{PHI}}{P2WPL}$$

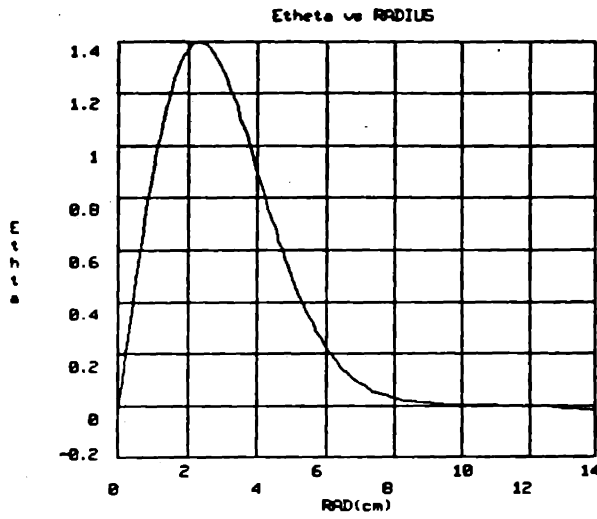
The effect of  $k_z$  on the damping length can be removed by dividing the Poynting flux ( $\text{PHI} \propto E^2 k_z$ ) by  $k_z$ , also shown in Figure 10.

From Figures 7, 8, and 9, it is clear that it is not possible to discern the  $m > 0$  from the  $m < 0$  modes by the relative phases between field components in the evanescent plasma edge (from either E or B). Most of the wave fields are well confined to the plasma center, so that most of the Poynting flux is from  $r_{\text{PHI}} < 8$  cm (Figures 7-9 and 11-14). The ELD and  $2\omega_{ci}$  power deposition profiles are even more peaked, due to the narrow Gaussian temperature profile, so that  $r_{\text{ELD}} < 3$  cm and  $r_{2\omega_{ci}} < 4$  cm.

Figures 15 to 18 show the single parallel pass radiation resistance ( $R_o$ , IV-1.3.(5)), the toroidal resonance ( $F_R R_o$ , IV-6.2.(5)) and anti-resonance ( $F_A R_o$ , IV-6.2.(6)), corrected radiation resistance,  $k_{||}$ ,  $\lambda_{2\omega_C}$  and  $\lambda_{\text{ELD}}$  as a function of density for the  $m = 0$ ,  $\mu = 3$  and 4 eigenmodes (the singularities at cutoff are, of course, unphysical, and are removed by letting  $k_{\perp i} \neq 0$  (Section IV-2.1.) .

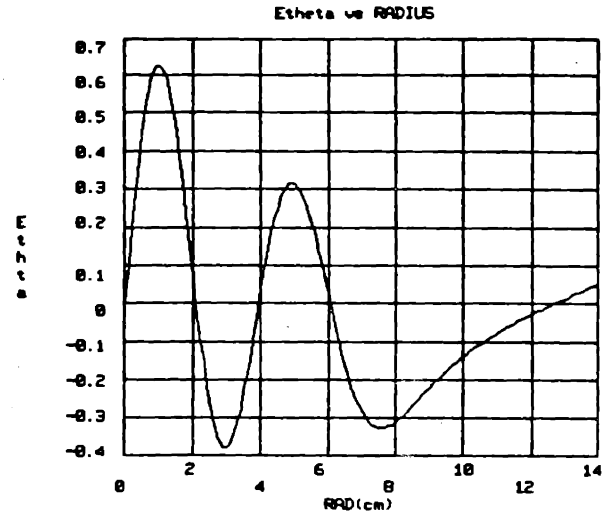


$m = 0, E_\theta$  profiles in Alcator and TFR



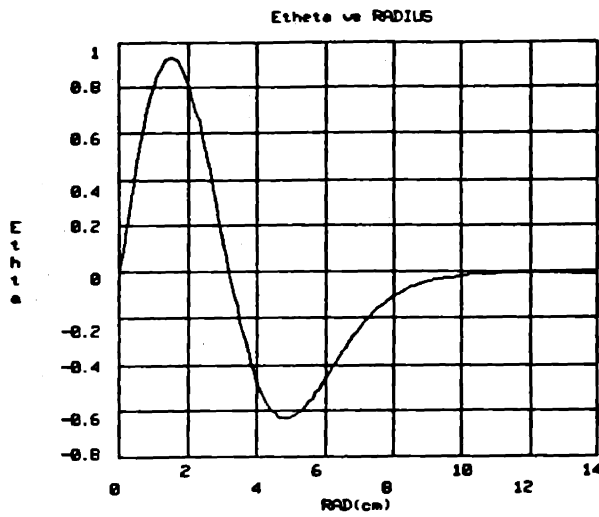
$\mu = 1$

Figure 1



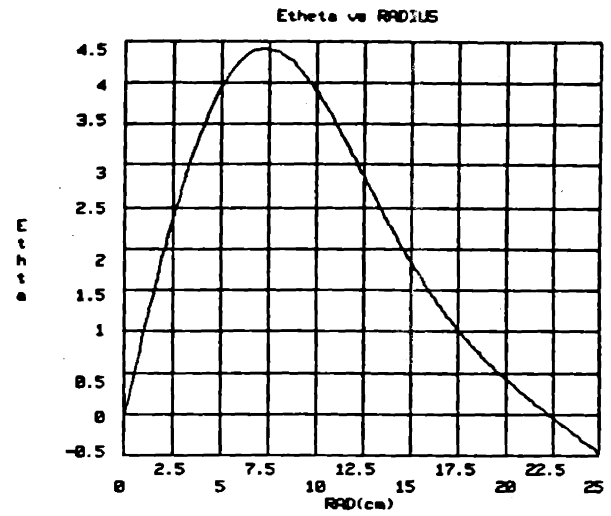
$\mu = 4$

Figure 4



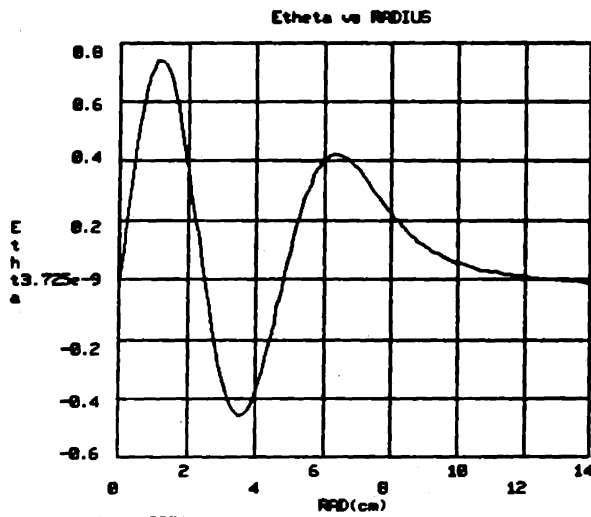
$\mu = 2$

Figure 2



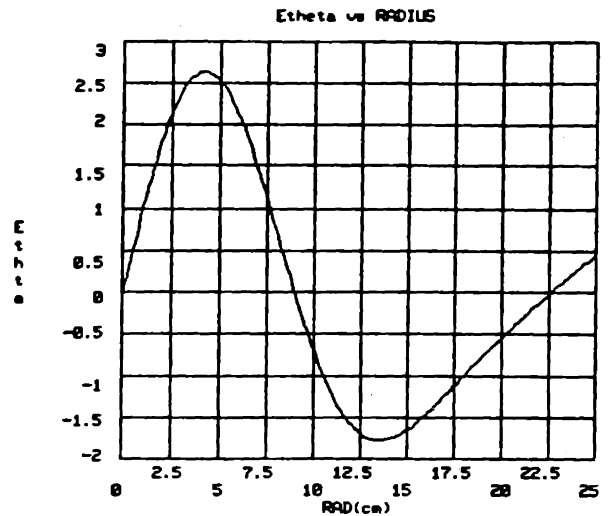
TFR  $\mu = 1$

Figure 5



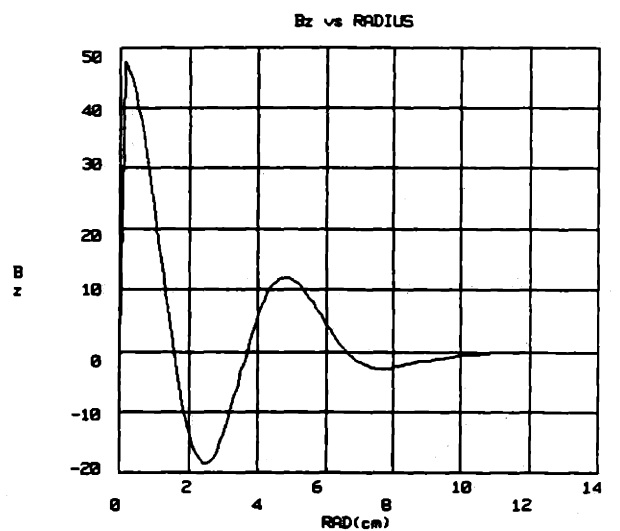
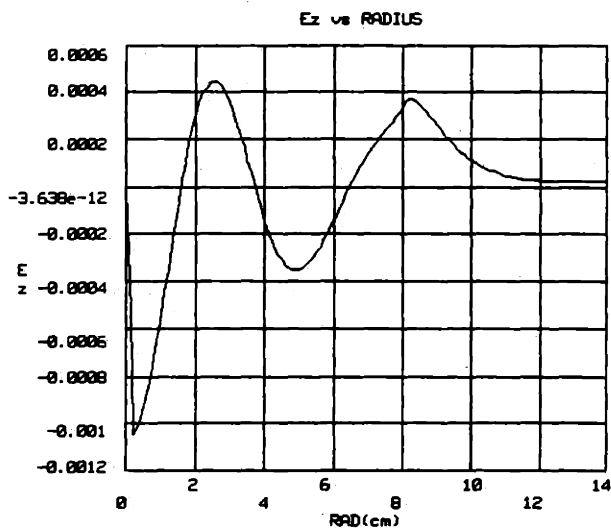
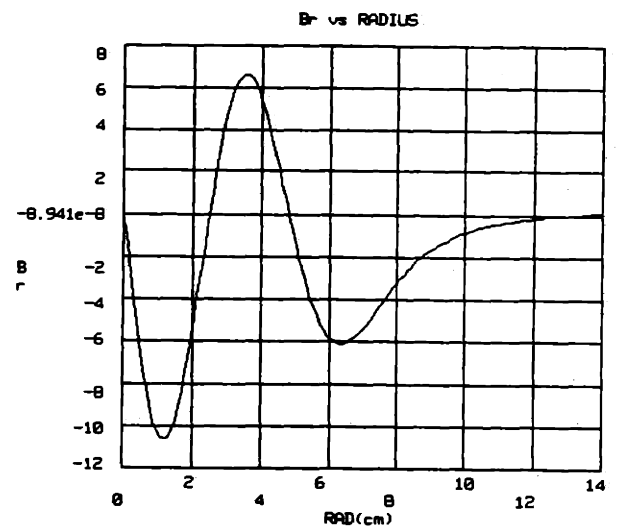
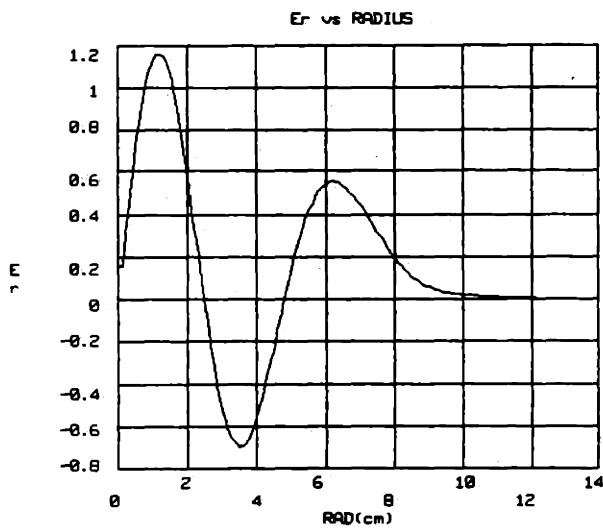
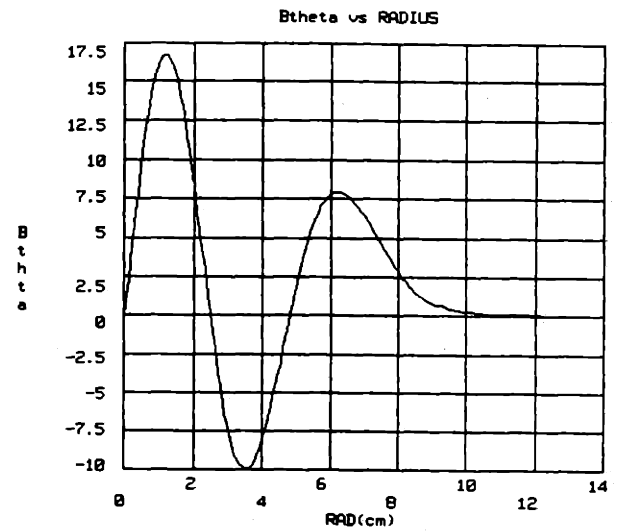
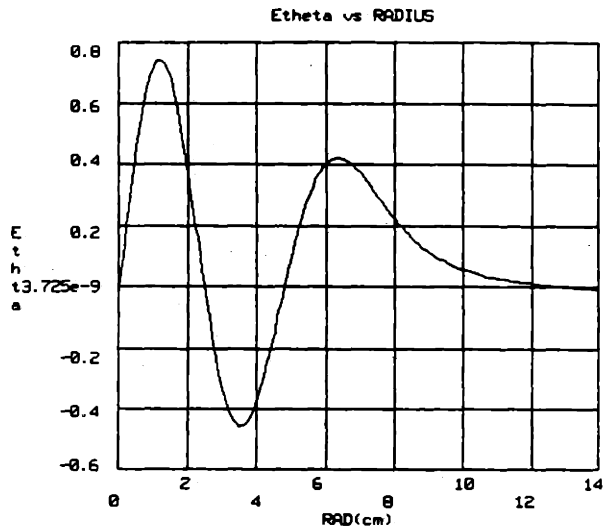
$\mu = 3$

Figure 3



TFR  $\mu = 2$

Figure 6



$m = 0, \mu = 3$  field profiles  
at high density

Figure 7

$\omega a = 2.0$      $\omega b = 2.0$      $n_a = .50000E+15$      $n_b = .00000E+00$   
 $m_a = 1.00$      $m_b = 1.00$      $T_i = 1000.0$      $T_e = 1000.0$   
 $kpar = .60291$      $m = 0.0$      $r_{max} = 14.00$      $pp = 9.00$   
 $lent = 20.00$      $f0 = .20000E+09$      $r_{maj} = 54.00$      $q0 = 0.90$   
 $q1 = 5.00$      $r_{ant} = 11.00$      $pas = 0.100$   
 $R = .6340E-01$      $e1dd1 = .17296E+05$      $p2ud1 = .22304E+04$

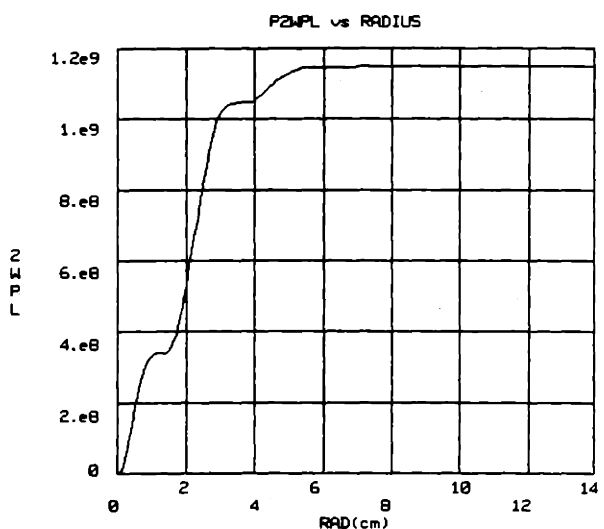
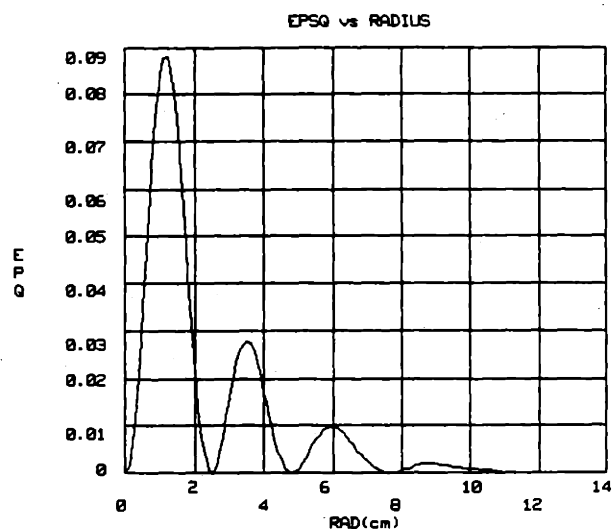
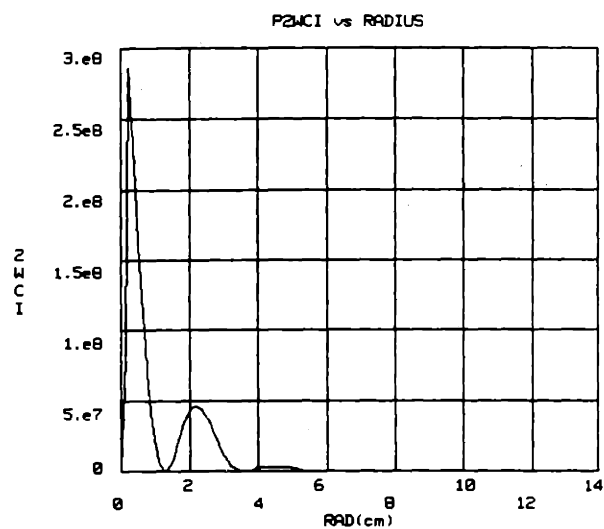
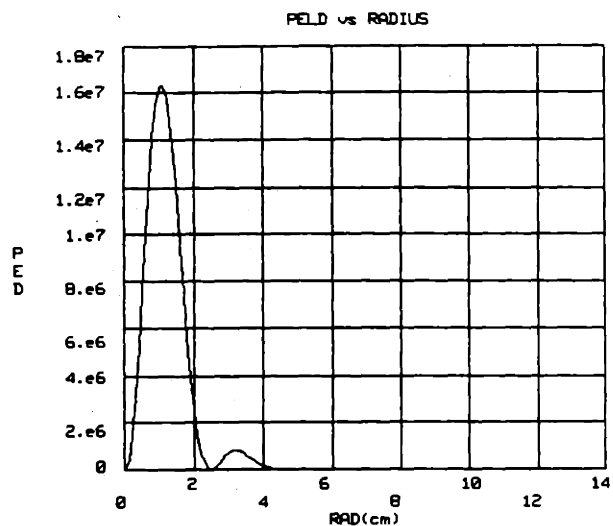
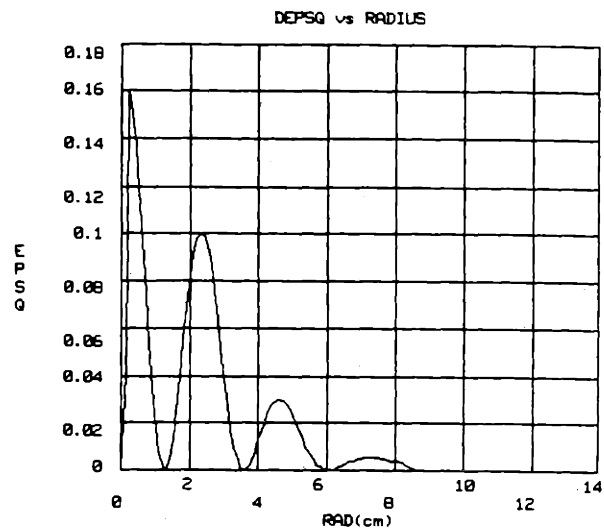
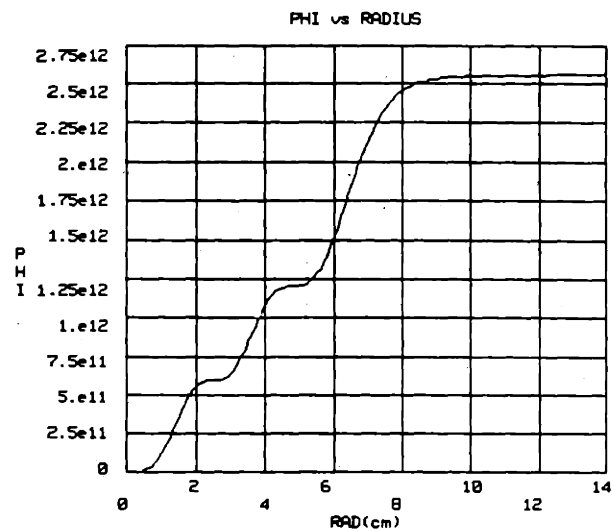
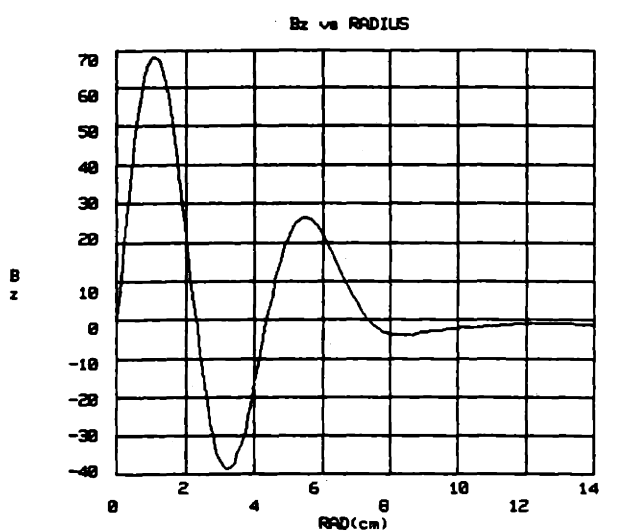
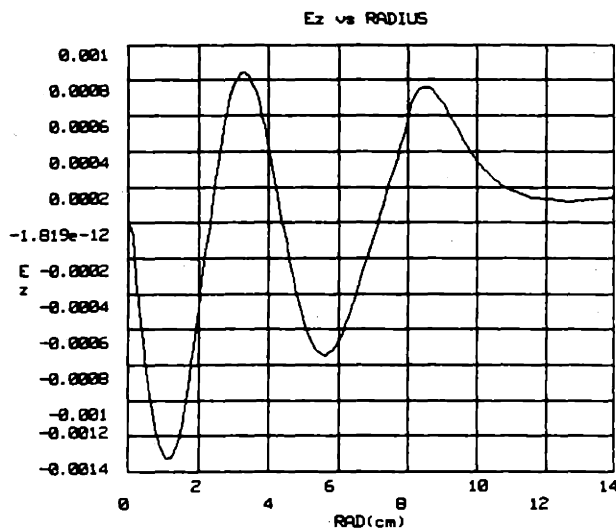
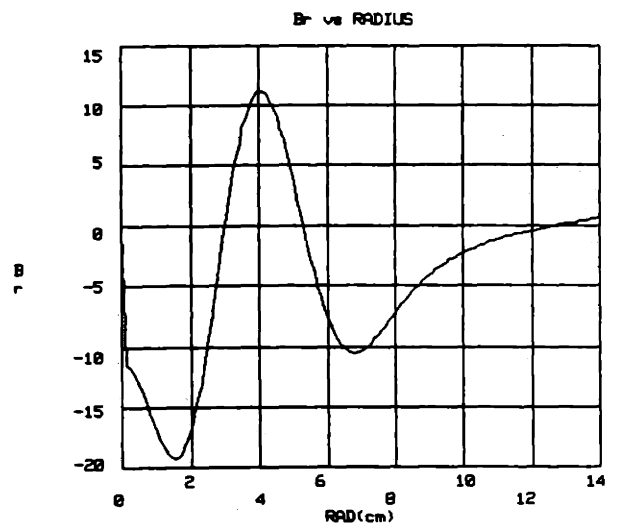
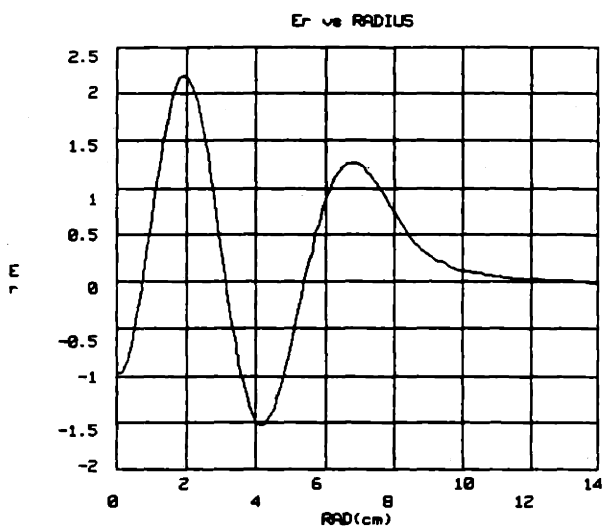
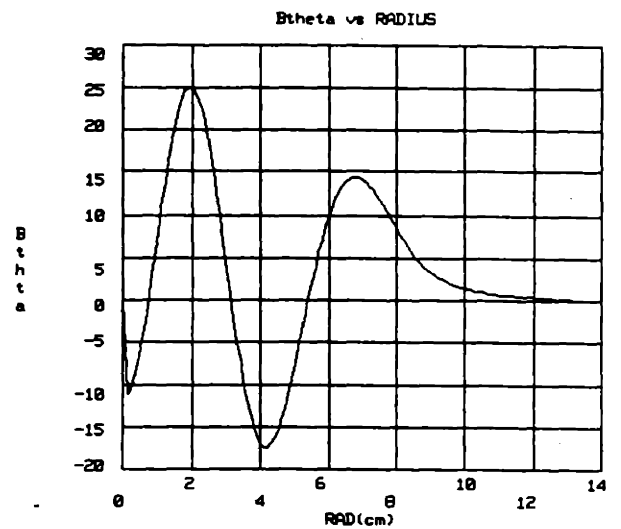
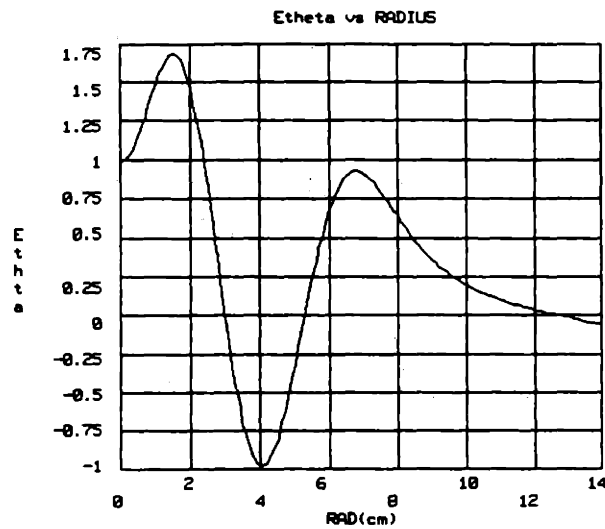


Figure 7 continued

omega a= 2.0	omega b= 2.0	na=.50000E+15	nb=.00000E+00
ma=1.00	mb=1.00	Ti=1000.0	Te=1000.0
Kpar=.68291	m= 0.0	rmax=14.00	pp= 9.00
lant=20.00	f0=.20000E+09	rmaJ=54.00	q0= 0.90
ql= 5.00	rant=11.00	pas=0.100	

R=-.63400E-01    eldd1=.17296E+05    p2wd1=.22304E+04



$m = -1, \mu = 3$  field profiles  
at high density

Figure 8

$\omega a = 2.0$      $\omega b = 2.0$      $n_a = .50000E+15$      $n_b = .00000E+00$   
 $m = 1.00$      $m = 1.00$      $T_i = 1000.0$      $T_e = 1000.0$   
 $K_{per} = .47800$      $m = -1.0$      $r_{max} = 14.00$      $p_p = 9.00$   
 $l_{ent} = 20.00$      $f_0 = .20000E+09$      $r_{maj} = 54.00$      $q_0 = 0.50$   
 $q_1 = 5.00$      $r_{ent} = 11.00$      $p_{as} = 0.100$   
 $R_r = -.17488E+00$      $e_{ldd1} = .40135E+05$      $p_{2nd1} = .65710E+03$

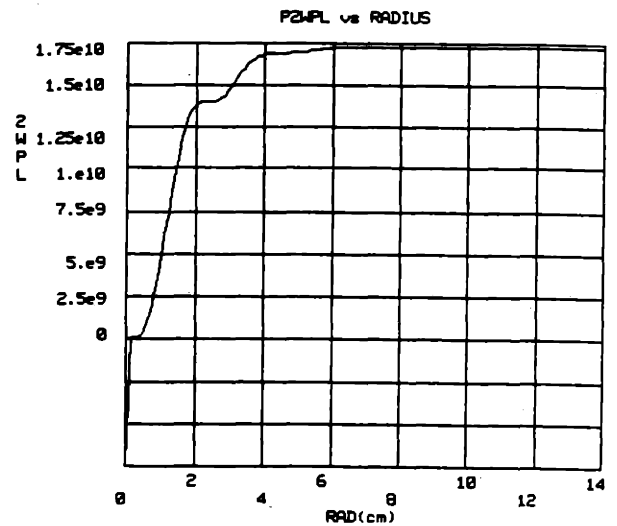
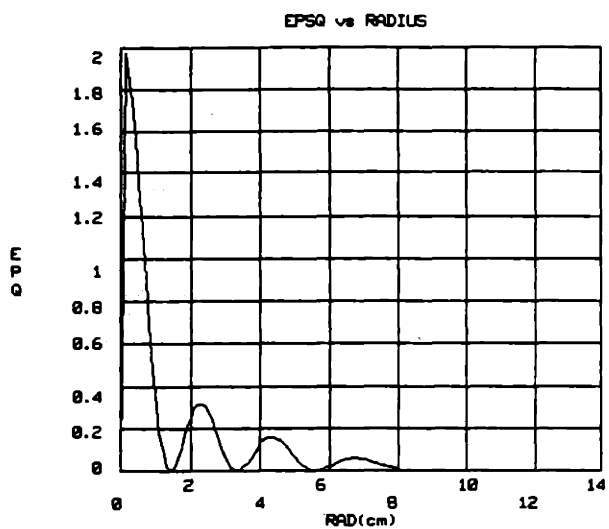
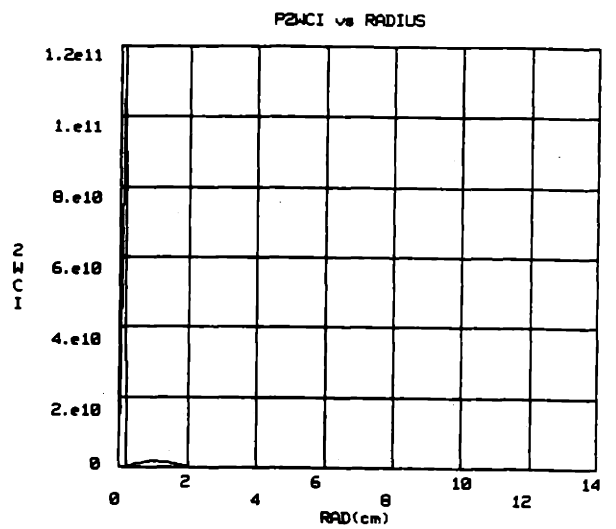
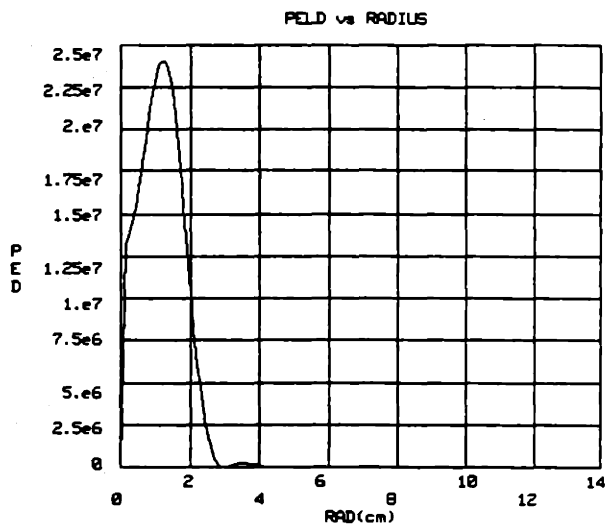
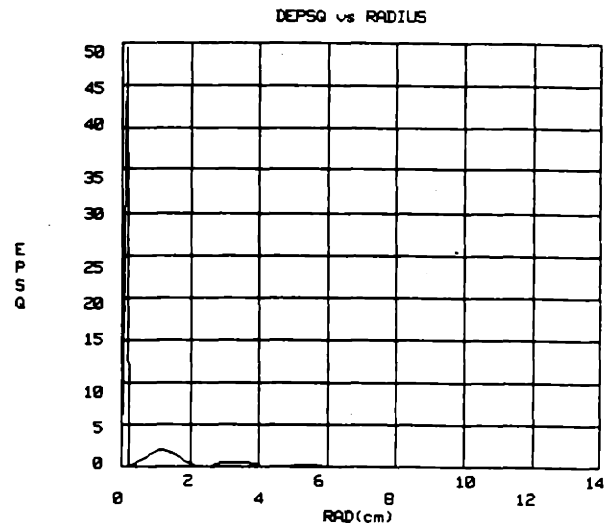
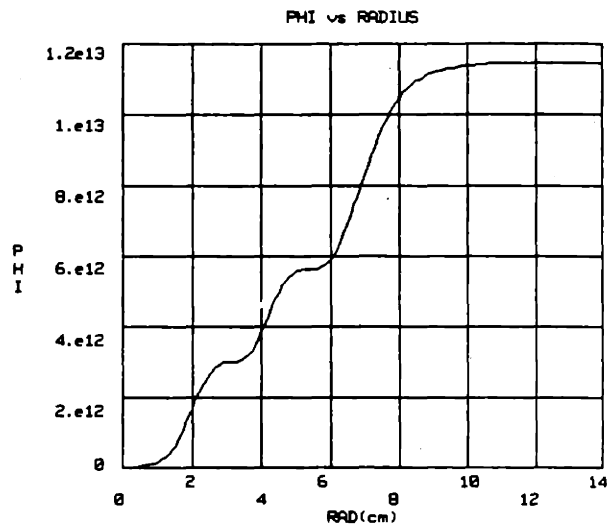
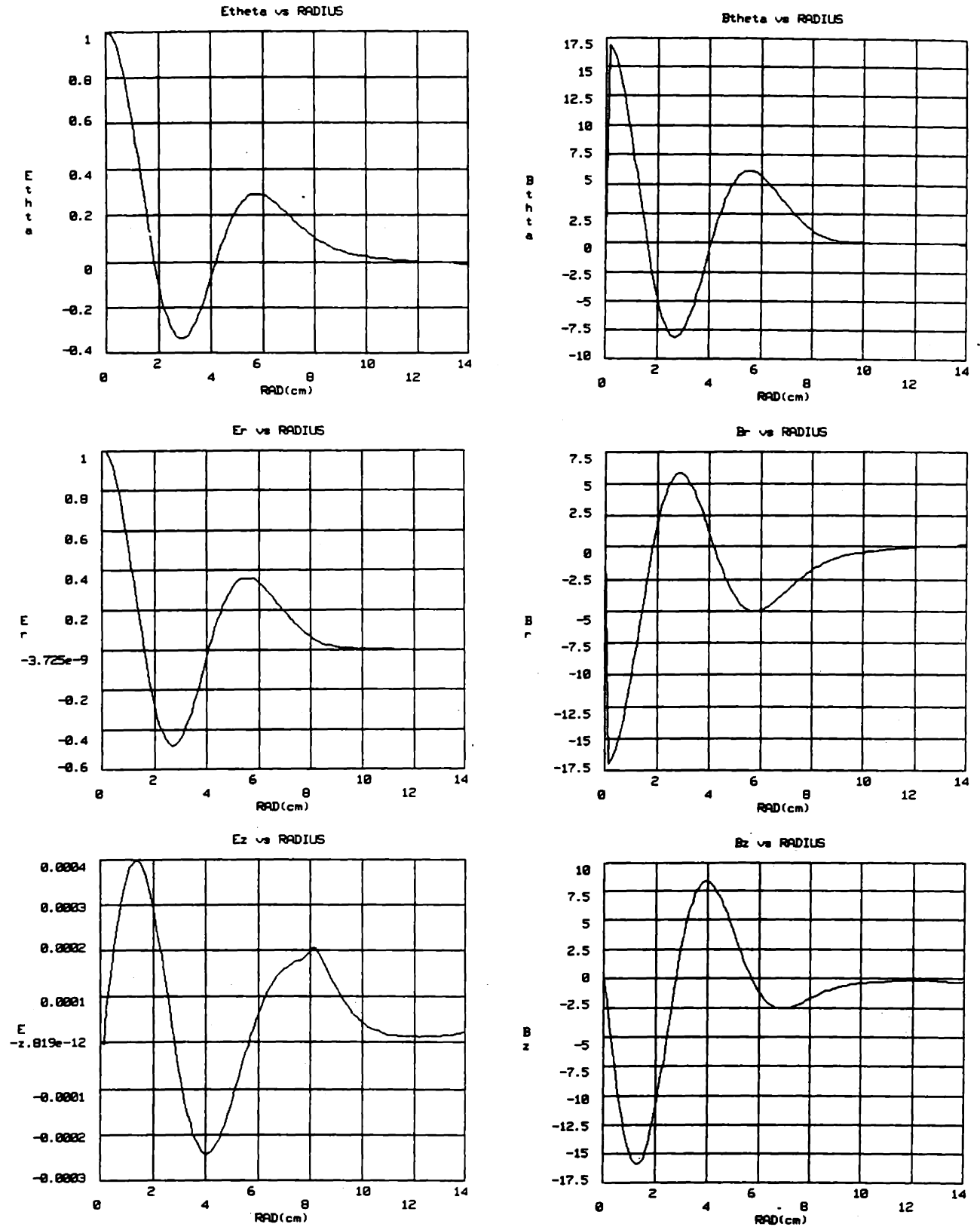


Figure 8 continued

omega a= 2.0	omega b= 2.0	na=.50000E+15	nb=.00000E+00
ma=1.00	mb=1.00	Ti=1000.0	Te=1000.0
Kpar=.47800	m=-1.0	rmax=14.00	pp= 9.00
lant=20.00	f0=.20000E+09	rma=54.00	q0= 0.90
q1= 5.00	rant=11.00	pas=0.100	
R=.17400E+00	e1dd1=.40135E+05	p2wd1=.65710E+03	*



$m = +1$ ,  $\mu = 3$  field profiles  
at high density

Figure 9

$\omega_a = 2.0$      $\omega_b = 2.0$      $n_a = .5000E+15$      $n_b = .0000E+00$   
 $m = 1.00$      $m = 1.00$      $T_i = 1000.0$      $T_e = 1000.0$   
 $K_{par} = .71288$      $m = 1.0$      $r_{max} = 14.00$      $pp = 9.00$   
 $l_{ent} = 20.00$      $f_0 = .2000E+09$      $r_{maj} = 54.00$      $q_0 = 0.90$   
 $q_1 = 5.00$      $r_{ent} = 11.00$      $pas = 0.100$   
 $R = .1194E-01$      $e_{ldd1} = .7376E+04$      $p2wd1 = .9500E+04$

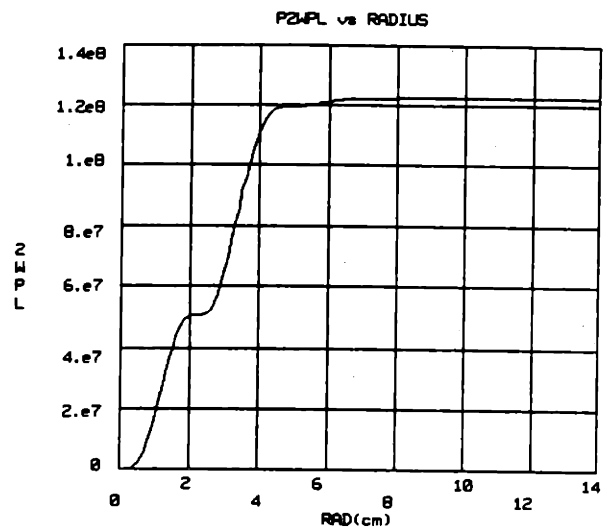
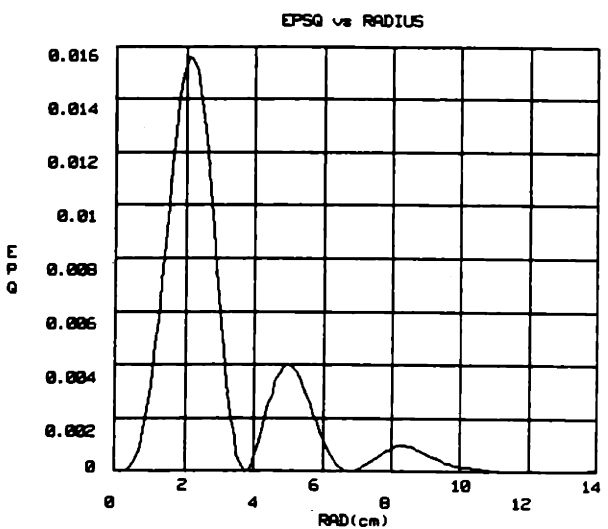
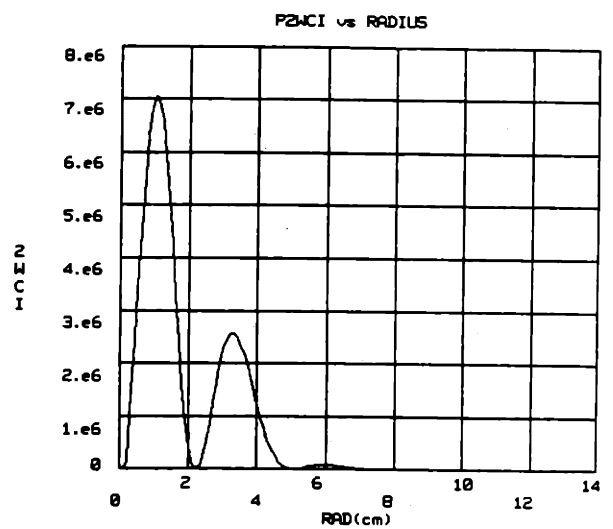
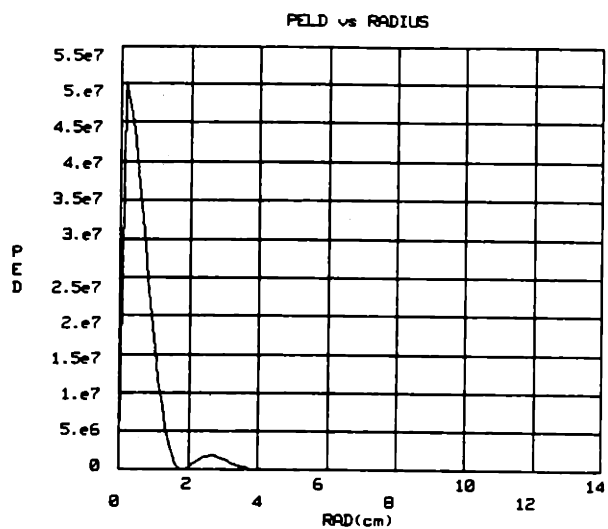
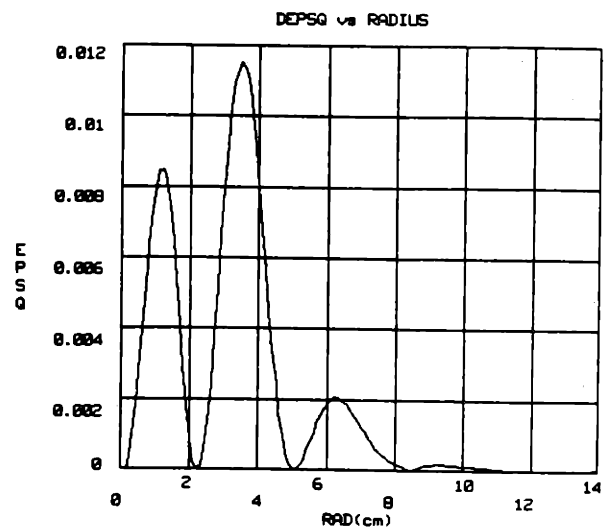
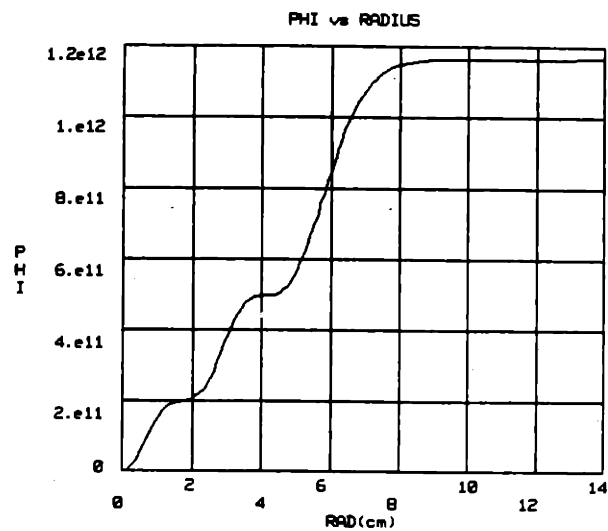


Figure 9 continued

omega a= 2.0	omega b= 2.0	na=.50000E+15	nb=.00000E+00
ma=1.00	mb=1.00	Ti=1000.0	Te=1000.0
Kpar=.71298	m= 1.0	rmax=14.00	pp= 9.00
lant=20.00	f0=.20000E+09	rmaJ=54.00	q0= 0.90
ql= 5.00	rant=11.00	pas=0.100	

R=.11944E-01    elddl=.73785E+04    p2wdl=.95003E+04

Damping Length for  $\mu = 3$ ,  $-3 \leq m \leq +3$  at High Density

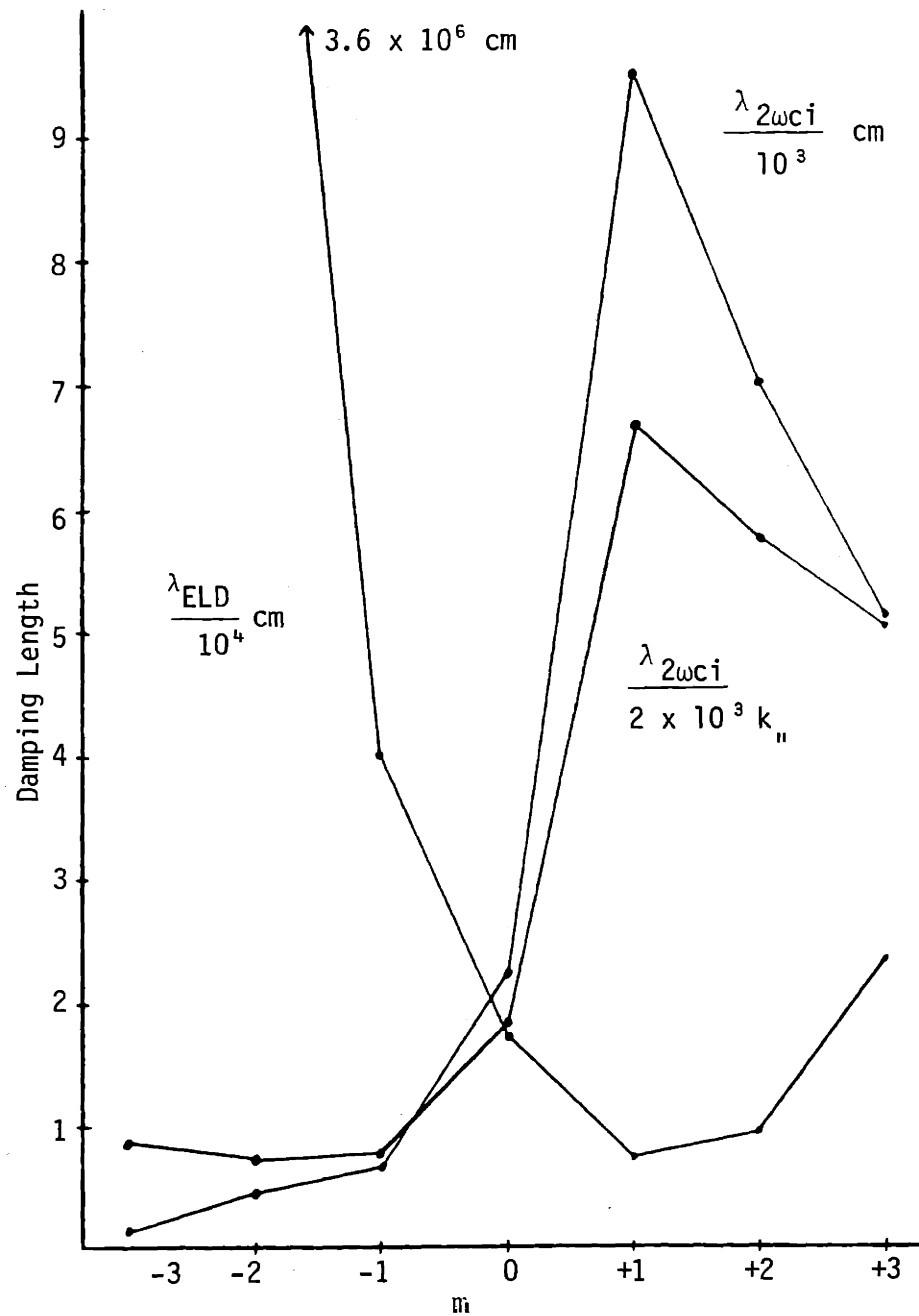
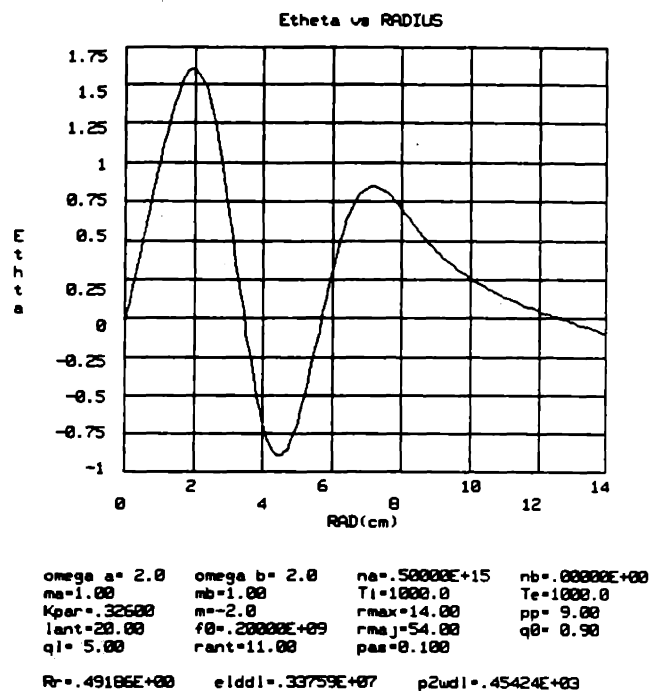


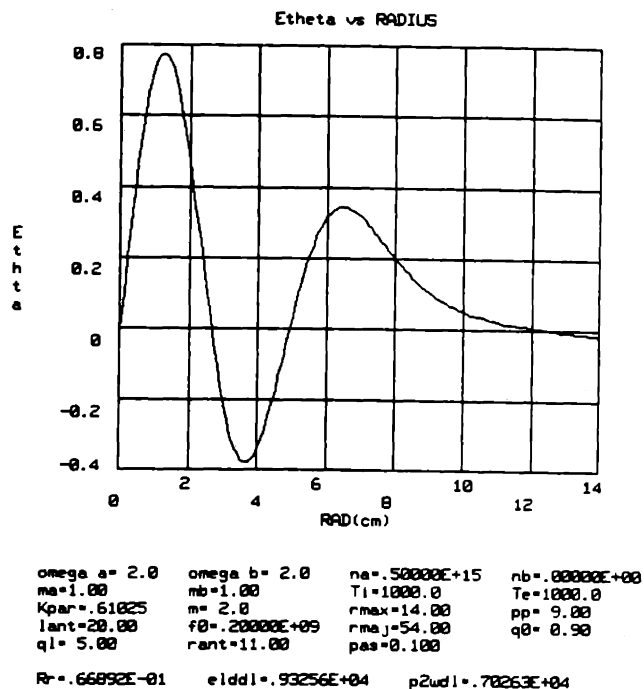
Figure 10





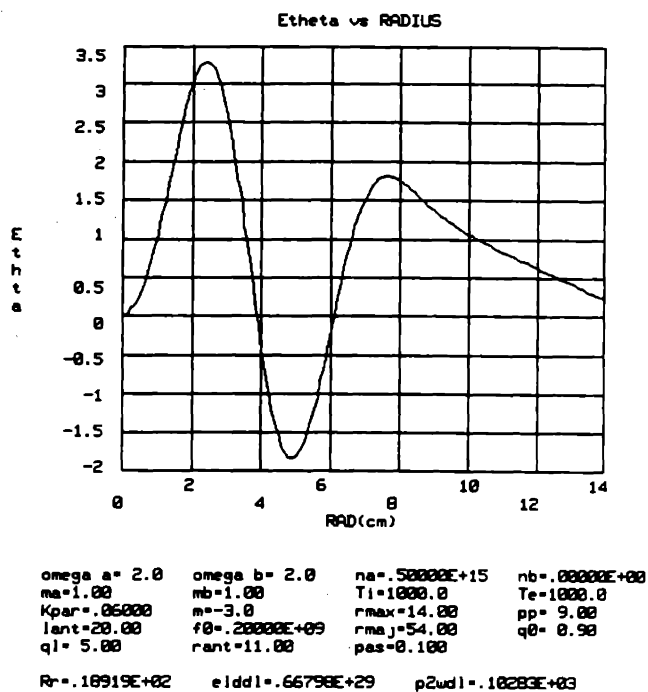
$m = -2, \mu = 3$  at high density

Figure 11



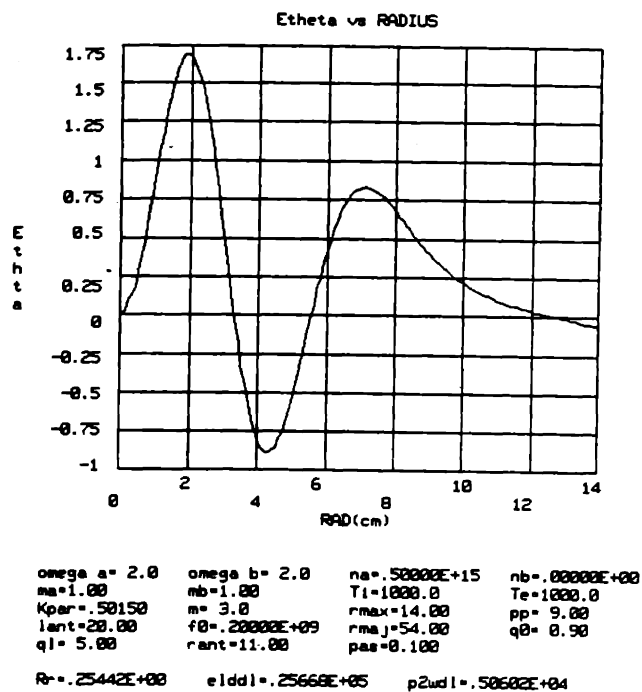
$m = +2, \mu = 3$  at high density

Figure 12



$m = -3, \mu = 3$  at high density

Figure 13



$m = +3, \mu = 3$  at high density

Figure 14

Radiation Resistance and Damping Lengths for  $\mu = 3$  and  $m = 0$   
as a Function of Density

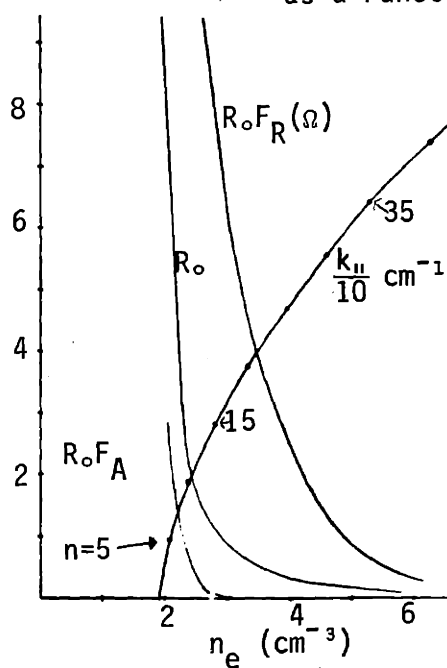


Figure 15

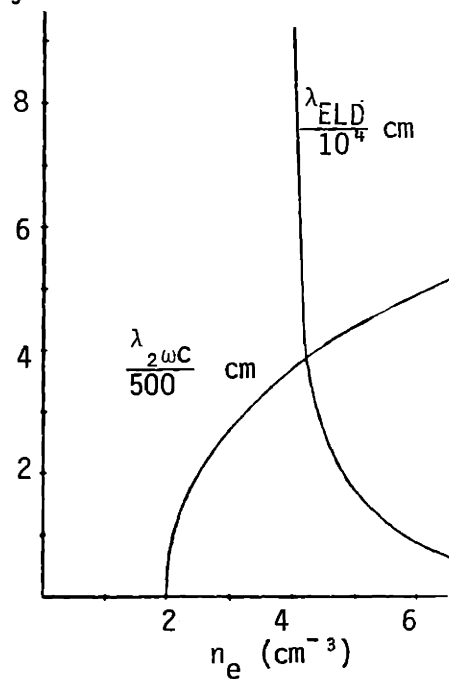


Figure 16

Radiation Resistance and Damping Lengths for  $\mu = 4$  and  $m = 0$   
as a Function of Density

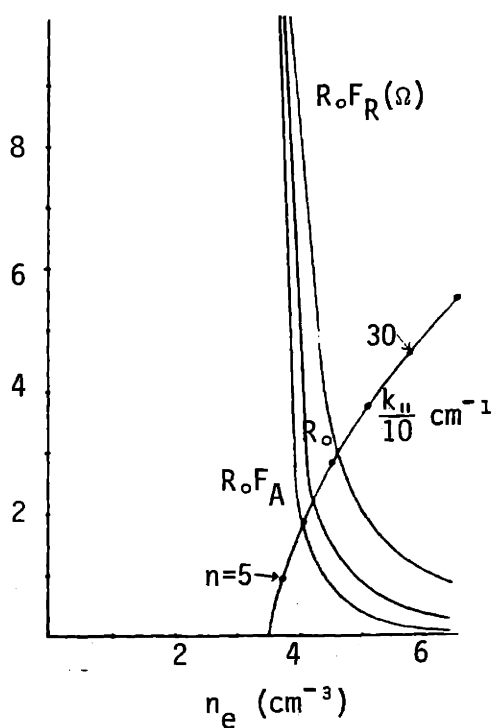


Figure 17

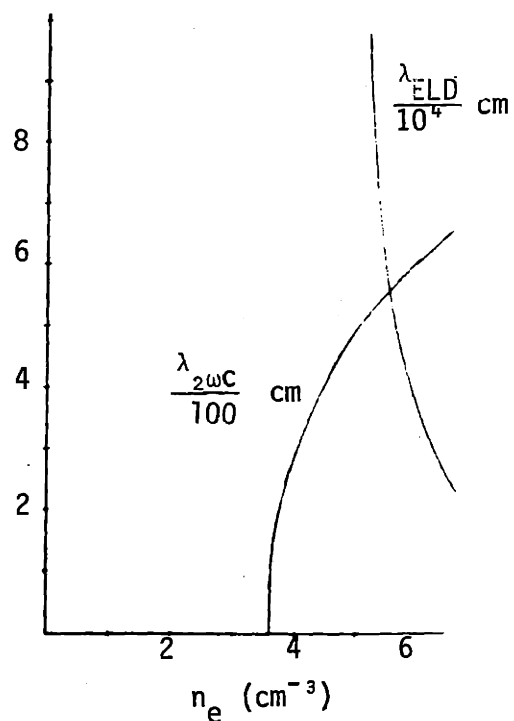


Figure 18

#### IV-5.4. Full inhomogeneous eigenmode dispersion relations

As we might expect, the full inhomogeneous eigenmode dispersion relation cannot be put in some simple form such as equation IV-3.1.(14). Numerically; however, TFR-EZ can be put in a loop that successively increases  $k_z$  and  $n_e$ , following the  $E_\theta(w) = 0$  boundary solution, and thus tracing the dispersion relation (Figure 1). Although simple in principle, this is a very large computation, and step sizes of the many nested loops must be judiciously chosen to ensure reasonable computation times, even on large computer systems such as Multics. The program is started with the usual boundary conditions ( $E_\theta, E'_\theta$ ), but with a very small  $n_e$  and  $k_z$ . After each TFR-EZ integration from the plasma center to the wall, the wall  $E_\theta$  is multiplied with its previous value. If the product is positive, no solution was crossed, and we increase  $n$  and rerun TFR-EZ. At some value of  $n$ , an onset (cutoff) will occur, and the product will be negative. We then increase  $k_z$  until the product is again positive, and so on. To find higher radial mode numbers, the program is simply started after a lower radial mode cutoff, but still with  $k_z = 0$ .

Figures 3-9 are the dispersion relation of the  $0, \pm 1, \pm 2, \pm 3$  eigenmodes for 200 MHz, 67 kG, pure hydrogen and peak densities up to  $6 \times 10^{14}/\text{cm}^3$ . As we saw in our cruder model (IV-3.1.), the  $m = +1$  is the fundamental mode, and can have a radial mode number as high as 5 at high density.  $k_z$  can also be as large as 1.2/cm, which corresponds to a toroidal mode number  $n = 65$ . Some 27 eigenmodes (Figure 10) are possible, which is in good agreement with Section IV-3.2. if we take into account that we did not include the poloidal modes with  $|m| > 3$ .

For the  $m > 0$  modes, we note an apparent lack of cutoff due to the tenuous edge plasma layer. Figures 11-13 show the details of the low

density end of the dispersion relations. The fundamental and higher toroidal resonances occur at

$$(1) \quad k_{||} = \frac{1}{R}, \quad \frac{2}{R}, \quad \dots$$

$$= .018, .037, .055, .074 \dots /cm$$

and further determine  $k_{||}$ , so that densities of the order of  $4 \times 10^{12}/cm^3$  are still necessary for the appearance of the lowest toroidal resonances.

$$(2) \quad \mu = 1, \quad m = \underline{+1}, \quad n = 3$$

Note that this corresponds to edge and even central densities at or below the lower hybrid resonance, and these simulations are probably not meaningful.

Figure 14 shows the  $m = 0, \mu = 1$  mode dispersion relation for pure hydrogen and deuterium plasmas, as well as a 50%  $H_2$ /50%  $D_2$  plasma, in the  $2\omega_{CH}$  regime.

Block diagram of inhomogeneous plasma eigenmode dispersion relation code

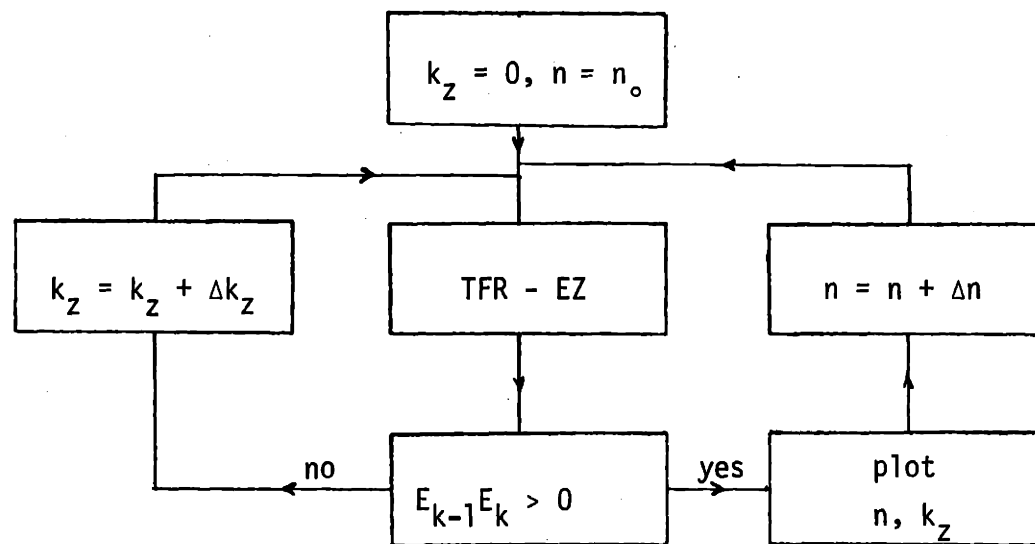


Figure 1

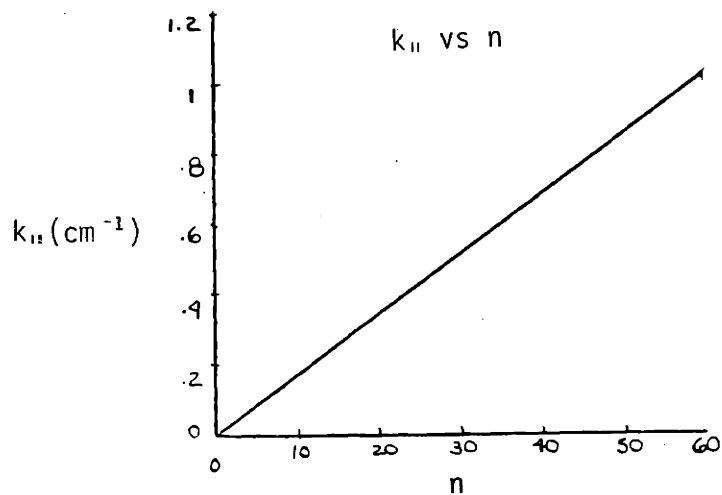


Figure 2

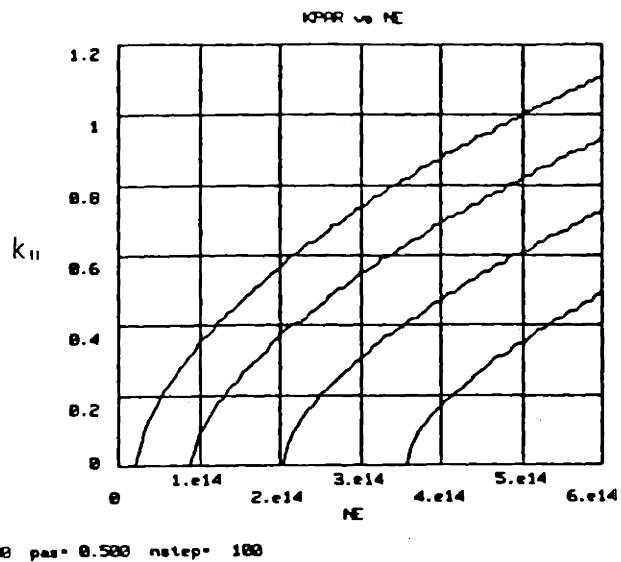


Figure 3

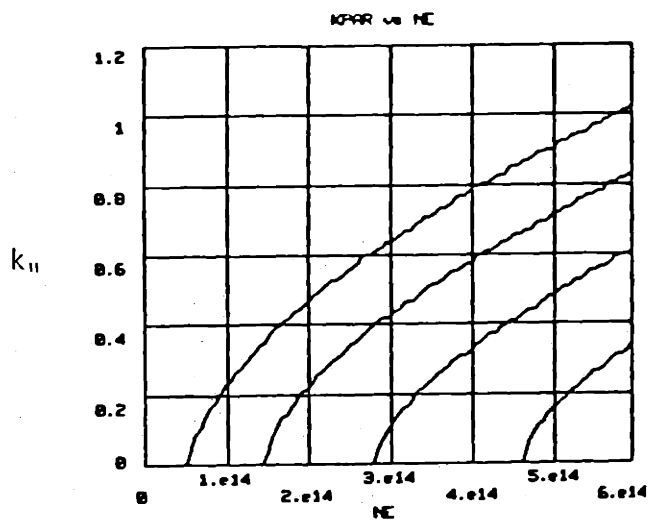


Figure 4

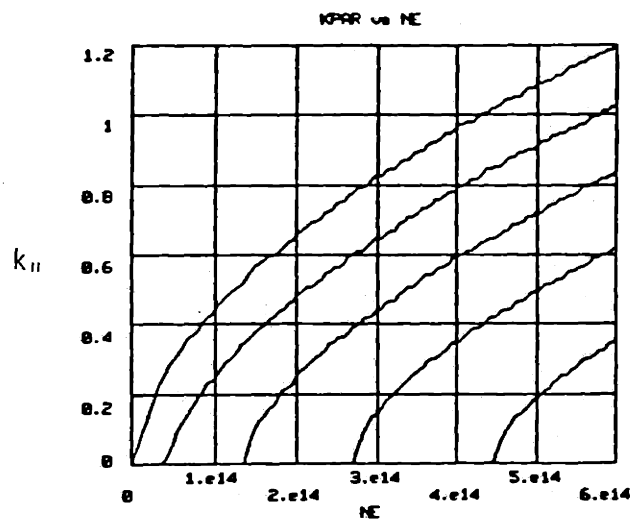


Figure 5

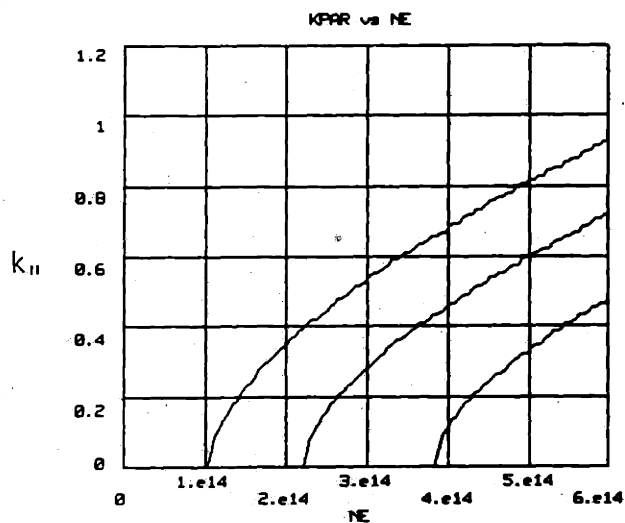


Figure 6

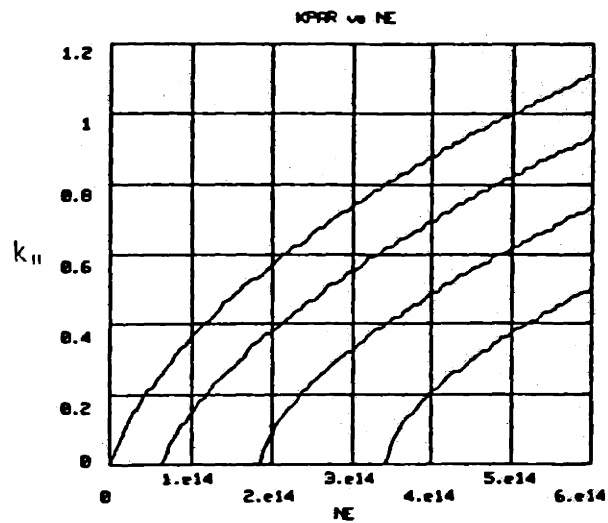
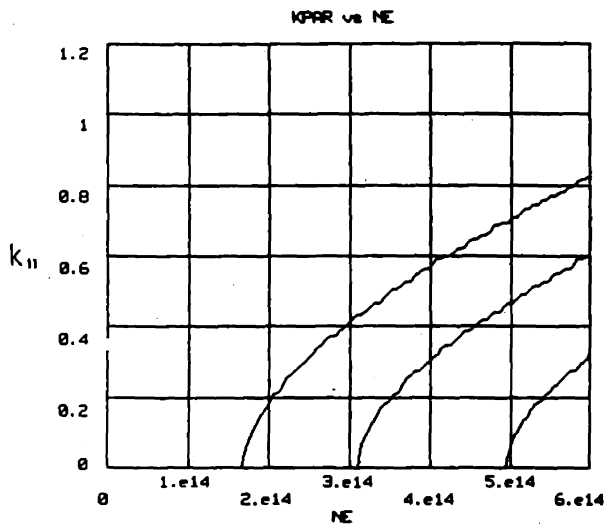
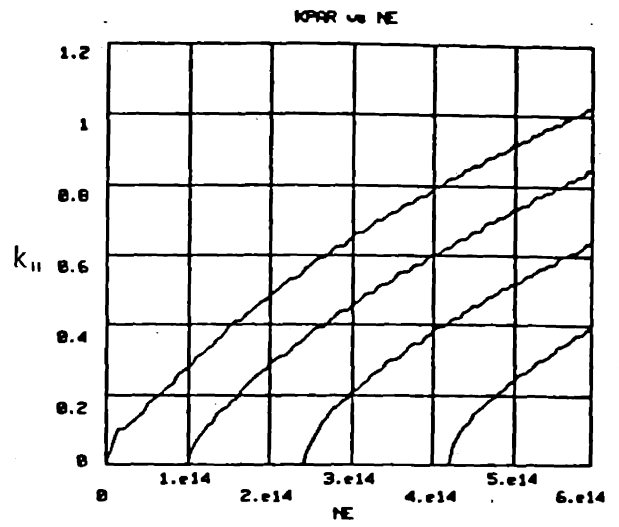


Figure 7



m=-3.000 pas= 0.500 nstep= 100 Figure 8



m= 3.000 pas= 0.500 nstep= 100 Figure 9

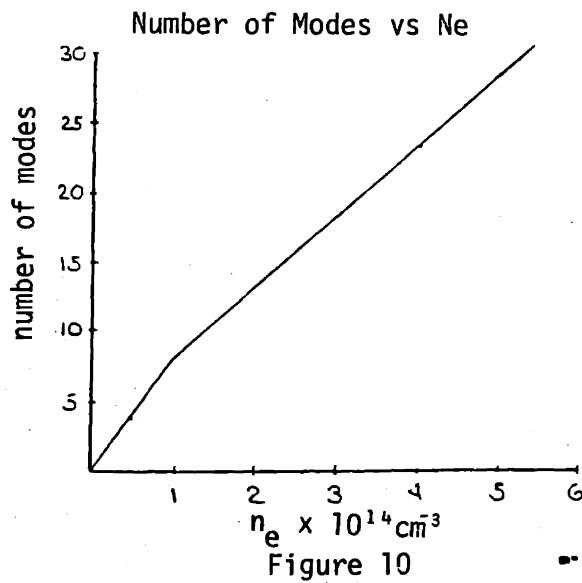
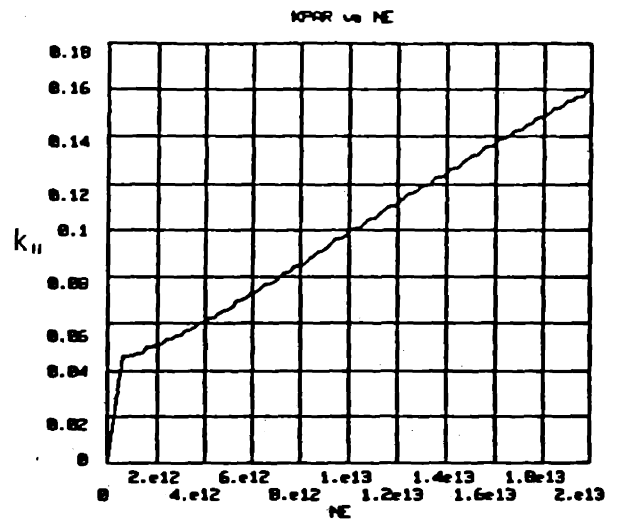
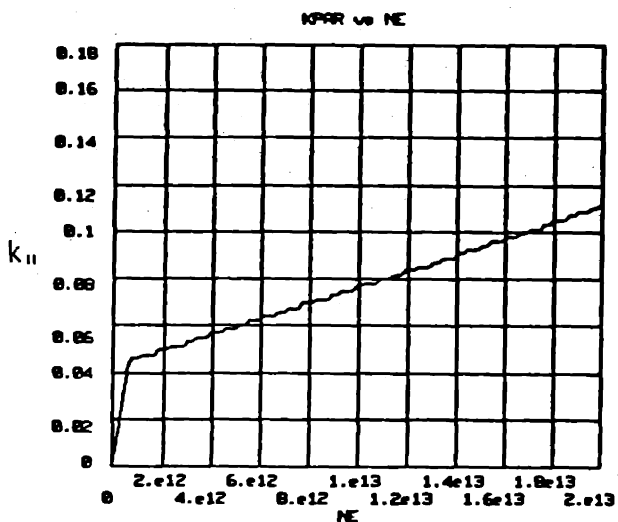


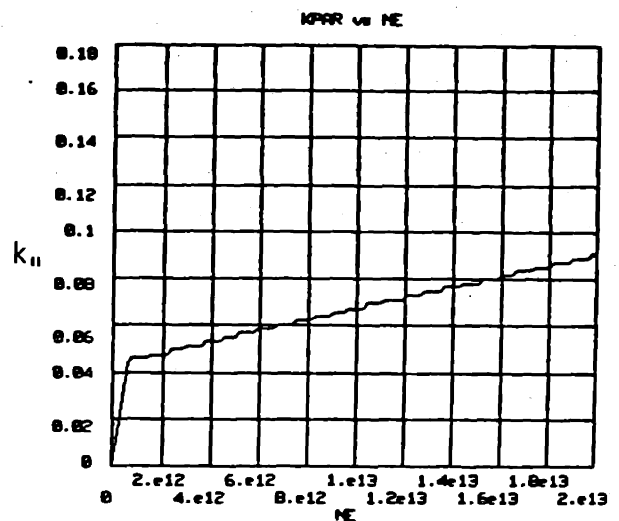
Figure 10



m= 1.000 pas= 0.500 nstep= 100 Figure 11



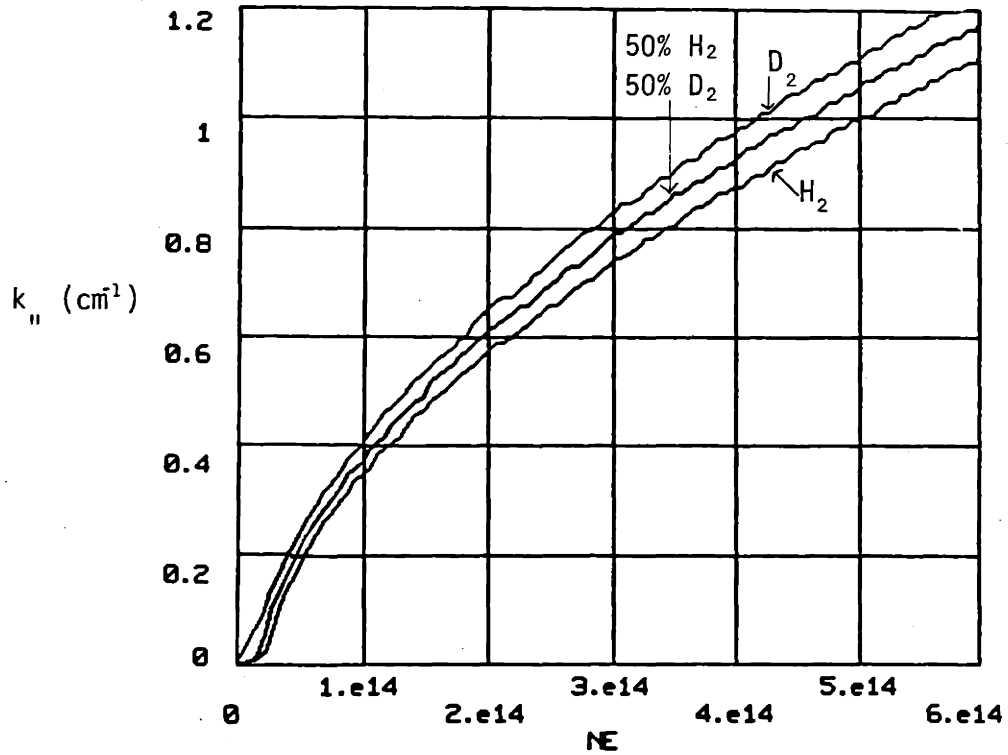
m= 2.000 pas= 0.500 nstep= 100 Figure 12



m= 3.000 pas= 0.500 nstep= 100 Figure 13

$m = 0, \mu = 1$  Dispersion Relations for Pure  $H_2$ , Pure  $D_2$ , and 50%  $H_2$  / 50%  $D_2$  Plasmas

KPAR vs NE



omega a=2.00 omega b=4.00 nb/na=1.000  
ma=1.00 mb=2.00 pm=0.00  
f0=.20000E+09 rmax=14.00 pp= 9.00 rw=12.50

Figure 14



#### IV-6. Stochastic Mode Stacking

##### IV-6.1. Introduction

The last theoretical aspect of eigenmode coupling, and one believed to be of importance at high density, is stochastic mode stacking in a multi-moded plasma cavity.<sup>24</sup> This somewhat new theory and original work, at least in the fusion field, is now discussed at length.<sup>23-25,7,93</sup>

In the Alcator program, our goal is to heat at high density, where a multitude of modes can propagate, many of which are coupled by the antenna, and several of which can be simultaneously resonant, due to their finite bandwidth in  $k$  space. This is fundamentally different from the regime of most previous experiments which were at low density, where only one mode at a time was important. Either a particular mode was actually tracked, or heating was efficient only for the short duration of the mode resonance.<sup>52</sup> Very large damping mechanisms can also be used to stop the toroidal resonances and stabilize the radiation resistance, but at a significant loss in antenna loading, a major engineering drawback.<sup>30</sup>

The first issue is the overlap of the skirts of the finite bandwidth modes. The coherent stacking of the many eigenmodes at the antenna will result in the radiation resistance increasing linearly with the number of modes present, whereas statistical interference away from the antenna will result in a less than linear increase in wave field and RF surface probe signal. This complicated multi- $k$  wave structure can be analyzed in analogy to noise theory<sup>63</sup> with many random frequencies and phases.<sup>64</sup> The key issues reside in the judicious choice of distribution functions of the statistical parameters. All previous theories of propagation, coupling, and damping of the various field components are used in making this choice

for various simulations.

The second major issue, which is closely related to the previous one, is the reactive component of the field structure and its effect on antenna loading and observed probe signal. The reactive wave components of the off resonance modes do not draw power from the antenna, and so do not contribute to radiation resistance, but are of utmost importance in the probe signal since, usually, only the magnitude is observed. If sophisticated phase detection is employed, the reactive skirts of the modes will be of importance in understanding the detailed phase structure of the measurements. This way, apparent magnetic probe signal phase jumps over a short time scale, as well as bulk continuous phase increase with density, are explained by the average phase increase of the finite Q toroidal modes. Appropriate analytical results from simplified models are backed up by detailed computational simulations.

The effect of internal plasma density perturbations<sup>65</sup> on effective Q is also discussed. The injection of a narrow spectrum of wave frequencies could also simulate lower Q, and increase the importance of the stochastic stacking.

It is also shown that, on the average, the total radiation resistance is independent of damping, is roughly proportional to the antenna length, and is independent of antenna phasing.

Finally, the above theory is used to tentatively explain three new experimental findings that cannot be explained satisfactorily without stochastic mode stacking. The first and most important is the somewhat linear increase of loading resistance with density.<sup>13</sup> The second is the saturation of the probe signal with electron density,<sup>41,42</sup> and the last is the noisy but monotonic phase increase between RF probes distributed around

the torus. The first two findings could be explained somewhat by some near-field loading, but the fact that the probe phase is continuous is most likely of the realm of mode stacking.

A further, important, theoretically explained experimental finding is that the measurement of the bulk parallel and perpendicular phase (hence wave number) suggests a small propagation angle (referred to the toroidal field), which would make second harmonic damping small. A corollary to this effect is that these low  $k_{\perp}$  modes will have large  $Q$  and enhanced resonant fields.

#### IV-6.2. Coherent and stochastic stacking

For a given perpendicular eigenmode field structure, two independent waves propagate away from the antenna along +z and -z around the torus, and add to the original electric field. Assuming (Figure 1)

$$(1) \quad k_{\pm} = k_{z\pm} + i k_{zi}$$

$$(2) \quad l = 2\pi R$$

we have the normalized electric field ( $z = 0$  at antenna)<sup>7,22</sup>

$$\begin{aligned} (2) \quad F = \frac{E_T}{E_0} &= \frac{e^{ik_+z}}{1 - e^{ik_+l}} + \frac{e^{ik_-(1-z)}}{1 - e^{ik_-l}} \\ &= \frac{e^{ik_+z}}{1 - e^{ik_+l}} + \frac{e^{ik_-(1-z)}}{1 - e^{ik_-l}} \\ &= F_+ + F_- \end{aligned}$$

where we used the binomial expansion

$$(3) \quad \frac{1}{1-x} = 1 + x + x^2 + \dots + x^n$$

Without mode splitting,  $k_{z+} = k_{z-}$ , and<sup>56</sup>

$$(4) \quad F = \frac{e^{ikz} + e^{ik(1-z)}}{1 - e^{ikl}} = i \frac{\cos[(z - \pi R)k]}{\sin[\pi Rk]} = i \cotan[\pi Rk] \text{ for } z = 0$$

At a toroidal resonance,  $k_z = n/R$ , and  $z = 0$

$$(5) \quad F_R = \frac{1 + e^{-k_i 2\pi R}}{1 - e^{-k_i 2\pi R}} \approx \frac{1}{k_i \pi R} \text{ for } k_i 2\pi R \ll 1$$

$$\approx \frac{2P_f}{P_{dis} \pi R} \propto \frac{k_{ii}}{\sigma}$$

At antiresonance,  $k_z = \frac{n + 1/2}{R}$  and

$$(6) \quad F_A = \frac{1 - e^{-ik_i 2\pi R}}{1 + e^{-ik_i 2\pi R}} \approx k_i \pi R \text{ for } k_i 2\pi R \ll 1$$

Figure 2 is a plot of  $F_R$  and  $F_A$  for large damping. Finally, the ratio of the maximum to minimum is

$$(7) \quad \frac{F_R}{F_A} = \left( \frac{1}{k_i \pi R} \right)^2 \text{ for } k_i 2\pi R \ll 1$$

The average value of  $F$  at the antenna between resonances can be calculated from (4) as

$$(8) \quad \langle F \rangle = R \int_{n/R}^{(n+1)/R} F dk = \langle F_+ \rangle + \langle F_- \rangle$$

and using the integration formulas<sup>137</sup>

$$(9) \quad \int \frac{dx}{a + b e^{px}} = \frac{x}{a} - \frac{1}{ap} \log_e(a + b e^{px})$$

$$\int \frac{e^{ax} dx}{b + c e^{ax}} = \frac{1}{ac} \log_e(b + c e^{ax})$$

we find

$$(10) \quad \begin{aligned} \langle F_+ \rangle &= 1 \\ \langle F_- \rangle &= 0 \end{aligned}$$

independently of  $k_i$ . The average of  $F_-$  is zero because  $F_-$  is negative half the time.  $F_+$ , on the other hand, is always positive.  $R_R$  is proportional to  $F$  (electric field), and thus  $\langle R_R \rangle$  is also independent of damping.

# Coherent Toroidal Wave Stacking Geometry

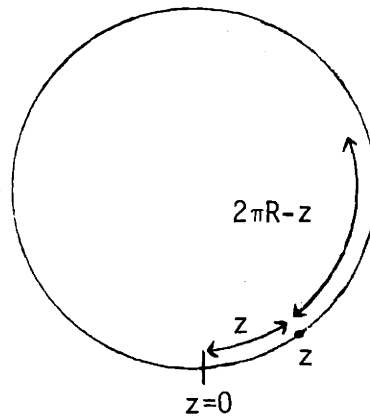


Figure 1

## $F_R$ and $F_A$ as a Function of Damping

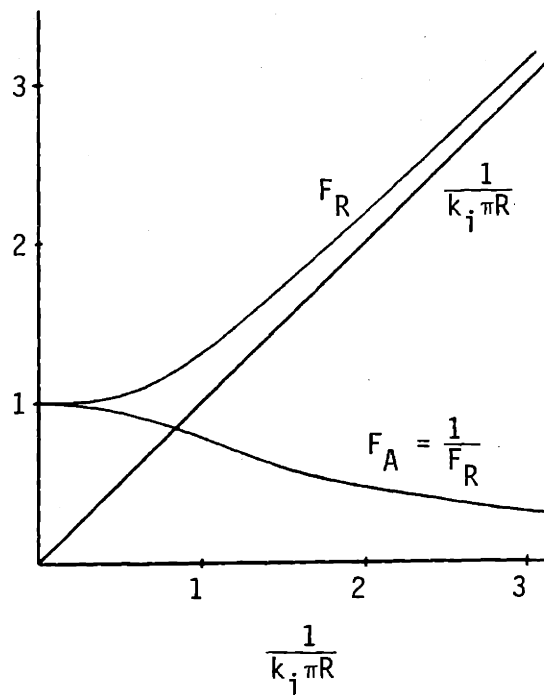


Figure 2

#### IV-6.3. Mode spacing and onset

From Section IV-3.2, equation (5) and Table 1, we can calculate the total number of possible perpendicular eigenmodes (onsets) as  
 $(\Omega = 2, \mu = 1, n_e = 3 \times 10^{14}/\text{cm}^3, a = 10 \text{ cm}, R = 54 \text{ cm})$

$$(1) \quad N_{m\perp} = \nu(2\nu - 1) \approx 2\nu^2 \text{ for } \nu \gg 1$$

$$\approx 3.9 \times 10^{-16} \frac{\Omega^2 a^2}{\mu} n_e \approx 45$$

Also recalling that

$$(2) \quad k_{\perp}^2 + (1 + \Omega) k_{\parallel}^2 = k_A^2$$

$$(3) \quad k_{\perp} = \frac{\pi \nu}{a}$$

$$(4) \quad k_{\parallel} = \frac{N_{mT}}{R}$$

for a given perpendicular eigenmode with  $k_{\perp} \ll k_A$ , we calculate the number of toroidal eigenmodes as

$$(5) \quad N_{mT} = \frac{k_A R}{\sqrt{1 + \Omega}} = 4.4 \times 10^{-8} \frac{\Omega R}{\sqrt{\mu(1 + \Omega)}} \sqrt{n_e} \approx 50$$

The total number of toroidal resonances is then approximately<sup>93</sup>

$$(6) \quad N_{mS} \approx \int_0^{\nu_{\max}} 2\nu^2 N_{mT}(\nu) d\nu = \frac{2}{3} \pi^2 \frac{R a^2}{\sqrt{1 + \Omega}} k_A^3$$

$$= \frac{N_{m\perp} N_{mT}}{3} \approx 5.75 \times 10^{-24} \frac{\Omega^3 R a^2}{\sqrt{1 + \Omega} \mu^{3/2}} n^{3/2} \approx 750$$

The rate of onsets and resonances and spacing ( $\Delta N = 1$ ) in density is

calculated as

$$(7) \quad \frac{\partial N_{m\perp}}{\partial n_e} = \frac{N_{m\perp}}{n_e}$$

$$\Delta n_{em\perp} = 2.56 \times 10^{15} \frac{\mu}{\Omega^2 a^2} \approx 6.4 \times 10^{12}/\text{cm}^3$$

$$(8) \quad \frac{\partial N_{mT}}{\partial n_e} = \frac{N_{mT}}{2 n_e}$$

$$\Delta n_{emT} = 4.5 \times 10^7 \frac{\sqrt{\mu(1+\Omega)}}{\Omega R} \sqrt{n_e} \approx 1.25 \times 10^{13}/\text{cm}^3$$

$$(9) \quad \frac{\partial N_{mS}}{\partial n_e} = \frac{3}{2} \frac{N_{mS}}{n_e}$$

$$\Delta n_{emS} = 1.16 \times 10^{23} \frac{\sqrt{1+\Omega} \mu^{3/2}}{\Omega^3 R a^2} \frac{1}{\sqrt{n_e}} \approx 2.7 \times 10^{11}/\text{cm}^3$$

Finally, we can calculate the mode spacing in time (typically  $\frac{\partial n}{\partial t} = 5 \times 10^{15} \text{ cm}^{-3} \text{ sec}^{-1}$ )

$$(10) \quad \Delta t = \Delta n \left( \frac{\partial n}{\partial t} \right)^{-1} \approx 1.3 \text{ msec for } m\perp \\ \approx 2.5 \text{ msec for } mT \\ \approx 54 \text{ } \mu\text{sec for } mS$$



#### IV-6.4. Quality factor

Wave or circuit quality factor is a criteria commonly used in electrodynamics and electrical engineering.<sup>133,134</sup> From first principles (previous Sections and IV-7.2), we have<sup>144</sup>

$$(1) \quad \frac{1}{k_i} = \frac{2 P_f}{P_{dis}}$$

$$(2) \quad P_f = W v_{g_{\parallel}}$$

$$(3) \quad v_{g_{\parallel}} = \frac{1 + \Omega}{1 - \Omega N_{\parallel}^2/2} N_{\parallel} V_A \approx 2V_A \quad \text{for } N_{\parallel} = .5 \text{ and } \Omega = 2$$

$$(4) \quad Q = \frac{\omega W}{P_{dis}}$$

$$(5) \quad Q = \frac{\omega}{\Delta\omega}$$

Combining (1) to (4), and for the rest of this Section, assuming a plasma with  $\Omega = 2$ ,  $\mu = 1$ ,  $n_e = 3 \times 10^{14}/\text{cm}^3$ ,  $a = 10 \text{ cm}$ ,  $R = 54 \text{ cm}$ , we have

$$(6) \quad Q = \frac{\omega}{2 k_i v_{g_{\parallel}}} \approx \frac{\omega}{2 k_i N_{\parallel} V_A} \frac{1 - \Omega N_{\parallel}^2/2}{1 + \Omega} \approx \frac{k_A}{4k_i} \quad \text{for } N_{\parallel} = .5$$

$$\approx 250 \text{ for } k_A = 1/\text{cm}, 1/k_i = 10 \text{ meters}$$

In comparison, we would have for a "normal" dielectric<sup>134</sup>

$$(7) \quad \frac{1}{k_i} = \frac{2 P_f}{P_{dis}} = \frac{\frac{2kE^2}{2\omega\mu}}{\frac{E^2}{2\eta}} = 2\eta \sqrt{\frac{\epsilon}{\mu}}$$

$$(8) \quad Q = \frac{\omega W}{P_{dis}} = \frac{\frac{\omega \epsilon E^2}{2}}{\frac{E^2}{2\eta}} = \eta \omega \epsilon$$

$$= \frac{\omega W}{k_i 2P_f} = \frac{\omega}{k_i 2v_g} = \frac{k}{2k_i} \quad \text{for } v_g = v_p$$

The quality factor can also be derived from the complex frequency

$$(9) \quad E = E_0 e^{-i(\omega - i\omega_i)t}$$

$$(10) \quad W = W_0 e^{-2\omega_i t}$$

$$(11) \quad P_{\text{dis}} = -\frac{\partial W}{\partial t} = 2\omega_i W_0$$

$$(12) \quad Q = \frac{\omega}{2\omega_i}$$

in agreement with (8) for nondispersive waves, where

$$(13) \quad \omega \propto k$$

$$\omega_i \propto k_i$$

Further assuming  $k_{\perp} = k_A$  and  $T_e = T_i = 1 \text{ keV}$ , we can calculate (Section IV-4.2.) the second harmonic damping  $Q^7$

$$(14) \quad W/\text{cm} \approx \frac{1}{8\pi} \epsilon_A E_y^2 \pi a^2$$

$$(15) \quad E_+ = \frac{E_x + i E_y}{2} = \frac{\Omega - 1}{2i} E_y = \frac{E_y}{2i}$$

$$(16) \quad P_{\text{dis}}/\text{cm} = \int_S P_{\text{dis}}/\text{cm}^3 ds = 2\pi a r P_{\text{dis}}/\text{cm}^3$$

$$(17) \quad Q_{2\omega c} = \frac{\omega W/\text{cm}}{P_{\text{dis}}/\text{cm}} \approx \frac{a}{R} \frac{2}{\beta_i} \approx 5 \times 10^{10} \frac{a}{R} \frac{B_0^2}{n_e T_i} \approx 110$$

and assuming  $\alpha = 1.9$ , we have the electron Landau damping  $Q$

$$(18) \quad Q_{\text{ELD}} = \frac{2}{\sqrt{\pi}} \frac{1}{\alpha e^{-\alpha^2}} \frac{1}{\beta_e} \approx 6500$$

$$\geq \frac{2.63}{2} \frac{R}{a} Q_{2\omega c}$$

Similarly, we can calculate collisional<sup>90,96</sup> and wall damping<sup>93,134,144</sup>

Q as

$$(19) \quad Q_{\text{coll}} = \frac{\omega W}{P_{\text{dis}}} = \frac{\omega \epsilon_A E^2 / 2}{\eta J^2 / 2} \approx \frac{1}{\omega \epsilon_A \eta}$$

$$\approx \frac{k_A}{2k_i} \approx 5500$$

$$(20) \quad Q_w \approx \frac{\omega W}{P_{\text{dis}}} \approx \frac{a}{\delta} \frac{\langle B^2 \rangle}{B_{\text{edge}}}$$

$$\approx \frac{k_A}{2k_i} \approx 37,000$$

where we note the difference between equations (8) and (19).

From (5), we calculate the half power width in density as<sup>10</sup>

$$(21) \quad Q = \frac{\omega}{\Delta\omega} \frac{\pi_i}{\Delta\pi_i} = \frac{2n_e}{\Delta n_e}$$

$$(22) \quad \Delta n_e = \frac{2n_e}{Q} \approx 6 \times 10^{12} / \text{cm}^3 \quad \text{for } Q = 100$$

Mode stacking becomes important when the mode width is comparable to the mode spacing. Following the notation of IV-6.3., and using (20), we can calculate the maximum allowable Q for mode stacking (overlapping skirts).

$$(23) \quad Q_{m_\perp} = \frac{2n_e}{\Delta n_{em_\perp}} = 2 N_{m_\perp} \approx 90$$

$$(24) \quad Q_{mT} = \frac{2n_e}{\Delta n_{emT}} = N_{mT} \approx 50$$

$$(25) \quad Q_{mS} = \frac{2n_e}{\Delta n_{emS}} = 3 N_{mS} = 2250$$

Summarizing these results, we write

$$(26) \quad Q_{mT} < Q_{m\perp} < Q_{2\omega e} < Q_{mS} < Q_{coll} < Q_{ELD} < Q_{wall}$$

and onsets and high radiation resistance perpendicular eigenmodes are expected to be distinguishable from the background. On the other hand, the average toroidal resonance is expected to be swamped out by stochastic mode stacking, especially with the resonant layer in the plasma center.

Finally, for a single mode, we can calculate the ratio of the maximum to minimum field from IV-6.2.(7), as

$$(27) \quad \frac{F_R}{F_A} = \left( \frac{1}{k_i \pi R} \right)^2 \approx \left( \frac{4 Q}{k_A \pi R} \right)^2 \approx 5.6 \text{ for } Q = 100, k_A = 1/\text{cm}$$

#### IV-6.5. Poloidal mode stacking

From Section IV-1.3., we calculated that, in general,

$$(1) \quad R_m = \frac{V_m^2}{4 P_{fm}}$$

$$(2) \quad R_T = \sum R_m$$


In this Section, we will consider the poloidal mode coupling effects of two independently fed antenna loops, as shown in Figure III-2.1.(3). Figures 1-4 show the usual discrete Fourier (sin, cos,  $m > 0$ ) components<sup>137</sup> of a 360° (TFR) and 130° (Alcator) antenna, which is fed either in or out of phase (Section III-2.1.).

For simplicity, we will assume that the ratio of the Poynting flux to the square of  $E_\theta$  at the antenna radius is independent of mode number ( $\beta_{am} \approx \beta_{an}$ , Section IV-1.3.). For any particular set of eigenmodes, this may be a very crude approximation (especially due to the evanescent edge), but will lead to two simple, important, statistical scaling laws.

Allowing  $m$  to be both positive and negative (Section IV-1.2.), we have

$$(3) \quad V_m = \int_{-\theta_a}^{+\theta_a} E_m \epsilon_a \cos (m\theta - \omega t) a \, d\theta$$

where

(4)  $\epsilon_a =$   for push-pull

for push-push

Using the trigonometric identity

$$(5) \quad \cos (m\theta - \omega t) = \cos m\theta \cos \omega t + \sin m\theta \sin \omega t$$

and assuming a push-pull fed antenna, we have

$$(6) \quad V_m = a E_m 2 \int_0^{\theta_a} \cos m\theta d\theta \cos \omega t \quad -\infty < m < +\infty$$

$$= 2 a E_m \frac{\sin m \theta_a}{m} \quad m \neq 0$$

$$(7) \quad R_T = \frac{a^2 E_m^2}{P_{fm}} \left[ \theta_a^2 + 2 \sum_{m=1}^{\infty} \frac{\sin^2 m\theta_a}{m^2} \right]$$

$$\propto \theta_a^2 + 2 \sum_{m=1}^{\infty} \frac{\sin^2 m\theta_a}{m^2} \approx 9.87 \text{ for } \theta_a = 180^\circ, m < 10$$

$$\approx 3.47 \text{ for } \theta_a = 65^\circ, m < 10$$

Similarly, for push-push we have

$$(8) \quad R_T = \frac{a^2 E_m^2}{P_{fm}} 2 \sum_{m=1}^{\infty} \left[ \frac{1 - \cos m\theta_a}{m} \right]^2$$

$$\propto 2 \sum_{m=1}^{\infty} \left[ \frac{1 - \cos m\theta_a}{m} \right]^2 \approx 9.47 \text{ for } \theta_a = 180^\circ, m < 10$$

$$\approx 3.29 \text{ for } \theta_a = 65^\circ, m < 10$$

and we note that the total radiation resistance is independent of antenna phasing. A closer look at (7) and (8) further shows that

$$(9) \quad R_T \approx \frac{a^2 E_m^2}{P_f} \pi \theta_a$$

Of course, the radiation resistances of the low poloidal mode numbers still increase as  $\theta_a^2$  (or even  $\theta_a^4$ ), but the spectrum width decreases with  $\theta_a$ , resulting in  $R_T \propto \theta_a$ .

# Antenna $k_\theta$ Spectrums for 360° and 130° Antennas

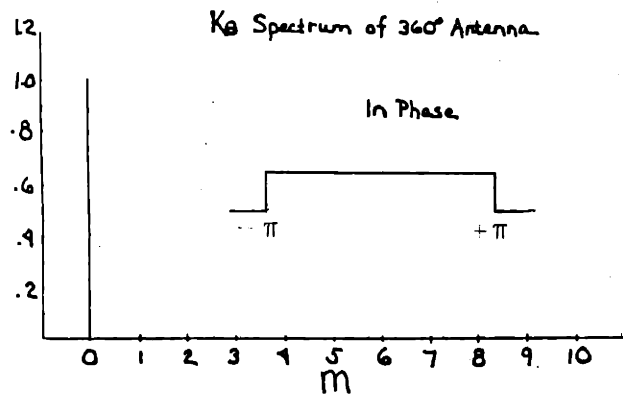


Figure 1

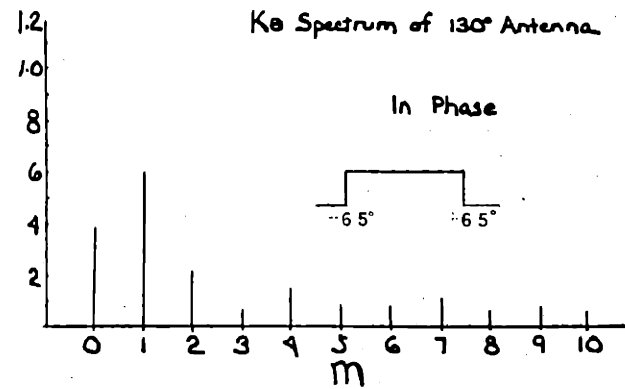


Figure 3

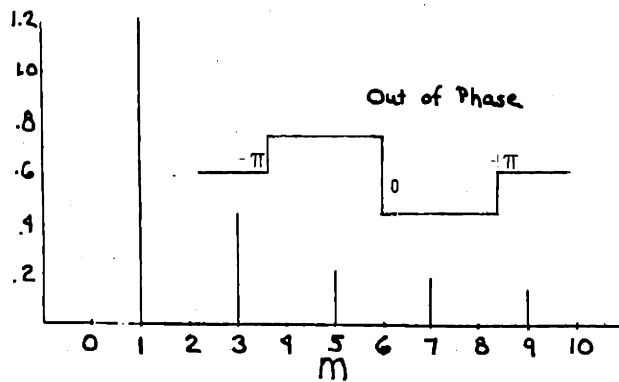


Figure 2

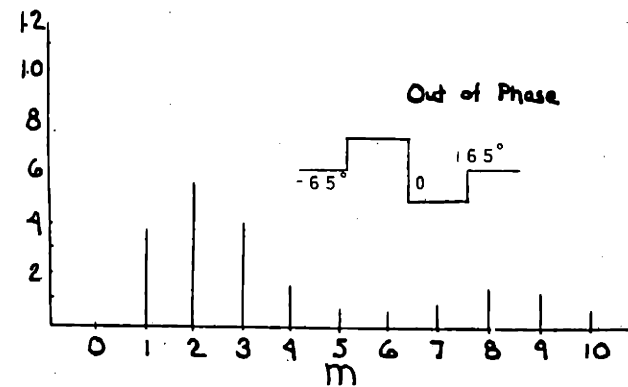


Figure 4

#### IV-6.6. Stochastic mode stacking

In this Section, we will examine the stochastic stacking of orthogonal eigenmodes, at and away from the antenna. At the antenna ( $E_a$ ), the eigenmodes are coherent (all in phase), and the electric fields simply add. Away ( $k_{\perp} z \gg 1$ ) from the antenna ( $E_p$ ), the modes become randomly phased with respect to each other (incoherent), and statistically interfere.

For perpendicular eigenmodes ( $k_{\perp j} = 0$ ) and high Q toroidal standing eigenmodes, the electric fields are either in phase or out of phase with respect to the antenna (for the same poloidal angle  $\theta$ ). This random phase can be modeled by a simple coin toss with heads = 1 and tails = -1. Thus for 1 coin (1 mode), two possibilities exist ( $2^1 = 2$ ), and

$$(1) \quad \langle E_a \rangle = \langle E_p \rangle = \frac{|1| + |-1|}{2} = 1$$

$$(2) \quad \frac{\langle E_p \rangle}{\langle E_a \rangle} = 1$$

For 2 coins, there are four possibilities ( $2^2$ ) and

$$(3) \quad \frac{\langle E_p \rangle}{\langle E_a \rangle} = \frac{|1+1| + |-1-1| + |-1+1| + |+1-1|}{[|+1| + |+1|] + [|-1| + |-1|] + [|-1| + |+1|] + [|+1| + |-1|]} = \frac{1}{2}$$

For 4 coins<sup>64</sup>

$$(4) \quad \frac{\langle E_p \rangle}{\langle E_a \rangle} = \frac{4 \left[ \binom{4}{4} + \binom{4}{0} \right] + 2 \left[ \binom{4}{1} + \binom{4}{3} \right] + 0 \left[ \binom{4}{2} \right]}{4(2^4)} = \frac{3}{8}$$

where

$$(5) \quad \binom{n}{r} = \frac{n!}{r!(n-r)!}$$



and in general ,

$$(6) \quad \frac{\langle E_p \rangle}{\langle E_a \rangle} = \frac{\left\langle \left| \sum_{m=1}^n x_m \right| \right\rangle}{\left\langle \sum_{m=1}^n |x_m| \right\rangle} \quad \text{for } x_m = \pm 1$$

$$= \frac{1}{\sqrt{n}} \quad \text{for } n \rightarrow \infty$$

as shown in Figure 1.

Considering the problem from another angle, we may write

$$(7) \quad P_{\text{antenna}} = \frac{I_o \ell}{2} [E_1 + E_2 + \dots + E_n] \approx \frac{n I_o E_o \ell}{2}$$

$$(8) \quad P_{\text{dis}} = \frac{E_p^2 \ell^2 \text{Vol}}{2n}$$

$$(9) \quad P_{\text{antenna}} = P_{\text{dis}}$$

and thus

$$(10) \quad E_a \propto n$$

$$(11) \quad E_p \propto \sqrt{n}$$

$$(12) \quad \frac{E_p}{E_a} = \frac{1}{\sqrt{n}}$$

in agreement with (6).

The electric field at the antenna is thus, on the average,  $\sqrt{n}$  times larger than the electric field around the torus, and is much more likely to produce edge heating (the edge evanescence even further deteriorates the situation).

To further investigate the stochastic field structure of a set of random modes, we define the following statistical operators.

$$(13) \text{ Average (mean) } = \langle p \rangle = \frac{\int p dq}{\int dq} = \frac{\sum p_n}{n}$$

$p$  = probe signal ( $|p(z)|$ ), radiation resistance ( $\text{Re } p(z = 0)$ )

$q = k, t, n_e$

$$(14) \text{ Deviation } = p - \langle p \rangle$$

$$(15) \text{ Average Deviation } = \langle |p - \langle p \rangle| \rangle$$

$$(16) \text{ Normalized Average Deviation } = \frac{\langle |p - \langle p \rangle| \rangle}{\langle p \rangle}$$

$$(17) \text{ Standard Deviation } = \sqrt{\langle (p - \langle p \rangle)^2 \rangle}$$

$$(18) \text{ Normalized Standard Deviation } = \frac{\sqrt{\langle (p - \langle p \rangle)^2 \rangle}}{\langle p \rangle}$$

$$(19) \text{ Range } = p_{\max} - p_{\min}$$

$$(20) \text{ Normalized Range } = \frac{p_{\max} - p_{\min}}{\langle p \rangle}$$

$$(21) \text{ Solidity}^{145} \text{ (normalized maximum) } = \frac{p_{\max}}{\langle p \rangle}$$

To illustrate the basic issues, we will use the following simple heuristic model. Consider the field solidity near the antenna (coherent  $R_R$ ) and away from the antenna (incoherent  $|V_p|$ ).

	<u>coherent</u>	<u>incoherent</u>
(22) $\langle p \rangle =$	$n$	$\sqrt{n}$
(23) $p_{\max} =$	$\alpha Q + (n - 1)$	$\alpha Q + \sqrt{n - 1}$
(24) Solidity (S) $\approx$	$\frac{\alpha Q + (n - 1)}{n}$	$\frac{\alpha Q + \sqrt{n - 1}}{\sqrt{n}}$

where  $n$  is the number of modes, and  $Q$  is the quality factor. Thus for the plausible cases

(25) $n = 1, \alpha Q = 10$	$S_c = 10$	$S_i = 10$
$n = 10, \alpha Q = 10$	$S_c = 1.9$	$S_i = 4.1$
$n = 10, \alpha Q = 2$	$S_c = 1.1$	$S_i = 1.6$

and, in general ,

$$(26) \quad S_i \geq S_c$$

$$(27) \quad S_i \approx S_c \sqrt{n} \quad \text{for } \alpha Q \gg n$$

which means that a probe signal (away from the antenna) will always have a "noisier" time history than the radiation resistance.

Although this simple model is a good approximation in the coherent case (since  $R_p$  is always positive), it does not take into account stochastic destructive interference (a high  $Q$  resonance with opposite phase can totally cancel out the background). The normalized range is a slightly more powerful operator (Range =  $p_{\max} - \langle p \rangle$  in coherent case), and further shows that the probe signal is statistically noisier than  $R_p$ . In the next Section, we will computationally use the average and standard deviations, again with the same results.

# Simple Stochastic Mode Stacking Model

$$\frac{\text{Total } E_{\text{probe}}}{\text{Total } E_{\text{antenna}}} = \left\langle \frac{\sum_{m=1}^n |x_m|}{\sum_{m=1}^n |x_m|} \right\rangle, x_m = \pm 1$$

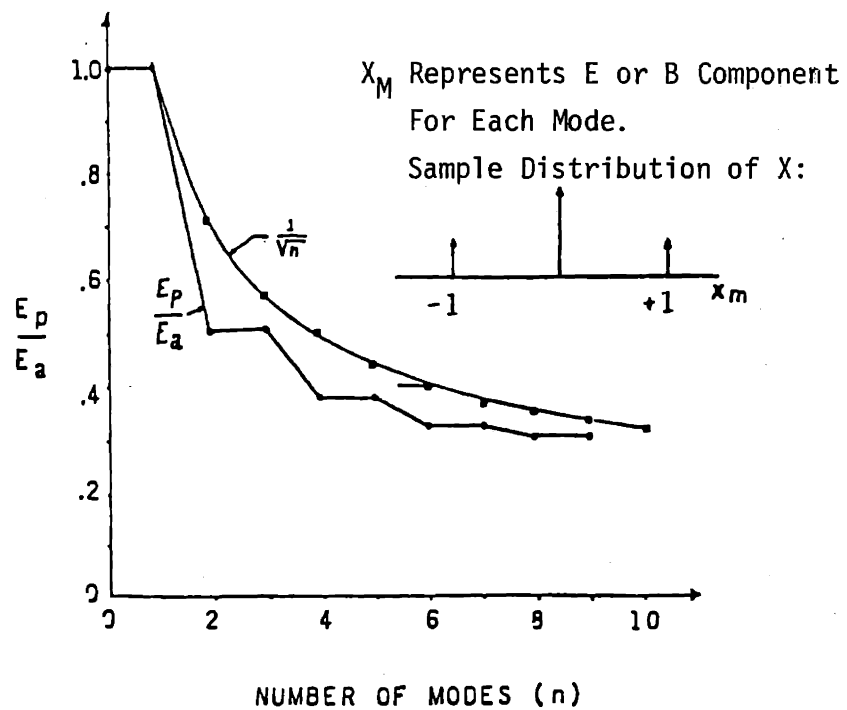


Figure 1

#### IV-6.7. Coherent and stochastic field simulations

In this Section, we will examine the results of realistic computer simulations of coherent and stochastic fields. A code called "Stochastic Mode Stacking" is listed in the appendix, and is versatile enough for almost any parameter scan. Figure 1 shows the many parameters that need to be chosen for a given set of simulations (multi-dimensional scans). Table 1 is an outline of the basic functions and variables used in the code, and is self explanatory with the code comments. Given the basic input data, the code calculates and plots the real, imaginary, magnitude and phase of the sum of  $s = 1, 3, 10$  and  $20$  modes, as a function of linearly increasing  $k_z$  of  $z$ .  $k_{z0}$  is some random number (i.e., between  $.5$  and  $.6/\text{cm}$ ) with random or fixed initial phase, and  $z$  is the distance from the antenna. The average, average deviation, and standard deviation of the various parameters are also outputted.

Figures 4 to 7 show the real, imaginary, magnitude and phase of the electric field at the antenna ( $z = 0$ ) versus  $k_z(k_z r)$ , without mode splitting ( $p = 0$ ) or fluctuations ( $k_z f_a = 0$ ), and one mode ( $s = 1$ ) and damping length  $= 10R$ . As expected from Section II-6.2,  $\langle \text{Re} \rangle = a_r \approx s = 1$ ,  $\langle \text{Im} \rangle = a_i \approx 0$ ,  $\langle \text{Mag} \rangle = a_m > a_r$  and  $\langle \phi \rangle = a_\phi \approx 0$ .

Figure 3 is a magnified view of Figure 4, where the maximum ( $F_R$ ), minimum ( $F_A$ ) and half power width ( $\Delta k_z$ ) are easily measured. Figure 8 shows eigenmode resonances produced by sweeping a multi-wavelength coax resonator through several resonances between  $150$  and  $220$  MHz (Figure 10). Figure 9 is similar, but with three different resonators in parallel.

Figures 11 and 12 are the same as 4 and 5, but with  $1\%$  mode splitting ( $p = .01$ ). Figures 13 and 14 are  $z$  scans (scan  $k = 0$ ) with large damping ( $k_z i = 1/R$ ). Note how the phase linearly increases near the

antenna ( $F_- \ll F_+$ ), but goes through nearly step-like transitions  $180^\circ$  around the torus ( $|F_+| \approx |F_-|$ ). This effect is further investigated in Figures 15 to 20, where the damping length is varied from  $R$  to  $100 R$ , and at a position near the HCN or Thompson ports ( $z = 60$ ). Note again the phase increase difference between the small and large damping cases. At high  $Q$ , the fields are almost always either in phase or out of phase, and the parallel wavelength is most easily calculated by measuring the distance between phase jumps or magnitude nodes. This situation, of course, does not occur if the mode is highly split (running wave, Figures 53 to 58).

Figures 21 to 24 show the radiation resistance ( $Re, z = 0$ ) for  $s = 1, 3, 10$  and  $20$  and  $k_z i = .1/R$ . Figures 25 to 30 show the same simulation, but with random initial phase, and  $z = 60$ . Note how the phase becomes progressively more difficult to measure with increasing number of modes. Table 2 summarizes the averages and normalized deviations, as a function of number of modes. Note, in particular, how the average of the magnitude of the probe signal ( $\approx \sqrt{s}$ ) and radiation resistance ( $\approx s$ ) agree with analytical theory. Also note how the normalized deviation of the radiation resistance decreases with  $s$ , while the probe signal is independent of  $s$ . For many modes, the radiation resistance has a smaller normalized deviation than the magnitude of the probe signal ( $R_R$  is less "noisy" than a probe signal).

Figures 31 to 36 show similar simulations with 10 modes, but with  $k_{||}$  spectrum widths of  $.01, .1$  and  $1/\text{cm}$ . Figures 37 to 40 show the effects of 1% and 10% high frequency fluctuations on one mode.

Figures 41 to 46 show the probe signal ( $z = 60$ ) for 10 modes as a function of increasing  $k_z$ , for  $k_i = 1/R, .1/R$  and  $.01/R$  (this is the same as Figures 15 to 20, but with scan  $k = 1$ ). Figures 47 to 52 show the effect of 1% and 10% mode splitting on the probe signal ( $z = 60$ ) and the radiation resistance ( $z = 0$ ), for one mode and  $k_i = .1/R$ .

Table 1.

Outline of basic functions and variables used in the  
stochastic mode stacking code.

#### INPUT DATA

$s$  = number of modes

$k = k_zr + i(k_zi)$  complex  $k$

$k_- = k_+ (1 + p)$  mode splitting

$k = k_zr[kzfa \cdot \cos(kzff \cdot kz)]$  fluctuations

$F = F_+ + F_-$  as calculated in II-6.2.

$kzro$  and  $pha$  = random set of initial  $k$  and phase

scan  $k$  linearly increases  $k$  or  $z$

$kzro = kmult (kzro - ksubt)$  changes the initial spectrum width

#### OUTPUT DATA

Plot either real, imaginary, magnitude or phase.

$ar = \langle Re \rangle$  ,  $ai = \langle Im \rangle$  ,  $am = \langle Mag \rangle$  ,  $ap = \langle \phi \rangle$

$adrs = \langle |Re - s| \rangle$  ;  $adrrs = \langle |Re - \sqrt{s}| \rangle$

$sdrs = \sqrt{\langle (Re - s)^2 \rangle}$  ;  $sdr rs = \sqrt{\langle (Re - \sqrt{s})^2 \rangle}$

$admrs = \left( \langle |Mag - \sqrt{s}| \rangle \right)$  average deviation of magnitude minus root s

Table 2

Summary of averages and normalized deviations as a function  
of number of modes for  $1/k_{zi} = 10R$ .

	$s = 1$	3	10	20
$z = 0$				
$\langle \text{Re} \rangle \approx s$	1.0029	3.026	9.989	20.04
$\frac{\langle  \text{Re} - s  \rangle}{\langle \text{Re} \rangle}$	.715	.448	.212	.108
$z = 60$				
$\langle \text{Mag} \rangle \approx \sqrt{s}$	1.079	2.056	2.936	4.043
$\frac{\langle  \text{Mag} - \sqrt{s}  \rangle}{\langle \text{Mag} \rangle}$	.448	.464	.476	.458



## Multi-Dimensional Scans

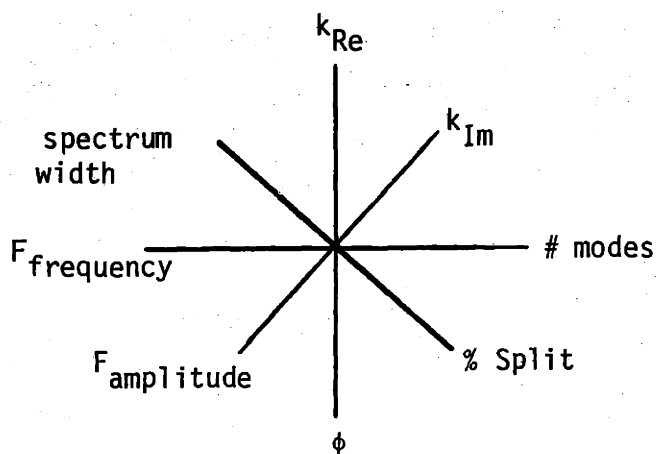


Figure 1

## Measurements

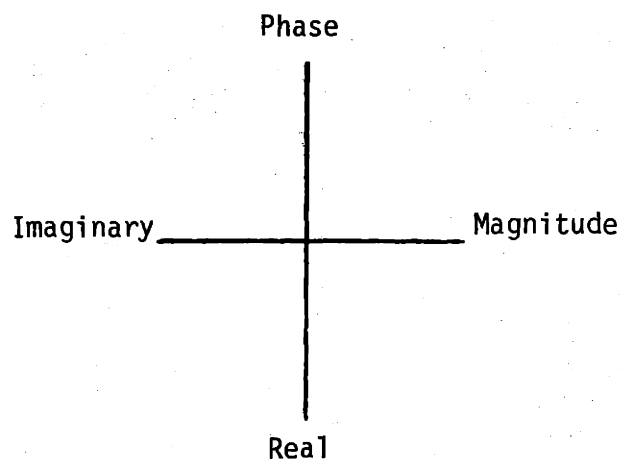
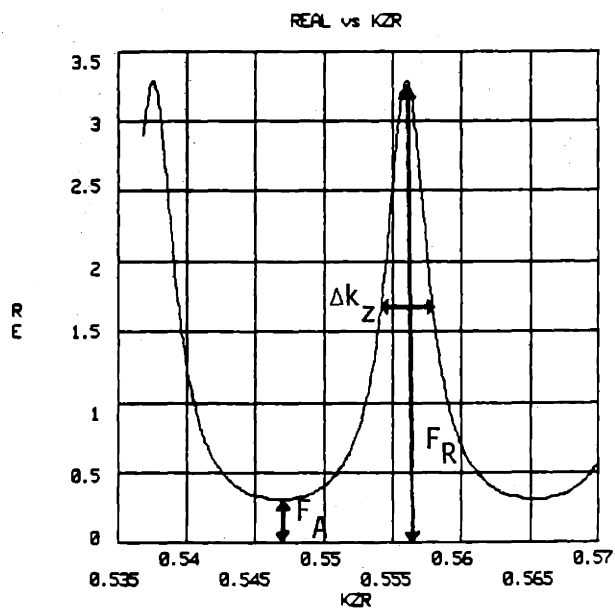


Figure 2

## Low Q Eigenmode Waveform Detail



```

INPUT  scan=1.0 s=1 phase=0.0 n=500 r=54.0
DATA   kzi=0.001652 p=0.00 z=0.00 zmin=50.00 zmax=75.00
        kminf=0.970 kmaxf=1.030 kmult=1.00 ksubt=0.000
        kzfa=0.000 kzff=1.0000E+04 kzo(1)=0.55340
AVERAGE  adr=0.9597 adi=0.1445 adm=1.2480 adp=0.0932
AVE DEV   adrs=0.7351 adrrs=0.7351 adms=0.7458 adms=0.7458
STAND DEV sdrs=0.9877 sdrrs=0.9877 sdms=0.9762 sdms=0.9762
    
```

Figure 3

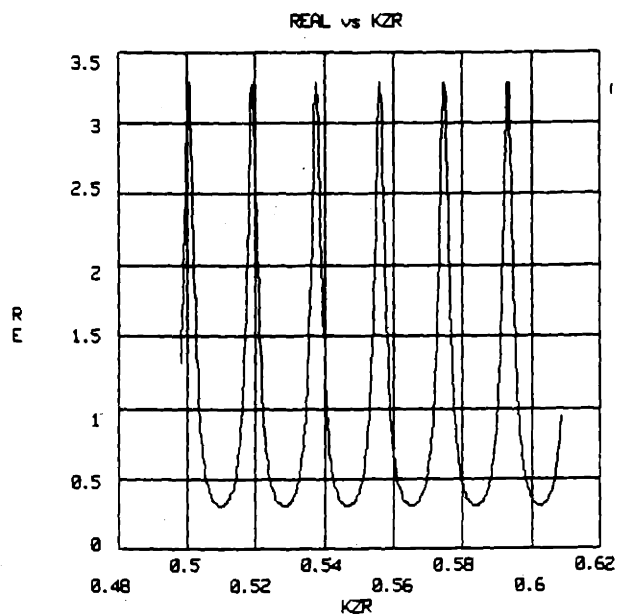


Figure 4

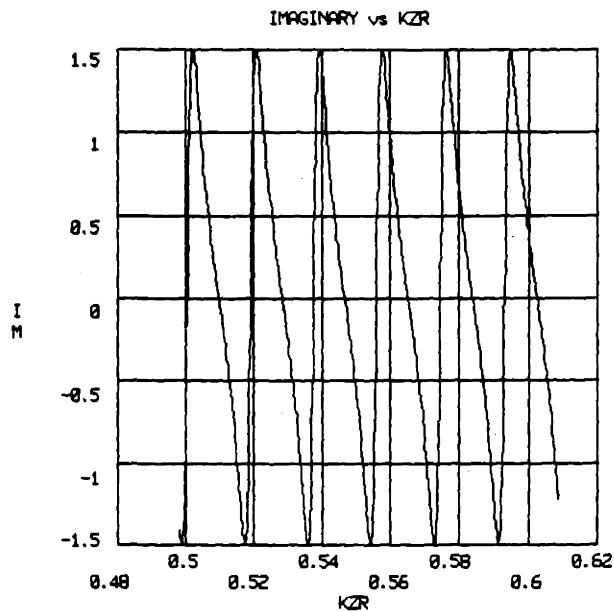


Figure 5

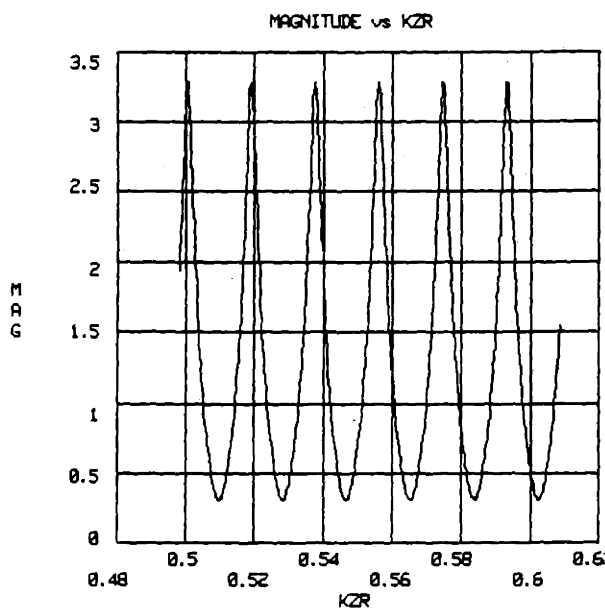


Figure 6

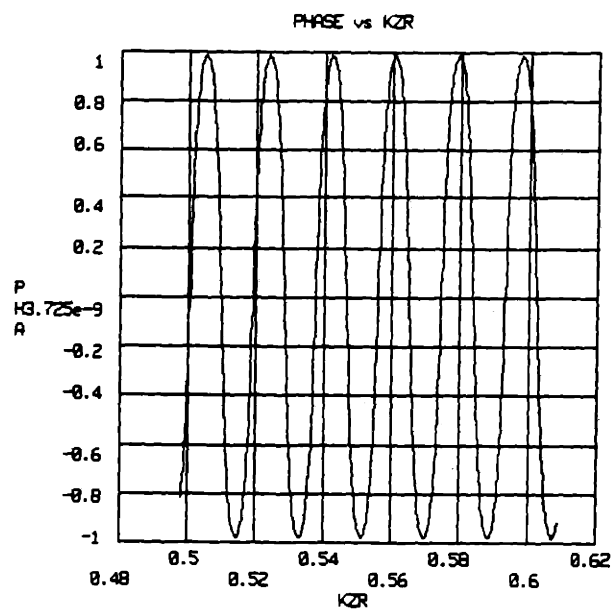


Figure 7

Real, Imaginary, Magnitude and Phase of one toroidal eigenmode at  $z = 0$ ,  
 $k_i = .1/R$ ,  $\phi = 0$  and no mode splitting

```

INPUT      scank=1.0 s= 1 phase=0.0 n= 500 r=54.0
DATA      kzi=0.001852 p= 0.00 z= 0.00 zmin= 0.00 zmax= 60.00
          kminf=0.900 kmaxf=1.100 kmult= 1.00 ksubt= 0.00
          kzfa=0.000 kzff=.10000E+04

```

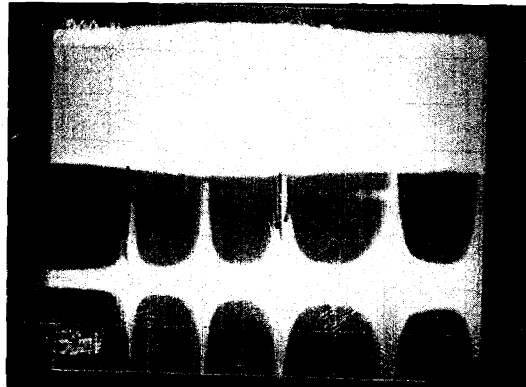
```

AVERAGE  adr= 0.9995 adi= 0.0063 adm= 1.3103 adp= 0.0042
AVE DEV   adrs= 0.7198 adrrs= 0.7198 adms= 0.7583 admps= 0.7583
STAND DEV sdrs= 0.6948 sdrrs= 0.6948 sdms= 0.9656 sdmrs= 0.9656

```

# Analog Eigenmode Simulations

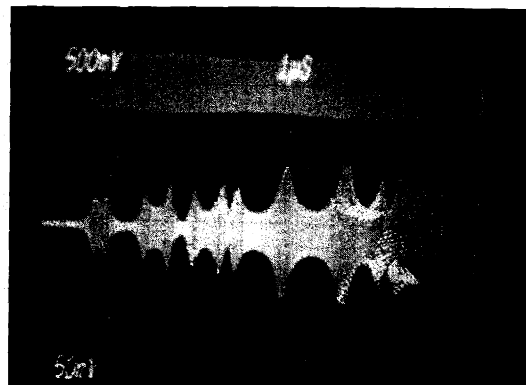
Figure 8



$V_{in}$  (A)

1 mode (B)

Figure 9

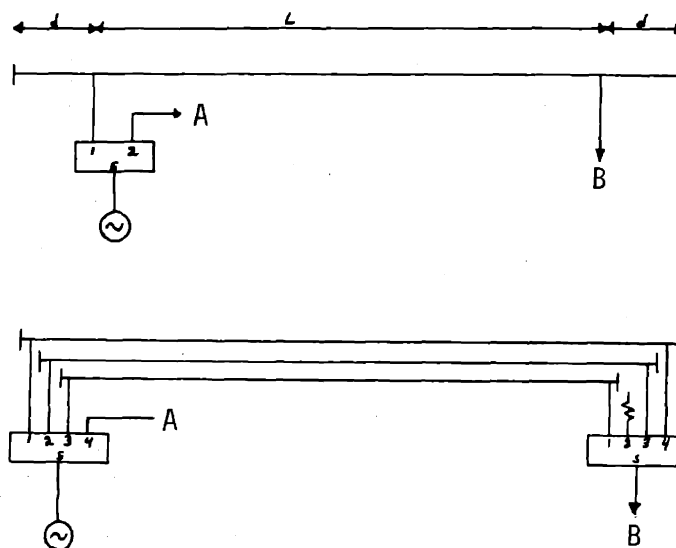


$V_{in}$  (A)

3 modes (B)

## Coax Eigenmode Simulators

Figure 10



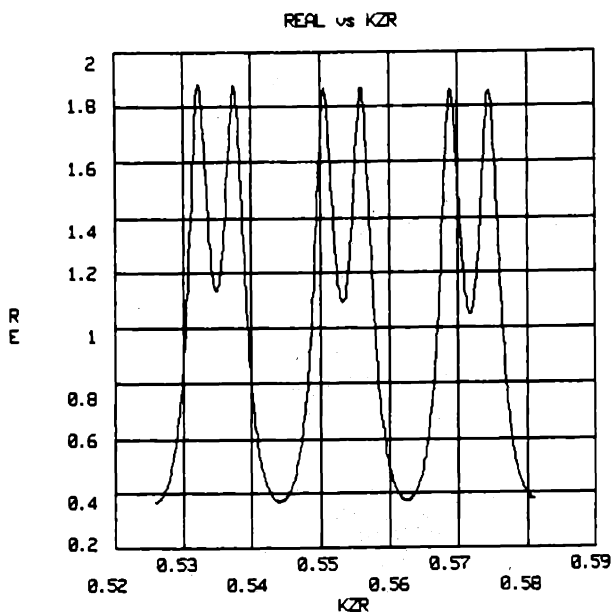


Figure 11

Same as Figures 4 and 5  
but with 1% splitting

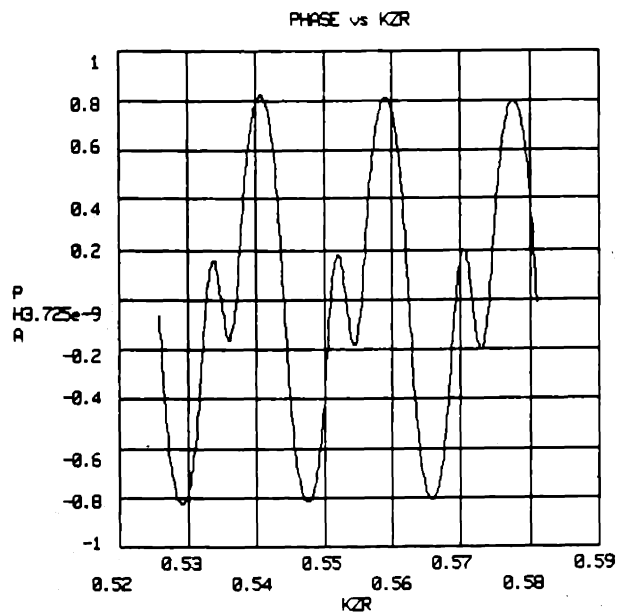


Figure 12

INPUT scank=1.0 s= 1 phase=0.0 n= 500 r=54.0  
DATA kzi=0.001852 p= 0.01 z= 0.00 zmin= 0.00 zmax= 60.00  
kminf=0.950 kmaxf=1.050 kmult= 1.00 ksubt= 0.00  
kzfa=.000 kzff=.10000E+04

AVERAGE adr= 0.9999 adi= -0.0022 adm= 1.1302 adp= -0.0032  
AVE DEV adrs= 0.4605 adrrs= 0.4605 adms= 0.4532 admps= 0.4532  
STAND DEV sdrs= 0.5153 sdrrs= 0.5153 sdms= 0.5200 sdmps= 0.5200

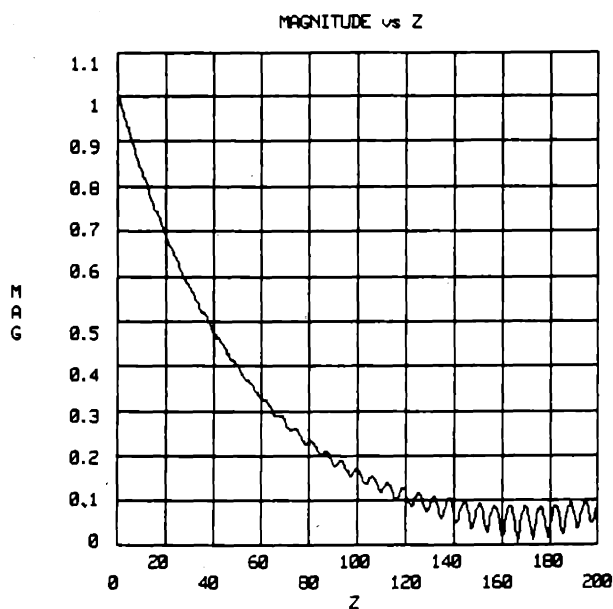


Figure 13

Toroidal position scan  
with  $k_i = 1/R$

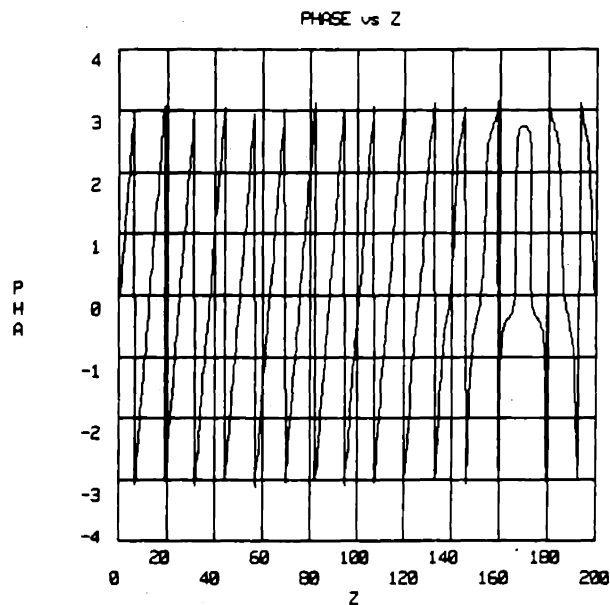


Figure 14

INPUT scank=0.0 s= 1 phase=0.0 n= 500 r=54.0  
DATA kzi=0.018519 p= 0.00 z= 0.00 zmin= 0.00 zmax= 200.00  
kminf=0.900 kmaxf=1.100 kmult= 1.00 ksubt= 0.000  
kzfa=.000 kzff=.10000E+04 kzo(1)=0.55340

AVERAGE adr= 0.0010 adi= 0.0106 adm= 0.2702 adp= 0.2649  
AVE DEV adrs= 0.9990 adrrs= 0.9990 adms= 0.7290 admps= 0.7290  
STAND DEV sdrs= 1.0336 sdrrs= 1.0336 sdms= 0.7725 sdmps= 0.7725

$$k_i = 1/R$$

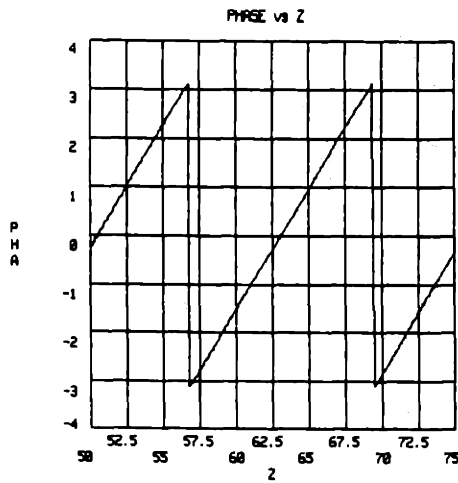


Figure 15

$$k_i = .1/R$$

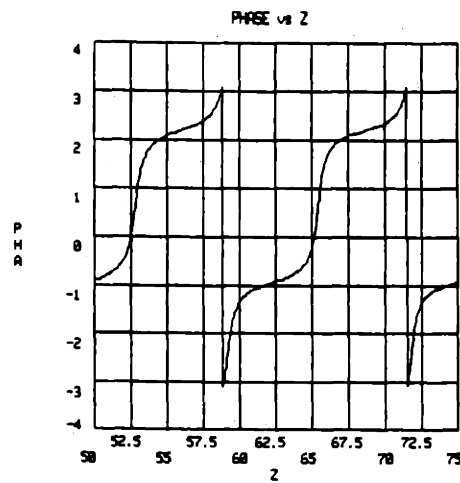


Figure 17

$$k_i = .01/R$$

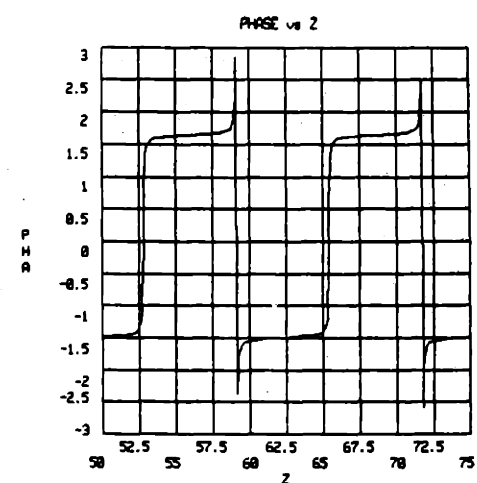


Figure 19

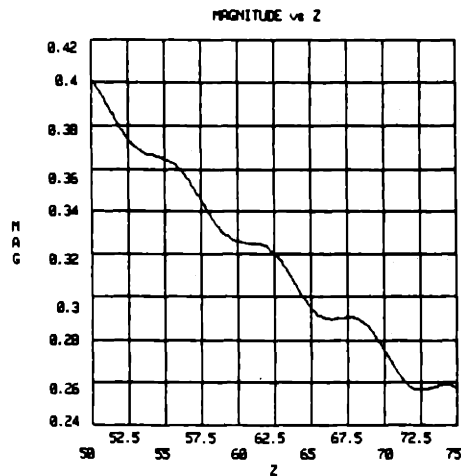


Figure 16

INPUT scank=0.0 s=1 phase=0.0 n=500 r=54.0  
DATA kzi=0.016519 p=0.00 z=0.00 zmin=50.00 zmax=75.00  
kminf=0.900 kmaxf=1.100 kmult=1.00 ksubt=0.00  
kzfa=0.000 kzff=.10000E+04  
AVERAGE adr=0.0012 adi=0.0123 adm=0.3177 adp=0.0139  
AVE DEV adrs=0.9988 adrsr=0.9988 adms=0.6823 admsr=0.6823  
STAND DEV adrs=1.0252 adrsr=1.0252 adms=0.6836 admsr=0.6836

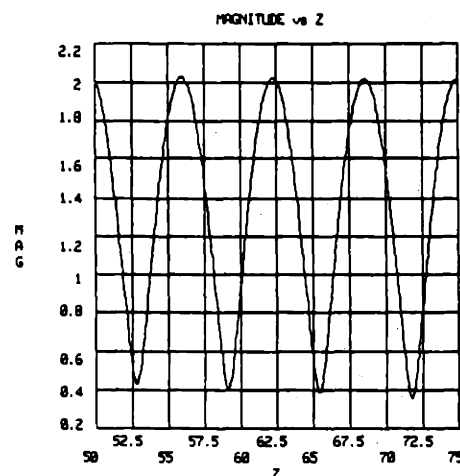


Figure 18

INPUT scank=0.0 s=1 phase=0.0 n=500 r=54.0  
DATA kzi=0.001652 p=0.00 z=0.00 zmin=50.00 zmax=75.00  
kminf=0.900 kmaxf=1.100 kmult=1.00 ksubt=0.00  
kzfa=0.000 kzff=.10000E+04  
AVERAGE adr=-0.0050 adi=0.0193 adm=1.3431 adp=0.4996  
AVE DEV adrs=1.0603 adrsr=1.0603 adms=0.5650 admsr=0.5650  
STAND DEV adrs=1.3229 adrsr=1.3229 adms=0.6447 admsr=0.6447

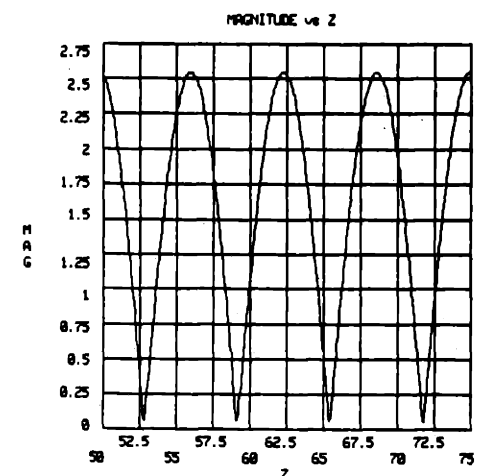
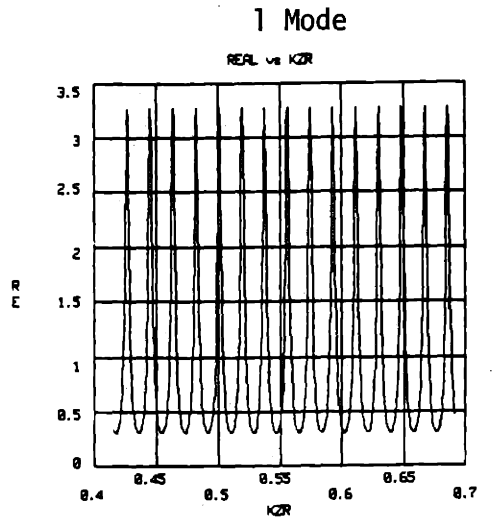


Figure 20

INPUT scank=0.0 s=1 phase=0.0 n=500 r=54.0  
DATA kzi=0.000165 p=0.00 z=0.00 zmin=50.00 zmax=75.00  
kminf=0.900 kmaxf=1.100 kmult=1.00 ksubt=0.00  
kzfa=0.000 kzff=.10000E+04  
AVERAGE adr=-0.0000 adi=0.0234 adm=1.6160 adp=0.0091  
AVE DEV adrs=1.0000 adrsr=1.0000 adms=0.8660 admsr=0.8660  
STAND DEV adrs=1.0182 adrsr=1.0182 adms=0.9946 admsr=0.9946

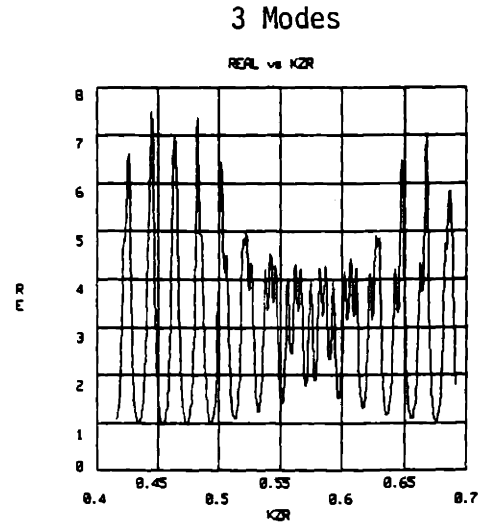
Phase and Magnitude Toroidal Position Scan for  $k_i = 1/R$ ,  $.1/R$  and  $.01/R$



INPUT scank=1.0 s=1 phase=0.0 n=500 r=54.0  
 DATA kzi=0.001852 p=0.00 z=0.00 zmin=0.00 zmax=60.00  
 kminf=0.750 kmaxf=1.250 kmult=1.00 ksult=0.000  
 kzfa=0.000 kzff=.10000E+04 kzo(1)=0.55340

AVERAGE adr= 1.0029 adi= -0.0023 ade= 1.3156 adp= -0.0042  
 AVE DEV adrs= 0.7174 adrrs= 0.7174 adms= 0.7599 adars= 0.7599  
 STAND DEV sdrs= 0.8938 sdrrs= 0.8938 sdms= 0.9865 sdars= 0.9865

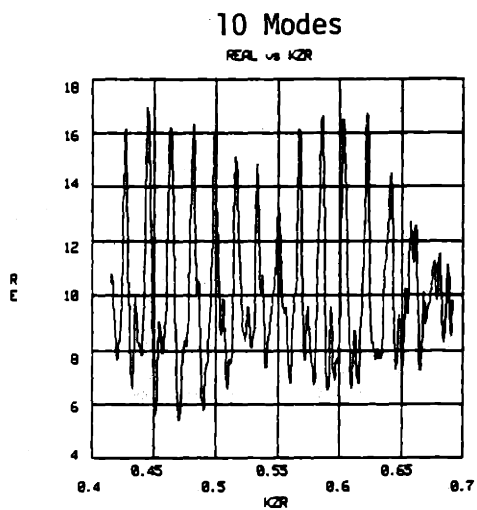
Figure 21



INPUT scank=1.0 s=3 phase=0.0 n=500 r=54.0  
 DATA kzi=0.001852 p=0.00 z=0.00 zmin=0.00 zmax=60.00  
 kminf=0.750 kmaxf=1.250 kmult=1.00 ksult=0.000  
 kzfa=0.000 kzff=.10000E+04 kzo(1)=0.55340

AVERAGE adr= 3.0264 adi= -0.0124 ade= 3.4294 adp= -0.0070  
 AVE DEV adrs= 1.3575 adrrs= 1.5671 adms= 1.3561 adars= 1.8519  
 STAND DEV sdrs= 1.5919 sdrrs= 2.0515 sdms= 1.6378 sdars= 2.3193

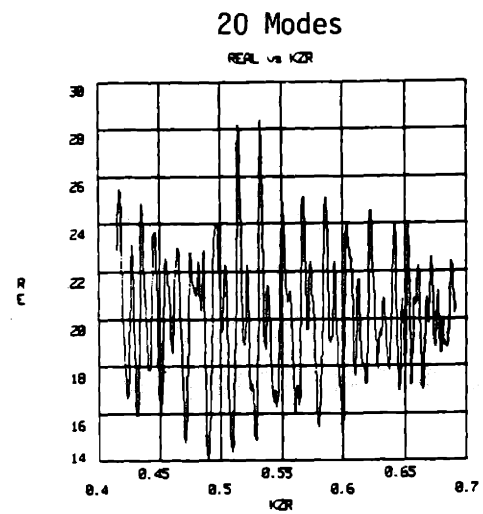
Figure 22



INPUT scank=1.0 s=10 phase=0.0 n=500 r=54.0  
 DATA kzi=0.001852 p=0.00 z=0.00 zmin=0.00 zmax=60.00  
 kminf=0.750 kmaxf=1.250 kmult=1.00 ksult=0.000  
 kzfa=0.000 kzff=.10000E+04 kzo(1)=0.55340

AVERAGE adr= 9.9892 adi= -0.0052 ade= 10.3294 adp= -0.0005  
 AVE DEV adrs= 2.1286 adrrs= 6.8278 adms= 2.1573 adars= 7.1581  
 STAND DEV sdrs= 2.6486 sdrrs= 7.3227 sdms= 2.7218 sdars= 7.6514

Figure 23



INPUT scank=1.0 s=20 phase=0.0 n=500 r=54.0  
 DATA kzi=0.001852 p=0.00 z=0.00 zmin=0.00 zmax=60.00  
 kminf=0.750 kmaxf=1.250 kmult=1.00 ksult=0.000  
 kzfa=0.000 kzff=.10000E+04 kzo(1)=0.55340

AVERAGE adr= 20.0391 adi= 0.0005 ade= 20.2136 adp= 0.0005  
 AVE DEV adrs= 2.1732 adrrs= 15.5669 adms= 2.1743 adars= 15.7414  
 STAND DEV sdrs= 2.6640 sdrrs= 15.7932 sdms= 2.6710 sdars= 15.9658

Figure 24

Radiation Resistance of the sum of 1, 3, 10 and 20 modes for  $k_i = .1/R$  and no mode splitting

1 mode

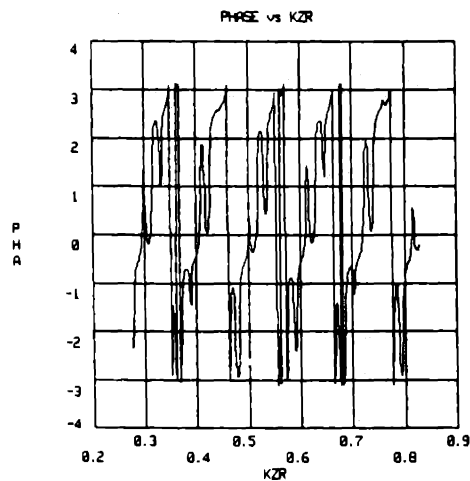


Figure 25

3 modes

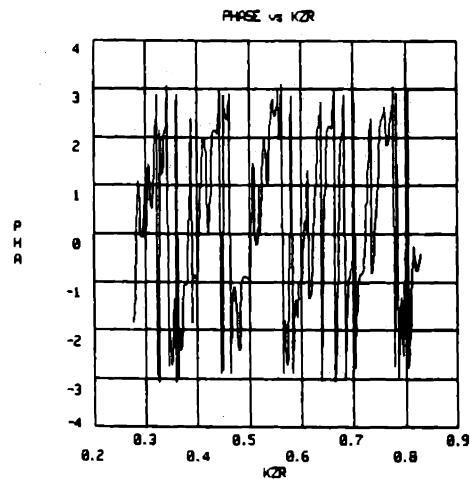


Figure 27

10 modes

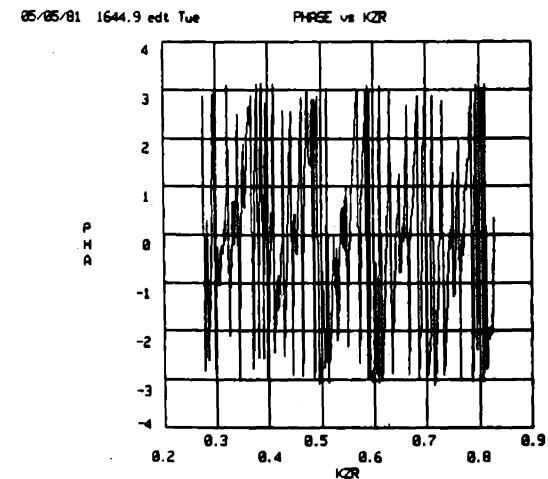


Figure 29

MAGNITUDE vs KZR

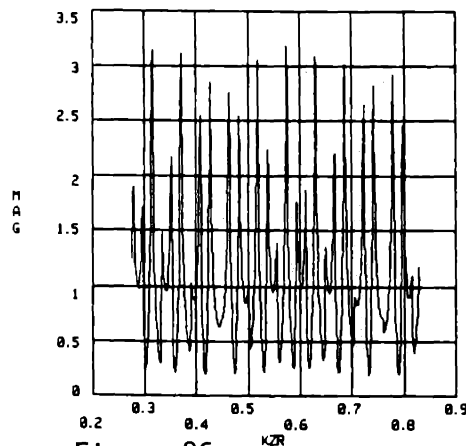


Figure 26

INPUT DATA  
 scan=1.0 s=1 phase=1.0 n=500 r=54.0  
 kzi=0.001852 p=0.00 z=60.00 zmin=0.00 zmax=60.00  
 kminf=0.500 kmaxf=1.500 kmult=1.00 ksubt=0.000  
 kzfa=0.000 kzff=-1.0000E+04 kzo(1)=0.55340

AVERAGE ar=0.0363 ai=-0.0234 am=1.0788 ap=0.3346  
 AVE DEV adrs=1.0679 adrs=1.0679 adms=0.4836 adms=0.4836  
 STAND DEV sdrs=1.3148 sdrs=1.3148 sdms=0.6684 sdms=0.6684

MAGNITUDE vs KZR

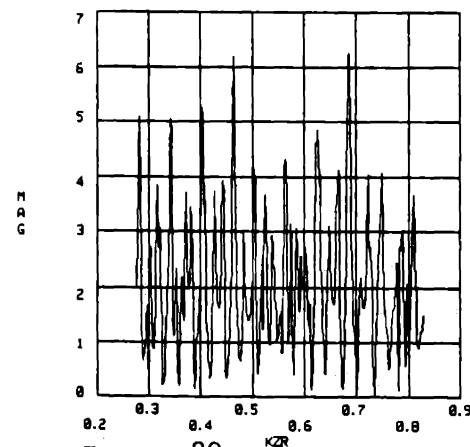


Figure 28

INPUT DATA  
 scan=1.0 s=3 phase=1.0 n=500 r=54.0  
 kzi=0.001852 p=0.00 z=60.00 zmin=0.00 zmax=60.00  
 kminf=0.500 kmaxf=1.500 kmult=1.00 ksubt=0.000  
 kzfa=0.000 kzff=-1.0000E+04 kzo(1)=0.55340

AVERAGE ar=0.0231 ai=-0.0128 am=2.0560 ap=0.1679  
 AVE DEV adrs=3.0547 adrs=1.9637 adms=1.3388 adms=0.9541  
 STAND DEV sdrs=3.4243 sdrs=2.4050 sdms=1.5436 sdms=1.2635

MAGNITUDE vs KZR

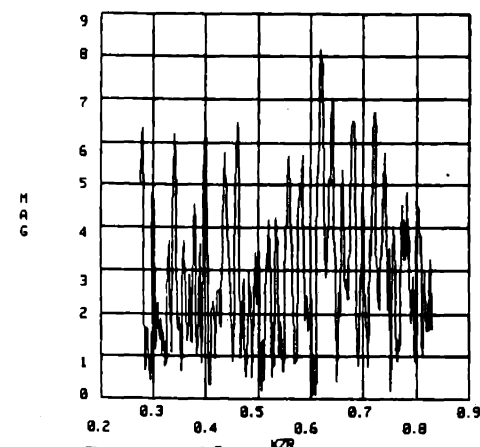


Figure 30

INPUT DATA  
 scan=1.0 s=10 phase=1.0 n=500 r=54.0  
 kzi=0.001852 p=0.00 z=60.00 zmin=0.00 zmax=60.00  
 kminf=0.500 kmaxf=1.500 kmult=1.00 ksubt=0.000  
 kzfa=0.000 kzff=-1.0000E+04 kzo(1)=0.55340

AVERAGE ar=-0.0160 ai=-0.0484 am=2.9365 ap=-0.0977  
 AVE DEV adrs=10.0160 adrs=3.3685 adms=7.0635 adms=1.3975  
 STAND DEV sdrs=10.2964 sdrs=3.9745 sdms=7.2567 sdms=1.6788

Phase and Magnitude of 1, 3, and 10 modes at  $r = 60$ , with Random Initial Phase,  $k_i = .1/R$  and No Mode Splitting.

# Probe Signal Simulations for 10 modes and Spectrum Widths of .01, .1 and 1/cm

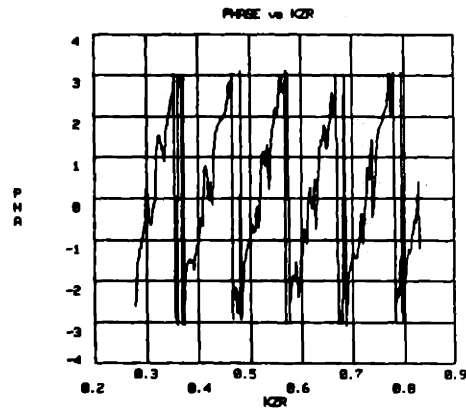


Figure 31

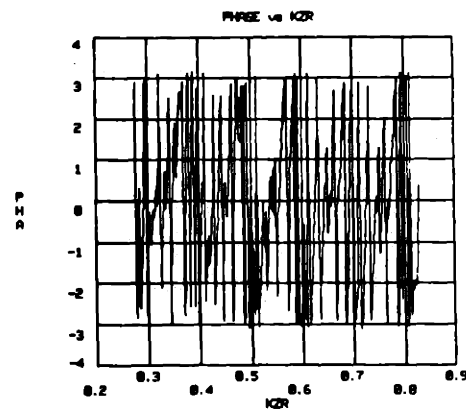


Figure 33

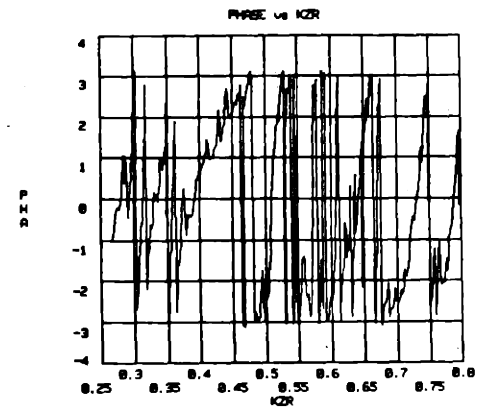
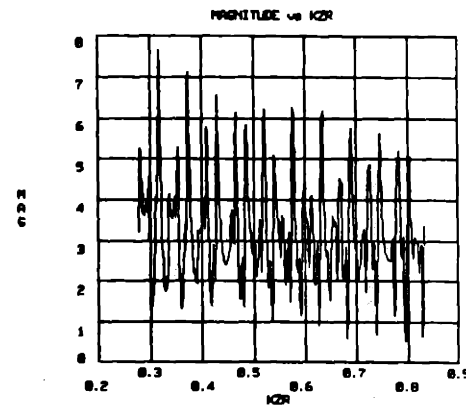
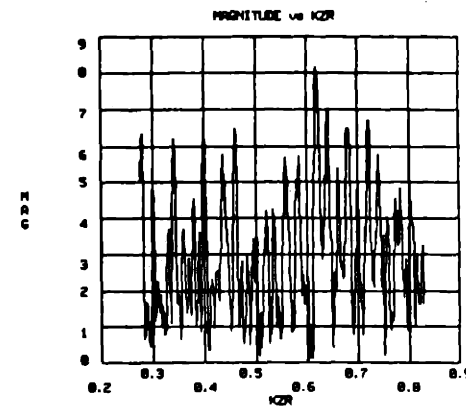


Figure 35



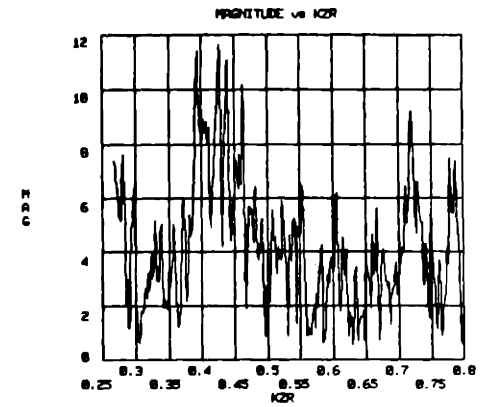
$$\Delta k_n = .01/\text{cm}$$

Figure 32



$$\Delta k_n = .1/\text{cm}$$

Figure 34



$$\Delta k_n = 1/\text{cm}$$

Figure 36

INPUT scank=1.0 s=10 phase=1.0 n=500 r=54.0  
DATA hzi=0.001052 p=0.00 z= 60.00 xmin= 0.00 xmax= 60.00  
hminf=0.500 hmaxf=1.500 hmult= 0.10 hsubt= -5.000  
hzfa=0.000 hzff=.10000E+04 hzr0(1)=0.55340  
AVERAGE sr= 0.0000 al= -0.1217 am= 3.2472 ap= 0.0175  
AVE DEV adrs= 9.9310 adrs= 3.3250 adrs= 6.7520 adrs= 0.9497  
STAND DEV adrs= 10.2249 adrs= 3.9360 adrs= 6.8616 adrs= 1.2203

INPUT scank=1.0 s=10 phase=1.0 n=500 r=54.0  
DATA hzi=0.001052 p=0.00 z= 60.00 xmin= 0.00 xmax= 60.00  
hminf=0.500 hmaxf=1.500 hmult= 1.00 hsubt= 0.000  
hzfa=0.000 hzff=.10000E+04 hzr0(1)=0.55340  
AVERAGE sr= -0.0168 al= -0.0404 am= 2.9365 ap= -0.0977  
AVE DEV adrs= 10.0160 adrs= 3.3605 adrs= 7.0635 adrs= 1.3975  
STAND DEV adrs= 10.2764 adrs= 3.9745 adrs= 7.2567 adrs= 1.6780

INPUT scank=1.0 s=10 phase=1.0 n=500 r=54.0  
DATA hzi=0.001052 p=0.00 z= 60.00 xmin= 0.00 xmax= 60.00  
hminf=0.500 hmaxf=1.500 hmult=10.00 hsubt= 0.500  
hzfa=0.000 hzff=.10000E+04 hzr0(1)=0.55340  
AVERAGE sr= 0.0460 al= 0.1996 am= 4.1905 ap= -0.2125  
AVE DEV adrs= 9.9615 adrs= 3.9533 adrs= 5.0406 adrs= 1.0755  
STAND DEV adrs= 10.5279 adrs= 4.6332 adrs= 6.2499 adrs= 2.5234



# Probe Signal Simulations for One Mode and 1% and 10% k Fluctuations

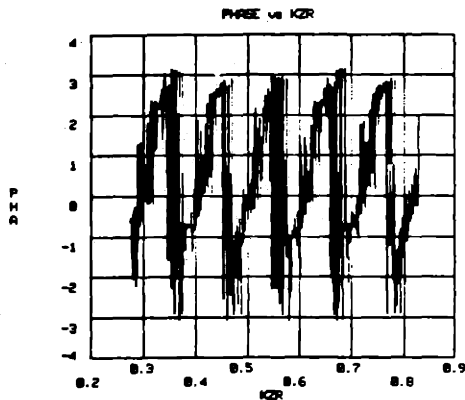


Figure 37

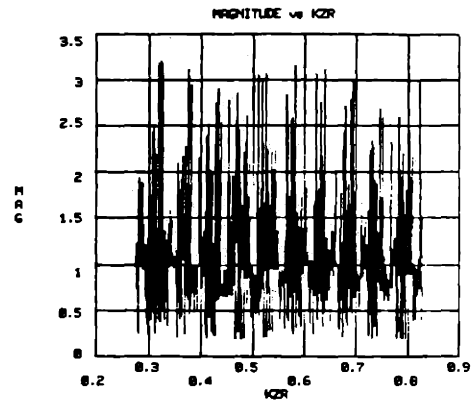


Figure 38

## 1% Fluctuations

```

INPUT  acen=1.0  a= 1  phase=1.0  n= 500  r=54.0
DATA   kzi=0.001052  p= 0.00  z= 60.00  zmin= 0.00  zmax= 60.00
        kaimf=0.500  kmaxf=1.500  kault= 1.00  kaubf= 0.000
        kafa=.010  kaff=.10000E+04  kaz=0(1)=0.55340

AVERAGE  ap= 0.6377  ai= -0.0052  ap= 1.1220  ap= 0.3906
AVE DEV  adrs= 1.0337  adrs= 1.0337  adrs= 0.5245  adrs= 0.5245
STAND DEV  adrs= 1.2557  adrs= 1.2557  adrs= 0.7155  adrs= 0.7155
    
```

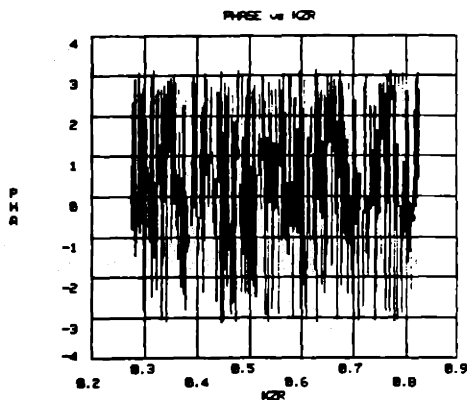


Figure 39

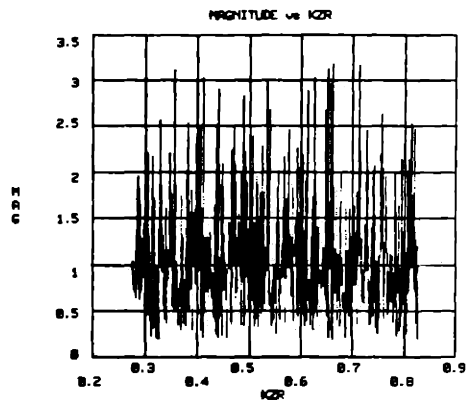


Figure 40

## 10% Fluctuations

```

INPUT  acen=1.0  a= 1  phase=1.0  n= 500  r=54.0
DATA   kzi=0.001052  p= 0.00  z= 60.00  zmin= 0.00  zmax= 60.00
        kaimf=0.500  kmaxf=1.500  kault= 1.00  kaubf= 0.000
        kafa=.100  kaff=.10000E+04  kaz=0(1)=0.55340

AVERAGE  ap= 0.6429  ai= 0.0126  ap= 1.0529  ap= 0.4316
AVE DEV  adrs= 1.0000  adrs= 1.0000  adrs= 0.4722  adrs= 0.4722
STAND DEV  adrs= 1.3002  adrs= 1.3002  adrs= 0.6310  adrs= 0.6310
    
```

# Probe Signal Simulations for 10 modes and $k_i = 1/R$ , $.1/R$ and $.01/R$

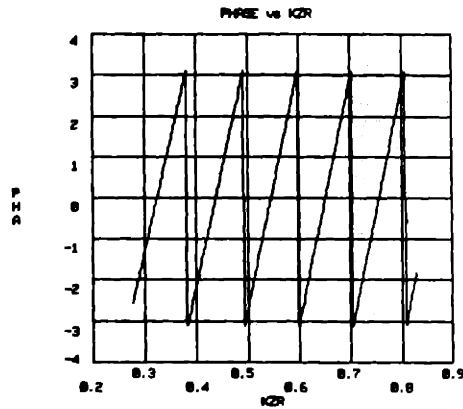


Figure 41

$$k_i = 1/R$$

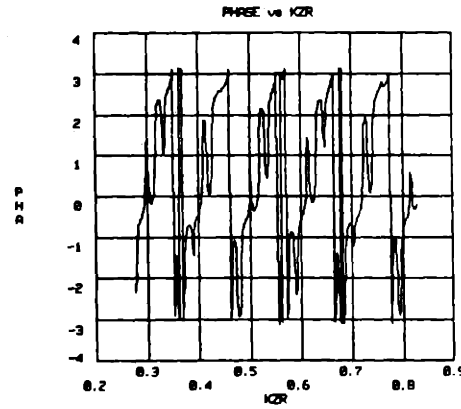


Figure 43

$$k_i = .1/R$$

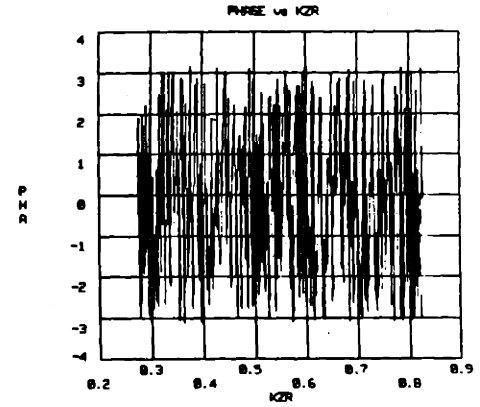


Figure 45

$$k_i = .01/R$$

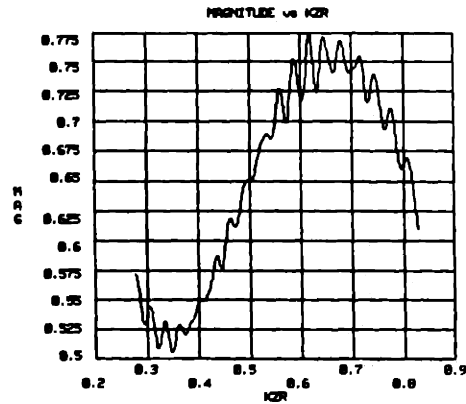


Figure 42

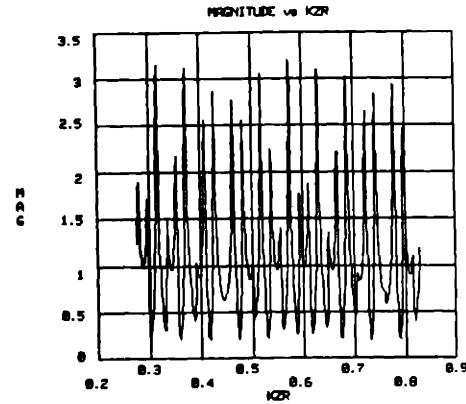


Figure 44

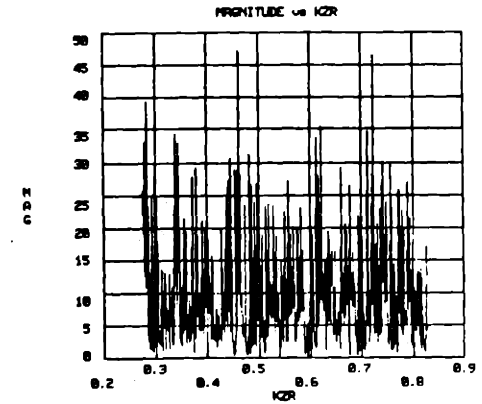


Figure 46

INPUT ecanh=1.0 a=10 phase=1.0 n=500 r=54.0  
DATA kzi=0.010520 p=0.00 z= 60.00 zmin= 0.00 zmax= 60.00  
kzinf=0.500 kzmax=1.500 kzult=1.00 kault= 0.000  
kzfa=.000 kzff=.10000E+04 kzo(1)=0.55340  
AVERAGE a= -0.0091 ai= -0.0115 am= 0.6521 ap= -0.0011  
AVE DEV adra= 10.0091 adrra= 3.1714 adma= 9.3479 adms= 2.5102  
STAND DEV adra= 10.0199 adrra= 3.2052 adma= 9.3483 adms= 2.5117

INPUT ecanh=1.0 a=1 phase=1.0 n=500 r=54.0  
DATA kzi=0.001052 p=0.00 z= 60.00 zmin= 0.00 zmax= 60.00  
kzinf=0.500 kzmax=1.500 kzult=1.00 kault= 0.000  
kzfa=.000 kzff=.10000E+04 kzo(1)=0.55340  
AVERAGE a= 0.0053 ai= -0.0234 am= 1.0708 ap= 0.3346  
AVE DEV adra= 1.0679 adrra= 1.0679 adma= 0.4936 adms= 0.4936  
STAND DEV adra= 1.3148 adrra= 1.3148 adma= 0.6604 adms= 0.6604

INPUT ecanh=1.0 a=10 phase=1.0 n=500 r=54.0  
DATA kzi=0.000105 p=0.00 z= 60.00 zmin= 0.00 zmax= 60.00  
kzinf=0.500 kzmax=1.500 kzult=1.00 kault= 0.000  
kzfa=.000 kzff=.10000E+04 kzo(1)=0.55340  
AVERAGE a= 0.1164 ai= 0.0709 am= 9.4967 ap= -0.0055  
AVE DEV adra= 11.2596 adrra= 6.0660 adma= 6.4056 adms= 6.0041  
STAND DEV adra= 13.3149 adrra= 9.4276 adma= 8.2590 adms= 10.3963

# Effect of Mode Splitting on One Mode

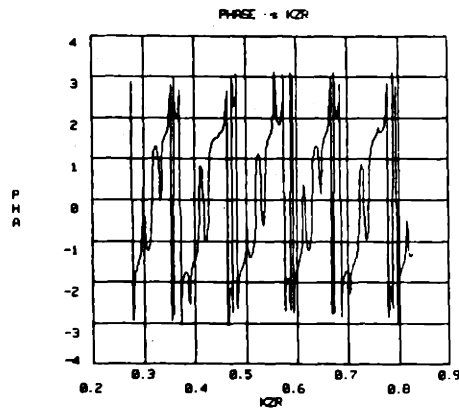


Figure 47

One Nonsplit Mode

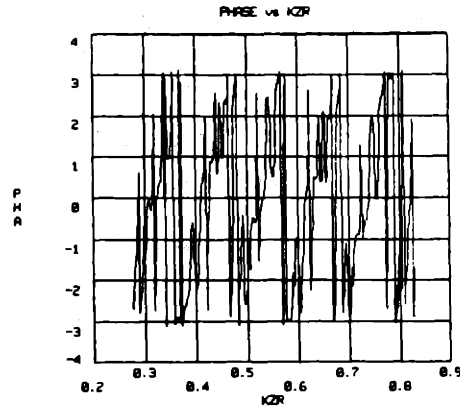


Figure 49

One 10% Split Mode

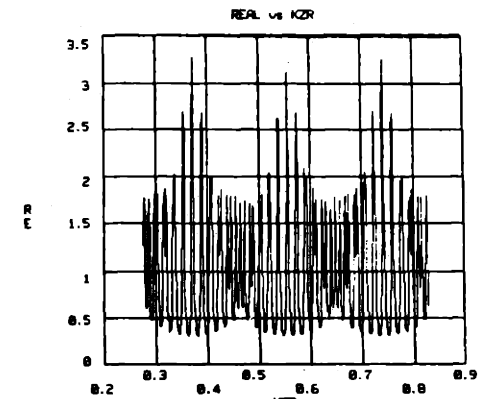


Figure 51

INPUT scank=1.0 s=1 phase=0.0 n=500 r=54.0  
DATA kzi=0.001852 p=0.10 z=0.00 zmin=0.00 zmax=60.00  
kminf=0.500 kmaxf=1.500 kmult=1.00 ksult=0.000  
kzfa=0.000 kzff=1.0000E+04 kzb(1)=0.55340  
AVERAGE ar=1.0014 ai=0.0014 am=1.1703 ap=0.0010  
AVE DEV adrs=0.5300 adrs=0.5300 adrs=0.5372 adrs=0.5372  
STAND DEV sdrs=0.6323 sdrs=0.6323 sdrs=0.6676 sdrs=0.6676

Split and Nonsplit  $R_R$  with  
One Mode

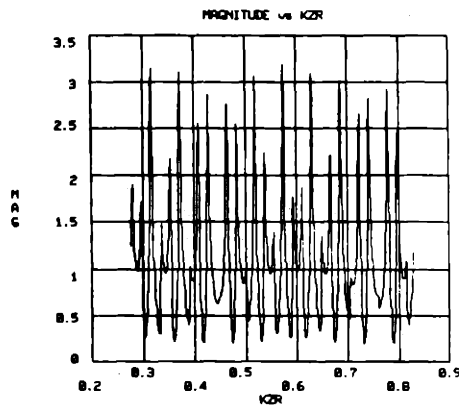


Figure 48

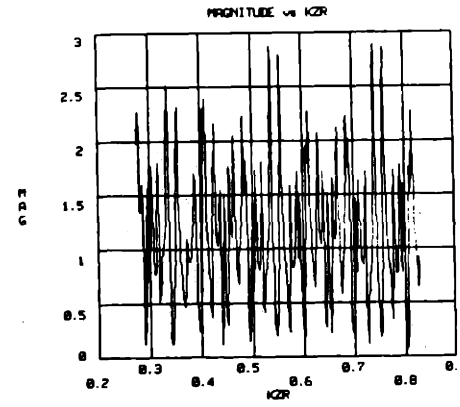


Figure 50

INPUT scank=1.0 s=1 phase=0.0 n=500 r=54.0  
DATA kzi=0.001852 p=0.10 z=60.00 zmin=0.00 zmax=60.00  
kminf=0.500 kmaxf=1.500 kmult=1.00 ksult=0.000  
kzfa=0.000 kzff=1.0000E+04 kzb(1)=0.55340  
AVERAGE ar=0.0050 ai=-0.0424 am=1.1384 ap=-0.0963  
AVE DEV adrs=1.1109 adrs=1.1109 adrs=0.4596 adrs=0.4596  
STAND DEV sdrs=1.3376 sdrs=1.3376 sdrs=0.5097 sdrs=0.5097

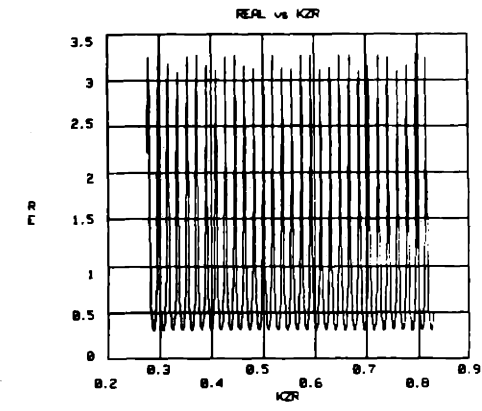


Figure 52

INPUT scank=1.0 s=1 phase=0.0 n=500 r=54.0  
DATA kzi=0.001852 p=0.10 z=0.00 zmin=0.00 zmax=60.00  
kminf=0.500 kmaxf=1.500 kmult=1.00 ksult=0.000  
kzfa=0.000 kzff=1.0000E+04 kzb(1)=0.55340  
AVERAGE ar=1.0001 ai=0.0007 am=1.3111 ap=0.0043  
AVE DEV adrs=0.7192 adrs=0.7192 adrs=0.7591 adrs=0.7591  
STAND DEV sdrs=0.6946 sdrs=0.6946 sdrs=0.9059 sdrs=0.9059

# Effect of Mode Splitting on One Mode for $k_i = .01/R$ and $S = 0, 1\%$ and $100\%$

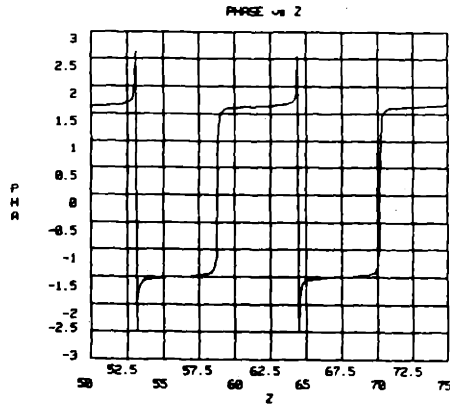


Figure 53

Nonsplit

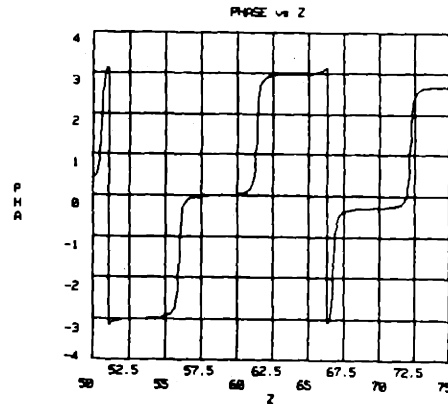


Figure 55

10% Split

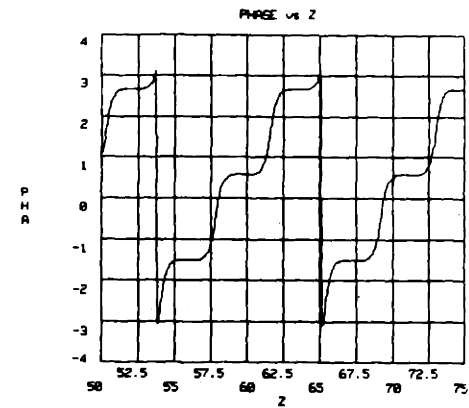
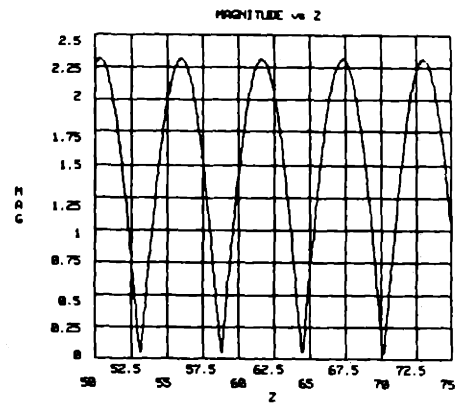


Figure 57

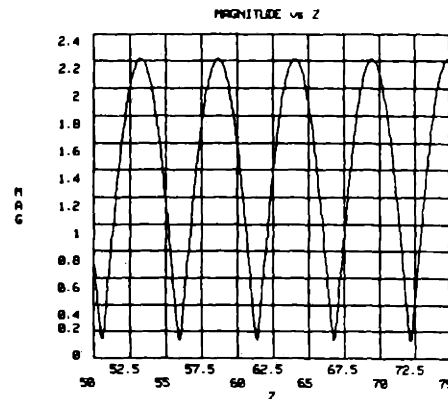
100% Split



INPUT DATA  
 $scank=0.0$   $s=1$   $phase=0.0$   $n=500$   $r=54.0$   
 $kz1=0.000105$   $p=0.00$   $z=0.00$   $zmin=50.00$   $zmax=75.00$   
 $kminf=0.900$   $kmaxf=1.100$   $kmult=1.00$   $ksubt=0.000$   
 $kzfa=0.000$   $kzff=1.0000E+04$   $kzr0(1)=0.55340$

AVERAGE  
 $ar=-0.0130$   $ai=0.1766$   $am=1.5143$   $ap=0.2103$   
 AVE DEV  $adrs=1.0130$   $adrs=1.0130$   $adrs=0.7665$   $adrs=0.7665$   
 STAND DEV  $sdrs=1.0202$   $sdrs=1.0202$   $sdms=0.0671$   $sdms=0.0671$

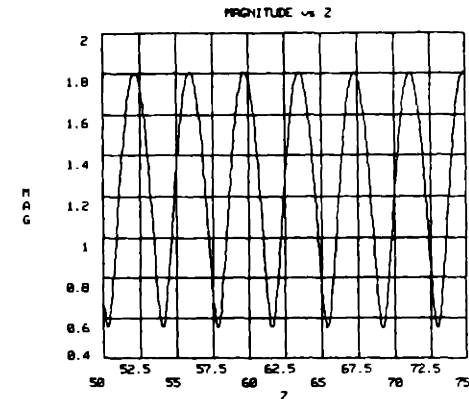
Figure 54



INPUT DATA  
 $scank=0.0$   $s=1$   $phase=0.0$   $n=500$   $r=54.0$   
 $kz1=0.000105$   $p=0.10$   $z=0.00$   $zmin=50.00$   $zmax=75.00$   
 $kminf=0.900$   $kmaxf=1.100$   $kmult=1.00$   $ksubt=0.000$   
 $kzfa=0.000$   $kzff=1.0000E+04$   $kzr0(1)=0.55340$

AVERAGE  
 $ar=-0.1426$   $ai=-0.0004$   $am=1.4003$   $ap=0.2490$   
 AVE DEV  $adrs=1.5900$   $adrs=1.5900$   $adms=0.6951$   $adms=0.6951$   
 STAND DEV  $sdrs=1.0006$   $sdms=1.0006$   $sdms=0.7874$   $sdms=0.7874$

Figure 56



INPUT DATA  
 $scank=0.0$   $s=1$   $phase=0.0$   $n=500$   $r=54.0$   
 $kz1=0.000105$   $p=1.00$   $z=0.00$   $zmin=50.00$   $zmax=75.00$   
 $kminf=0.900$   $kmaxf=1.100$   $kmult=1.00$   $ksubt=0.000$   
 $kzfa=0.000$   $kzff=1.0000E+04$   $kzr0(1)=0.55340$

AVERAGE  
 $ar=-0.0051$   $ai=0.0600$   $am=1.2584$   $ap=0.6020$   
 AVE DEV  $adrs=1.2032$   $adrs=1.2032$   $adms=0.4433$   $adms=0.4433$   
 STAND DEV  $sdrs=1.4469$   $sdms=1.4469$   $sdms=0.5067$   $sdms=0.5067$

Figure 58

#### IV-7. The Single Perpendicular Pass Regime

In the presence of very strong damping mechanisms, neither parallel or perpendicular eigenmodes can occur. Our previous waveguide treatment must be significantly modified to account for the excited spectrum of the antenna. The concept of group velocity becomes an important analytical and intuitive tool for estimating energy propagation and deposition in this regime.

##### IV-7.1. Single perpendicular pass radiation resistance

To estimate the single perpendicular pass radiation resistance, we will model the antenna and plasma in the semi-infinite Cartesian geometry shown in Figure 1. The antenna is modeled as a Fourier transformed current sheet at  $r_a$ <sup>126</sup>

$$(1) \quad J_k = J_0 e^{i(kz - \omega t)}$$

backed by an infinitely conducting wall at  $r_w$ . The semi-infinite plasma is modeled as a flat density profile starting at  $r_c$  (cutoff radius), which will be a function of  $k_{||}$ , as we shall see shortly. Only  $E$  in the direction of  $J$  needs to be considered, so that we define

$$(2) \quad E_k = E_y \approx E_{\theta}$$

Neglecting vacuum displacement currents, the electric field in the three regions a, b, and c can be written as

$$(3) \quad E_a = A e^{ikr}$$

$$(4) \quad E_b = B e^{kr} + C e^{-kr}$$

$$(5) \quad E_c = D e^{kr} + F e^{-kr}$$

where  $E_a$  is propagating and  $E_b$  and  $E_c$  are evanescent. From continuity in  $E_y$ , we have the boundary conditions

$$(6) \quad E_c(r_w) = 0$$

$$(7) \quad E_b(r_a) = E_c(r_a)$$

$$(8) \quad E_a(r_c) = E_b(r_c)$$

From continuity in  $H_z$  ( $H_z \propto \partial E / \partial r$ ), we have

$$(9) \quad \frac{d}{dr} [E_a(r_c) - E_b(r_c)] = 0$$

$$(10) \quad \frac{d}{dr} [E_b(r_a) - E_c(r_a)] = \frac{4\pi i \omega J_k}{c^2}$$

By making the judicious change of variables

$$(11) \quad \delta_1 = r_a - r_c$$

$$(12) \quad \delta_2 = r_w - r_a$$

$$(13) \quad \delta_3 = \delta_1 + \delta_2 = r_w - r_c$$

equations (3) through (10) can be reduced to five compact equations and five unknowns.

$$(14) \quad D = -F e^{-2k\delta_2}$$

$$(15) \quad E(r_a) = B + C = D + F$$

$$(16) \quad A e^{-ikr\delta_1} = B e^{-k\delta_1} + C e^{+k\delta_1}$$

$$(17) \quad ik_r A e^{-ikr\delta_1} = kBe^{-k\delta_1} - kCe^{k\delta_1}$$

$$(18) \quad B - D + F - C = \frac{4\pi i \omega J}{kc^2}$$

which, after some lengthy algebra, can be solved to give the Fourier transformed electric field at the antenna (15).

$$(19) \quad E_k(r_a) = \frac{2\pi i \omega J_k}{kc^2} (e^{k\delta_2} - e^{-k\delta_2}) \times \frac{k(e^{k\delta_1} + e^{-k\delta_1}) + ik_r(e^{k\delta_1} - e^{-k\delta_1})}{k(e^{k\delta_3} + e^{-k\delta_3}) + ik_r(e^{k\delta_3} - e^{-k\delta_3})}$$

Fourier transforming the antenna current strip, we have

$$(20) \quad J_k = \frac{1}{2\pi} \int_{-w/2}^{+w/2} J_o e^{-ikz} dz = \frac{J_o w \sin k w/2}{2\pi k w/2}$$

and

$$(21) \quad I_o = J_o w$$

The inverse transform of the electric field at the antenna radius is

$$(22) \quad E(r_a, z) = \int_{-\infty}^{+\infty} E_k(r_a) e^{ikz} dk$$

The Fourier spectrum of the antenna complex impedance is

$$(23) \quad Z_k = \frac{E_k(r_a)}{I_o}$$

and the inverse transform is

$$(24) \quad Z = \int_{-\infty}^{+\infty} \frac{E_k(r_a)}{I_o} e^{ikz} dk \approx 2 \int_0^{+\infty} \frac{E_k(r_a)}{I_o} dk$$

Radiation resistance and impedance per unit length are then simply

$$(25) \quad R_R/\text{cm} = \text{Re} [Z]$$

$$(26) \quad X/\text{cm} = \text{Im} [Z]$$

In this model, we furthermore let  $k_r$  and  $\delta_1$  be functions of  $k$ , which is of course,  $k_z$ . Assuming a parabolic density profile, the cutoff ( $k_{||} = 0$ ) density and radius ( $r_c$ ), and thus  $\delta_1$ , can be found by equations IV-2.5.(12) and IV-2.2.(6) as

$$(27) \quad n_c = \frac{1 + \Omega}{\Omega^2} 5.16 \times 10^{14} \mu k^2$$

$$(28) \quad \delta_1 = \delta_o + r_p \left( 1 - \sqrt{1 - \frac{n_c}{n_o}} \right)$$

The plasma density in region a is then assumed to be

$$(29) \quad n_a = \frac{n_o + n_c}{2}$$

so that  $k_r$  can be calculated from equation IV-2.2.(6), assuming  $k_{||} = k$  and  $n = n_a$ .

Figures 2 and 3 show the real and imaginary parts of the  $k$  spectrum for typical Alcator conditions

$$\mu = 2 \text{ (Deuterium)}$$

$$r_a = 11 \text{ cm}$$

$$\Omega = 2$$

$$r_p = 9 \text{ cm}$$

$$f. = 97 \text{ MHz}$$

$$w = 3 \text{ cm}$$

$$r_w = 12.5 \text{ cm}$$

$$n_o = 0, .5, 1, 5 \times 10^{14} \times 10^{14}/\text{cm}^3$$

The integral of the imaginary part of the spectrum for  $n_o = 0$  ( $Z_{INTI} = 2.2 \Omega/\text{cm}$ ) is in reasonable agreement with  $\omega L$ , as calculated from equations III-2.4.(1) and III-2.5.(9)

$$(30) \quad \omega L = \frac{\omega Z}{c} = 1.83 \Omega/\text{cm}$$



The reactance is also slightly decreased, as expected, when the high dielectric constant plasma is brought near the stripline antenna. The real part of the spectrum is concentrated at low  $k_{||}$ , and its integral ( $Z_{INTR} \approx .25 \Omega/\text{cm}$ ) is much smaller than the imaginary part, and is weakly dependent on density.

Figures 4 and 5 show the 200 MHz hydrogen case, where we note how the no-plasma reactance exactly doubled as expected. The real part also approximately doubled.

These computational findings can be more physically understood by separating the problem into three parts; wave production, attenuation and transmission. Wave production can be visualized as a wave from the antenna center conductor that interferes with a wave reflected from the wall.

$$(31) \quad E \propto 1 - e^{-2k_{||}\delta_2} \approx 2k_{||}\delta_2$$

Halving  $\delta_2 = r_w - r_a$  thus halves the electric field, and reduces the coupled power and radiation resistance by a factor of four ( $Z_{INTR} = .065$ ), just as in eigenmode coupling. This effect is clearly shown in Figure 6 where  $r_a$  was increased from 11 cm to 11.75 cm. The antenna reactance also decreased according to equation (30).

The attenuation is simply the evanescence between the antenna and the plasma

$$(32) \quad E \propto e^{-k_{||}\delta_1}$$

and limits  $k_{||} < 1/\delta_1$ . Decreasing  $r_p$  from 9 to 8 cm decreased  $R_R$  by about 40%, as shown in Figure 7.

The transmission factor between the evanescent wave in the vacuum region and the propagating wave in the plasma comes about from the mismatch between

the plasma and vacuum impedences. From continuity of E and H, we have  
(Transmitted, Incident, Reflected)

$$(33) \quad E_T e^{ik_{\perp}r} = E_I e^{-k_{\parallel}r} - E_R e^{+k_{\parallel}r}$$

$$(34) \quad E_T i k_{\perp} e^{ik_{\perp}r} = -E_I k_{\parallel} e^{-k_{\parallel}r} - E_R k_{\parallel} e^{k_{\parallel}r}$$

and assuming for simplicity  $r = 0$  and  $E_I = 1$ , we find

$$(35) \quad E_T \propto \frac{f(k_{\parallel})}{ik_{\perp} - k_{\parallel}}$$

For constant  $k_{\parallel}$ ,  $E_T$  is dependent on density when  $k_{\perp} \ll k_{\parallel}$ . On the other hand, at high density and small  $k_{\parallel}$ ,  $k_{\perp} \gg k_{\parallel}$  and

$$(36) \quad E_T \propto \frac{f(k_{\parallel})}{ik_{\perp}} \propto \frac{1}{\sqrt{n}}$$

Figure 8 shows a similar case, but with deuterium at high density ( $1 - 5 \times 10^{14}/\text{cm}^3$ ). For small  $k_{\parallel}$  ( $\approx .1/\text{cm}$ ), the cutoff layer is nearly fixed to the plasma edge ( $r_c = r_p$ ), so that equation (36) is valid, and

$$(37) \quad R_R \propto EXH \propto k_{\perp} E_T^2 \propto \frac{1}{k_{\perp}} \propto \frac{1}{\sqrt{n}}$$

which is in good agreement with Figure 8.

If a density gradient were included, the transmission coefficient would be slightly larger, but since

$$(38) \quad \lambda = \frac{2\pi}{k_{\perp}} \gtrsim k_{\perp} \left[ \frac{\partial k_{\perp}}{\partial r} \right]^{-1}$$

our computational model is a good approximation.

The last variable is the width of the antenna. Since  $w < 1/k_{\text{real}}$ , the width has little affect on  $R_R$ . On the other hand, decreasing  $w$  increases the high end of the imaginary part of the  $k_{\parallel}$  spectrum (Figure 9), and thus the impedance, also in agreement with equation (30).

# Single Perpendicular Pass Regime Fast Wave Coupling Model

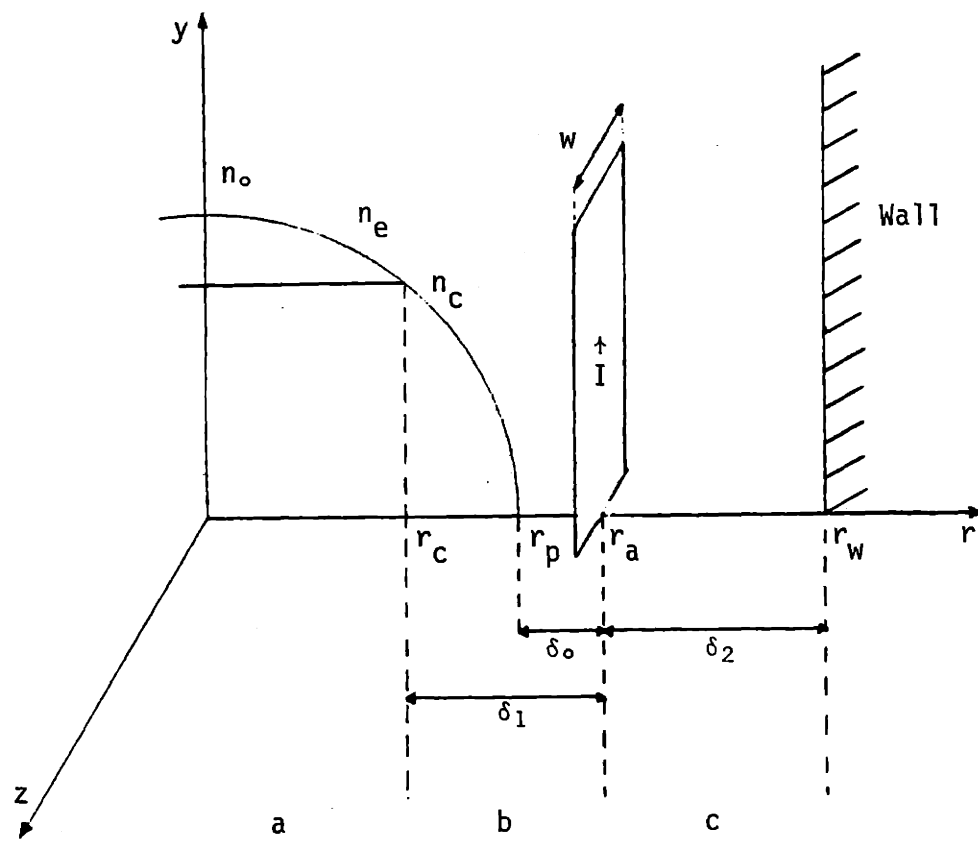


Figure 1

# $k_{||}$ Spectrum for $D_2$ at 97 MHz

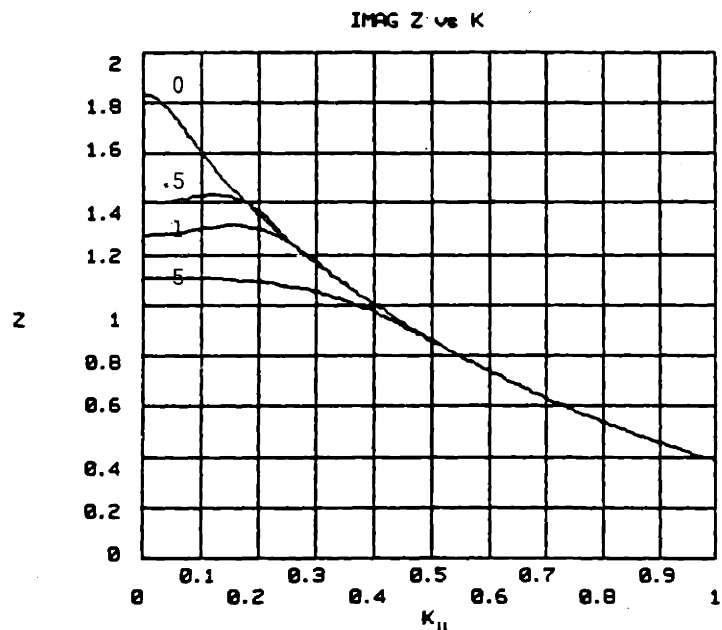


Figure 2

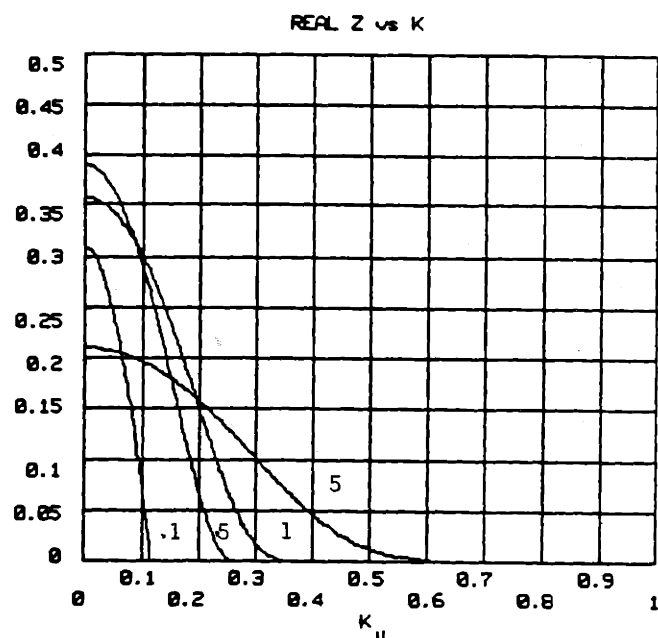


Figure 3

$\mu = 2.000$   $\omega = 2.000$   $f = .97000E+08$   
 $r = 12.500$   $a = 11.000$   $r_p = 9.000$   $w = 3.000$   
 $n0 = .00000E+00$   $z_{intr} = 0.000$   $z_{int1} = 2.204$   
 $n0 = .50000E+14$   $z_{intr} = 0.108$   $z_{int1} = 2.131$   
 $n0 = .10000E+15$   $z_{intr} = 0.128$   $z_{int1} = 2.081$   
 $n0 = .50000E+15$   $z_{intr} = 0.123$   $z_{int1} = 1.952$

# $k_{||}$ Spectrum for $H_2$ at 200 MHz

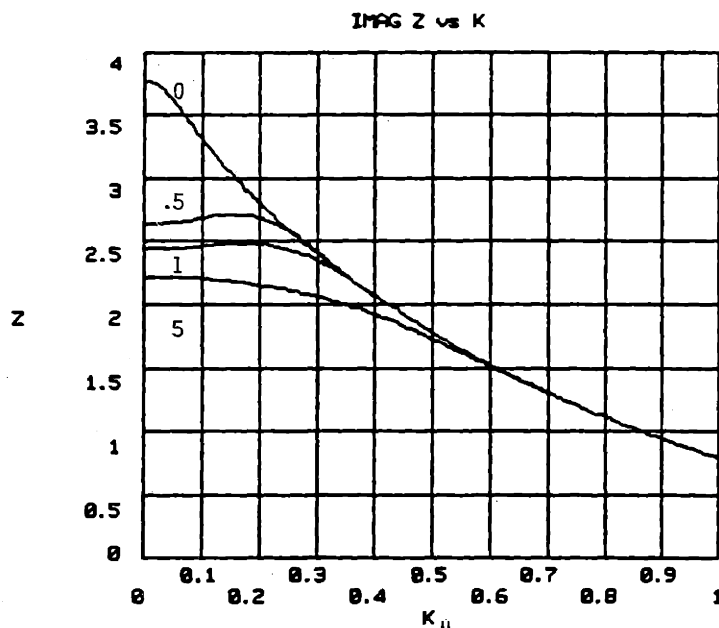


Figure 4

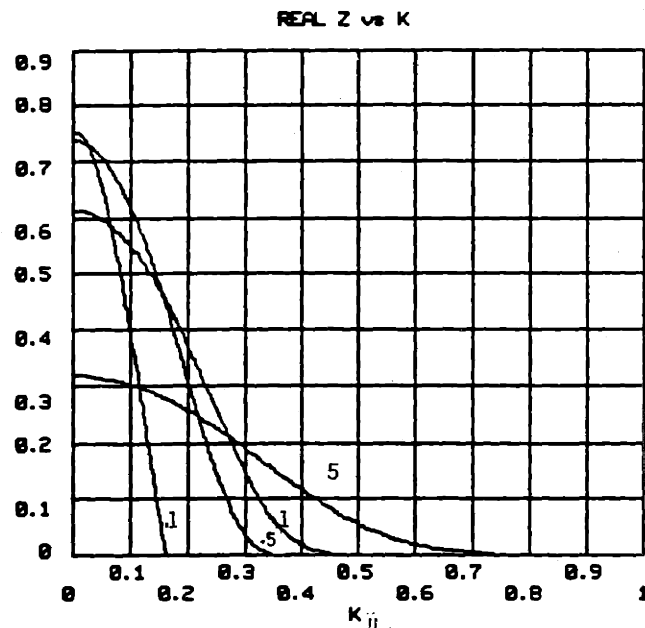
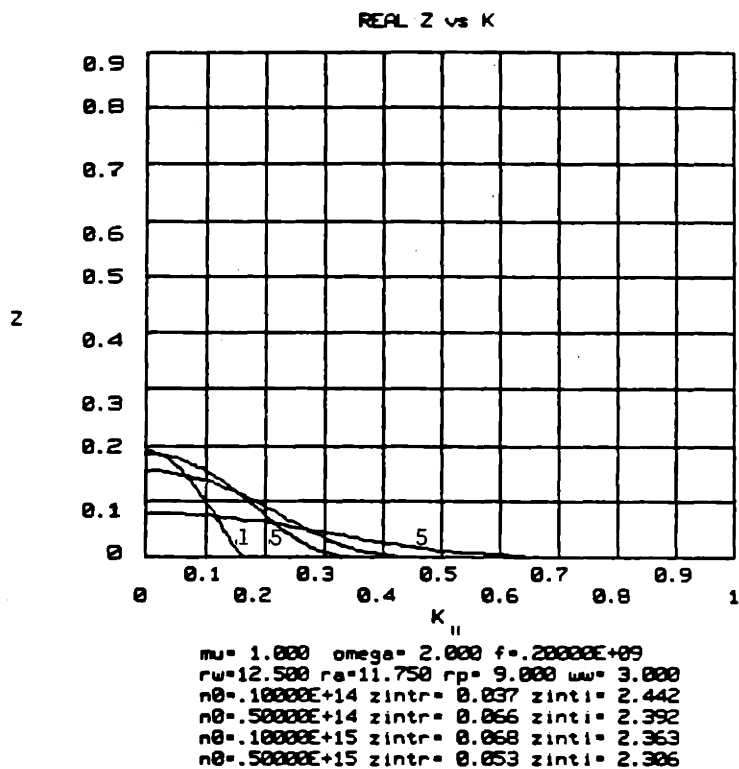
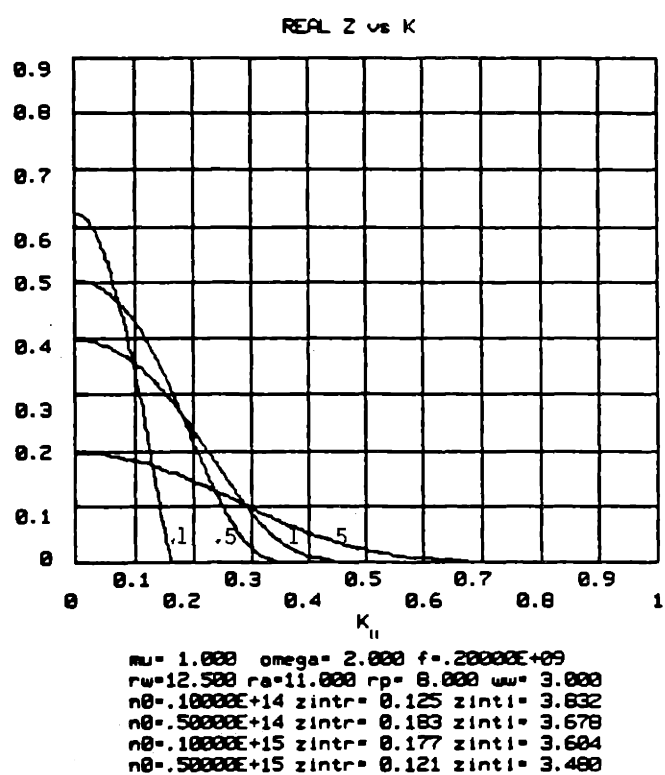


Figure 5

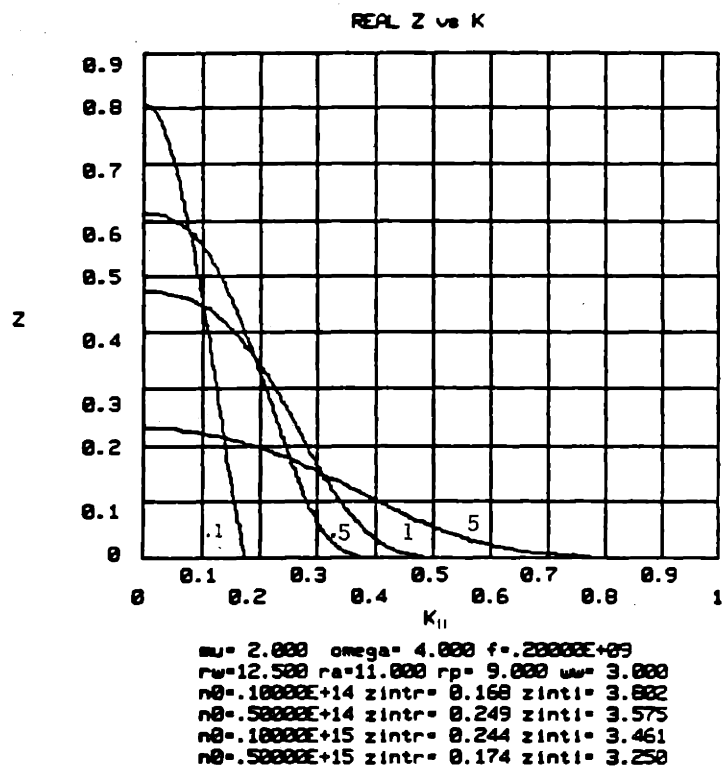
$\mu = 1.000$   $\omega = 2.000$   $f = .20000E+09$   
 $r = 12.500$   $a = 11.000$   $r_p = 9.000$   $w = 3.000$   
 $n0 = .00000E+00$   $z_{intr} = 0.000$   $z_{int1} = 4.544$   
 $n0 = .50000E+14$   $z_{intr} = 0.263$   $z_{int1} = 4.290$   
 $n0 = .10000E+15$   $z_{intr} = 0.275$   $z_{int1} = 4.172$   
 $n0 = .50000E+15$   $z_{intr} = 0.219$   $z_{int1} = 3.935$



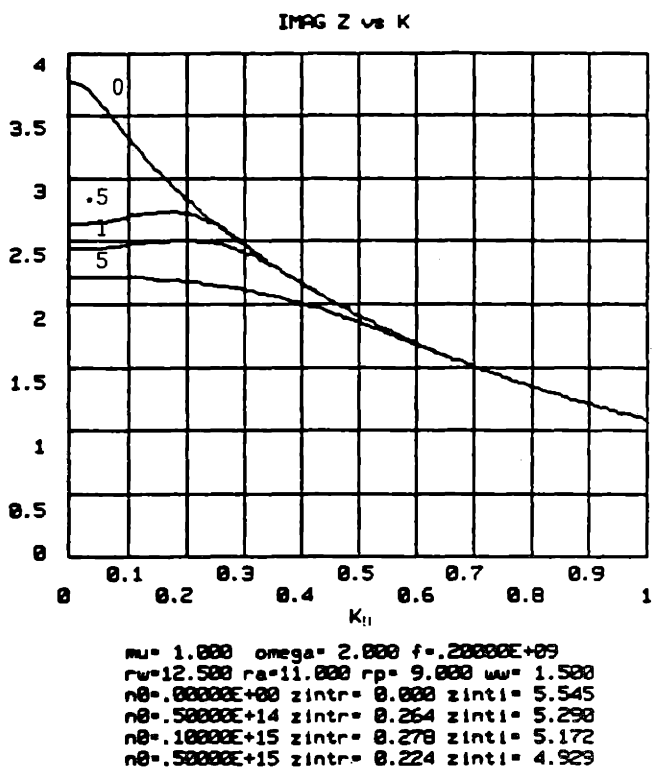
$d = .75$  cm Figure 6



$r_p = 8$  cm Figure 7



$D_2$  at 200 MHz Figure 8



$w = 1.5$  cm Figure 9

#### IV-7.2. Group velocity and simple ray tracing

The normalized fast wave dispersion relation is well approximated by (IV-2.2.(21))

$$(1) \quad N_{\perp}^2 + (1 + \Omega) N_{\parallel}^2 = 1$$

$$(2) \quad k_{\perp}^2 = k_x^2 + k_y^2 = \frac{\omega^2}{V_A^2} - (1 + \Omega) k_{\parallel}^2$$

The group velocity in the y direction ( $\theta$ ) is<sup>144</sup>

$$(3) \quad v_{gy} = \frac{\partial \omega}{\partial k_y} = \left( \frac{\partial k_y}{\partial \omega} \right)^{-1}$$

Combining (2) and (3), we have

$$(4) \quad \frac{1}{v_{gy}} = \frac{\partial k_y}{\partial \omega} = \frac{1}{2k_y} \left[ \frac{2\omega}{V_A^2} - \frac{k_{\parallel}^2}{\omega c_i} \right]$$

and similarly for  $v_{gx}$ , so that

$$(5) \quad \frac{v_{gy}}{v_{gx}} = \frac{k_y}{k_x} \ll 1$$

For practical purposes, this means that in the perpendicular plane, energy leaves the antenna at nearly right angles to the vacuum wall, as shown in Figure 1. The wave energy (rays) can even be slightly focused to the plasma center, due to the  $k_r$  gradient ( $n(r)$ ). In the single perpendicular pass regime, as opposed to eigenmode coupling, the antenna must of course be fed in the push-pull mode for plasma center heating.

Substituting  $k_{\perp} \rightarrow k_y$  in (4) gives the compact form

$$(6) \quad v_{g\perp} = \frac{N_{\perp}}{1 - \frac{\Omega}{2} N_{\parallel}^2} V_A$$

$$= \frac{N_{\perp}}{1 - N_{\parallel}^2} V_A \quad \text{for } \Omega = 2$$

$$= V_A \quad \text{for } k_{\parallel} = 0$$

Similarly<sup>11</sup>,

$$\begin{aligned}
 (7) \quad v_{g_{\parallel}} &= \frac{(1 + \Omega)N_{\parallel}}{1 - \frac{\Omega}{2} N_{\parallel}^2} V_A \\
 &= \frac{3N_{\parallel}}{1 - N_{\parallel}^2} V_A \quad \text{for } \Omega = 2 \\
 &= \frac{(1 + \Omega)^{3/2}}{1 + \Omega/2} V_A \quad \text{for } k_{\perp} = 0
 \end{aligned}$$

and finally, dividing (6) by (7), we have

$$(8) \quad \frac{v_{g_{\perp}}}{v_{g_{\parallel}}} = \frac{1}{1 + \Omega} \frac{N_{\perp}}{N_{\parallel}}$$

Figure 2 shows a graph of the normalized  $N_{2 \times 2}$  fast wave dispersion relation (IV-2.2.(18)), the normalized  $N$  elliptical (1) approximation, the normalized  $1/N$  wave normal surface (normalized phase velocity), and the group velocity for  $\Omega = 2$ . The normalized phase velocity  $\omega/kV_A$ , is simply  $\frac{1}{|N|}$  in the direction of  $k$ . Loci labeled by the number 4 for example, all refer to the case

$$\begin{aligned}
 (9) \quad k_{\perp} &= k_{\parallel} = .5k_A \\
 N_{\perp} &= N_{\parallel} = .5 \\
 v_{p_{\perp}} &= v_{p_{\parallel}} = \sqrt{2} V_A \\
 v_{g_{\parallel}} &= 3v_{g_{\perp}} = 2.0 V_A
 \end{aligned}$$

Simple ray tracing can be done graphically, as shown in Figure 3, by plotting the local group velocity on the wave normal surfaces for different times. The calculation and plot need to be done only once, since the copy can be photostatically reduced. The rays are then conformed to the group velocity vector field.

For the more realistic inhomogeneous plasma of the previous Section, we saw that almost all the power was coupled to small  $k_{\parallel}$ , due to the evanescent edge. Thus, outside the  $45^{\circ}$  wedge drawn in Figure 3,  $k_{\parallel} > k_{\perp}$ , and we need not worry about the more complicated boundary conditions at the plasma edge and vacuum chamber wall. Inside this wedge, our simple model is qualitatively correct, and we note in particular, how the phase fronts propagate at nearly right angles with the magnetic field. The group velocity, and thus the rays, on the other hand, have a very strong tendency to diverge in the parallel direction. The density gradient again tends to focus the rays by increasing  $k_{\perp}$ , so that most of the single pass energy at the plasma center should be enclosed in the  $45^{\circ}$  wedge, not that this wedge is in either  $k$  (wave normal) or  $x$  (wave front) space.

Since the fast wave is electromagnetic (Section IV-2.7.), power flow can also be followed with the Poynting flux vector (IV-1.3.)

$$(10) \quad \mathbf{S} = \frac{\mathbf{E} \times \mathbf{H}}{2}$$

and assuming  $k_y = E_z = 0$

$$(11) \quad S_x = \frac{k_x}{2\omega\mu_0} E_y^2$$

$$S_y = 0 \quad \text{since } E_x \propto i E_y$$

$$S_z = \frac{k_z}{2\omega\mu_0} (E_x^2 + E_y^2)$$

and

$$(12) \quad \frac{S_{\perp}}{S_{\parallel}} = \frac{E_y^2}{E_x^2 + E_y^2} \frac{k_{\perp}}{k_{\parallel}}$$



From the first line of the wave tensor equation, II-2.2.(14)

$$(13) \quad \frac{E_x^2}{E_y^2} = \left[ \frac{\Omega S}{S - n_{||}^2} \right]^2$$

which combined with (12), gives the energy propagation direction

$$(14) \quad \frac{S_{\perp}}{S_{||}} = \frac{1}{\left( \frac{\Omega}{1 - (1 - \Omega^2) N_{||}^2} \right)^2 + 1} \frac{N_{\perp}}{N_{||}}$$

$$= \frac{1}{5} \frac{N_{\perp}}{N_{||}} \quad \text{for } \Omega = 2 \text{ and } N_{||} = 0 \quad (90^\circ)$$

$$= \frac{1}{2.4} \frac{N_{\perp}}{N_{||}} \quad \text{for } \Omega = 2 \text{ and } N_{||} = .478 \quad (45^\circ)$$

$$= \frac{1}{2} \frac{N_{\perp}}{N_{||}} \quad \text{for } \Omega = 2 \text{ and } N_{||} = .58 \quad (0^\circ)$$

in qualitative agreement with the simple elliptical approximation of equation (8).

# Perpendicular Ray Training

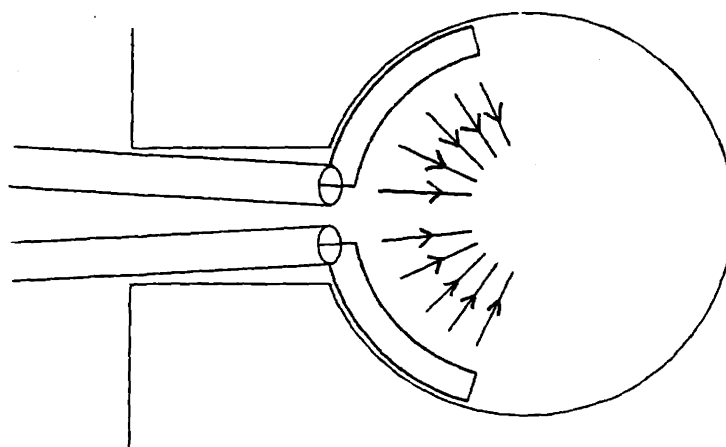


Figure 1

## First Order Phase and Group Velocities for $\Omega = 2$

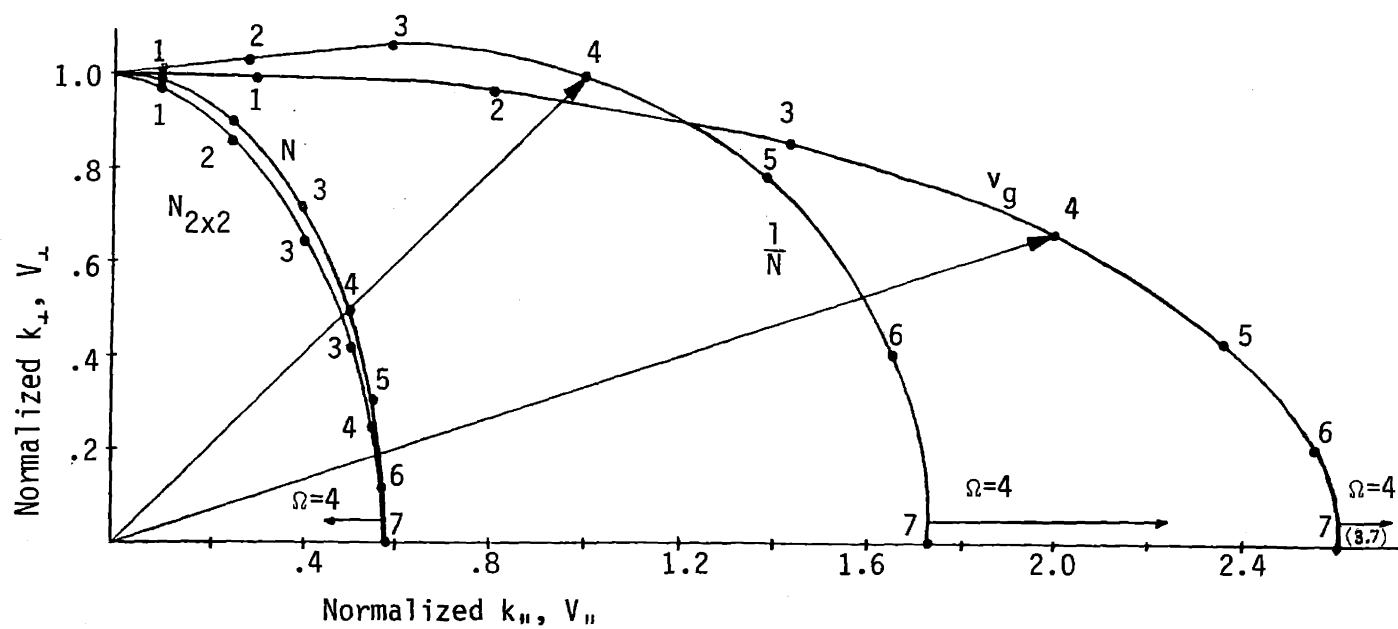


Figure 2

### Phase Fronts and Simple Ray Tracing

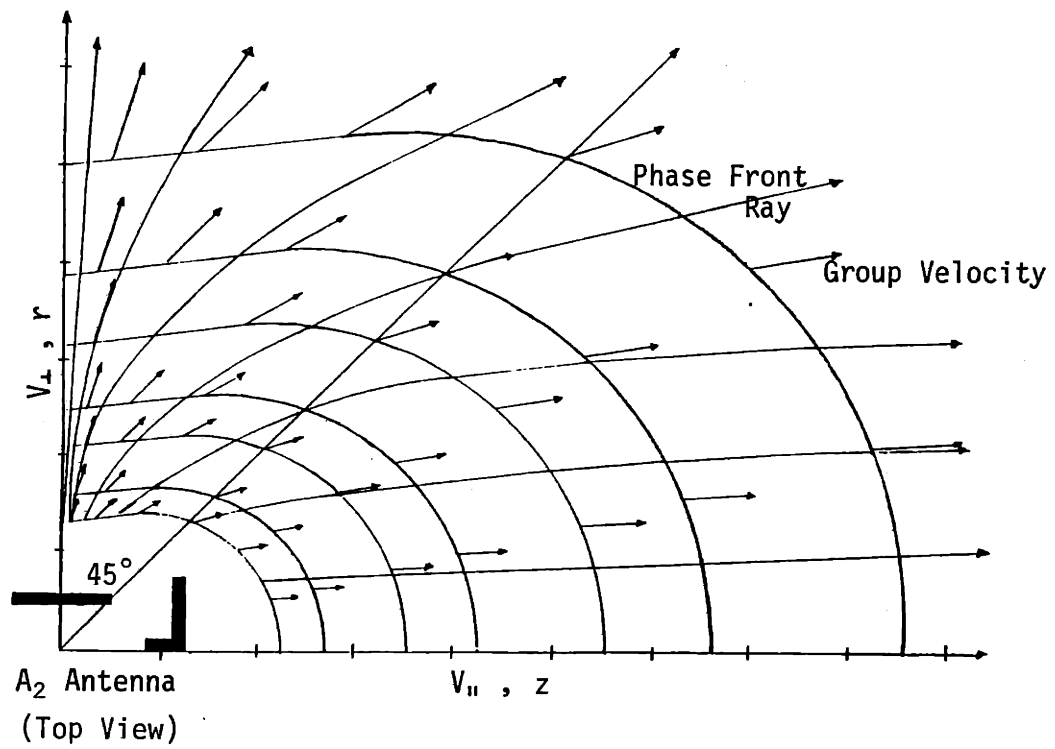


Figure 3

#### IV-7.3. Tunneling and poloidal magnetic field effects in the TIIH regime

Near the two ion-ion hybrid resonance, the wave equation can be approximated by<sup>10,48</sup>

$$(1) \quad \frac{d^2 E}{dr^2} + \left[ \frac{a}{r} + b \right] E = 0$$

where from equation IV-2.3.(17)

$$(2) \quad \frac{a}{r} + b = k_{\perp}^2(r)$$

and from simple curve fitting at and away from the TIIH layer.

$$(3) \quad b = k_{\perp}^2(r \rightarrow \infty) \approx 1$$

$$(4) \quad a = \Delta r b \approx .5$$

Where  $\Delta r$  is the thickness of the evanescent layer. Figure 1 is a close up of Figure IV-2.5.(3), and is in good agreement with Figure 2, which is a plot of equation (2).

Equation (1) can be solved using confluent hypergeometric functions<sup>81,131</sup> (instead of trigonometric functions, for  $k^2(r) = \text{constant}$ ). From principles of geometrical optics, we can define reflection (R), transmission (T), and absorption coefficients (A) at the TIIH layer. The tunneling coefficient

$$(5) \quad \eta = \Delta r k_{\perp}(\infty) = .5$$

is independent of damping and direction of propagation through the resonant layer. A wave incident from the high field side sees a resonance first, no reflection occurs, and most of the power is absorbed.

$$(6) \quad R = 0$$

$$(7) \quad T = e^{-\pi\eta/2} = .46$$

$$(8) \quad A_H = 1 - R^2 - T^2 = 1 - e^{-\pi\eta} = .8$$

A wave incident from the low field side, on the other hand, encounters an evanescent layer, and most of the power is reflected.

$$(9) \quad R = 1 - e^{-\pi\eta} = .8$$

$$(10) \quad T = e^{-\pi\eta/2} = .46$$

$$(11) \quad A_L = e^{-\pi\eta} + e^{-2\pi\eta} \approx .16$$

Figure 3 shows how absorption from the high field side ( $A_H$ ) is always greater than from the low field side ( $A_L$ ).

We might wonder how power is absorbed if the mechanism is independent damping. The answer resides in the fact that the perpendicular group velocity tends to zero near the resonance. Even for a small change in  $\omega$ , the resonant layer  $k_\perp$  can change drastically, so that

$$(12) \quad \frac{\partial k_\perp}{\partial \omega} \rightarrow \infty \quad \text{and} \quad v_{g\perp} \rightarrow 0$$

The parallel group velocity, on the other hand, is weakly dependent on  $\omega$ , so that energy is diverted along the parallel direction, and slowly dissipates over many wavelengths. Of course, mode conversion occurs here, so that this treatment is only approximate.

Another major difference between low and high field side incidence is the difference in species heated. High field incidence lets  $k_\perp \rightarrow \infty$  so that, from equation IV-2.6.(5),  $E_z$  is increased manyfold, and electron Landau damping (equation IV-4.3.(5)) can become very large. The inclusion of rotational transform effects (Figure 4) also increases  $k_\perp$ .

$$(13) \quad k_{\parallel} \approx k_z + \frac{B_{\theta}}{B_0} k_{\perp}$$

and further enhances electron heating through increased  $E_z$  and  $v_{the} / v_p$ .

At low minority concentrations, the TIIH layer becomes part of the finite thickness (IV-4.2.(36)), fundamental and second harmonic resonance layers and hot plasma effects swamp out the resonances, and electron heating stays small compared to ion heating.

# Detail of TIIH Resonance Layer Dispersion Relation

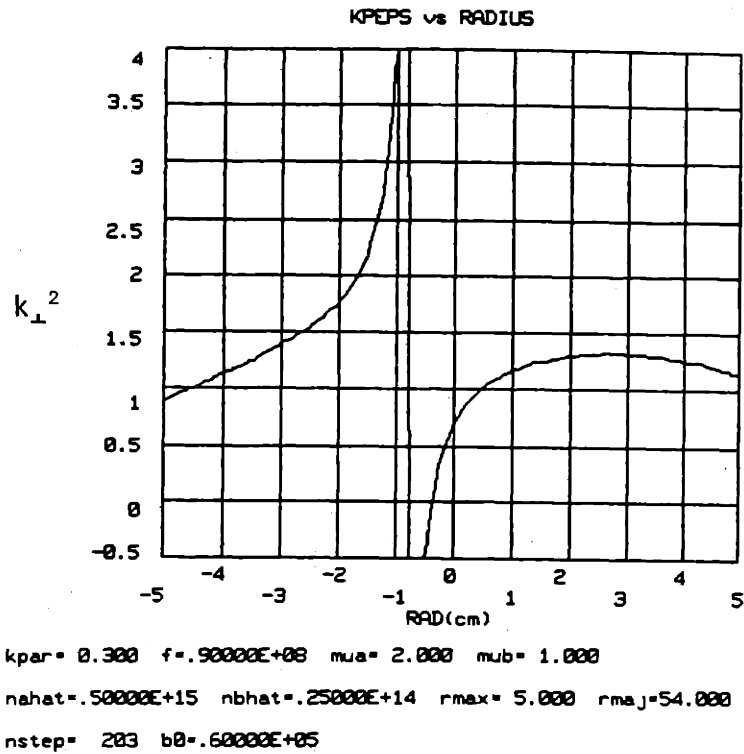


Figure 1

## Simplified TIIH Resonance Layer Dispersion Relation Model

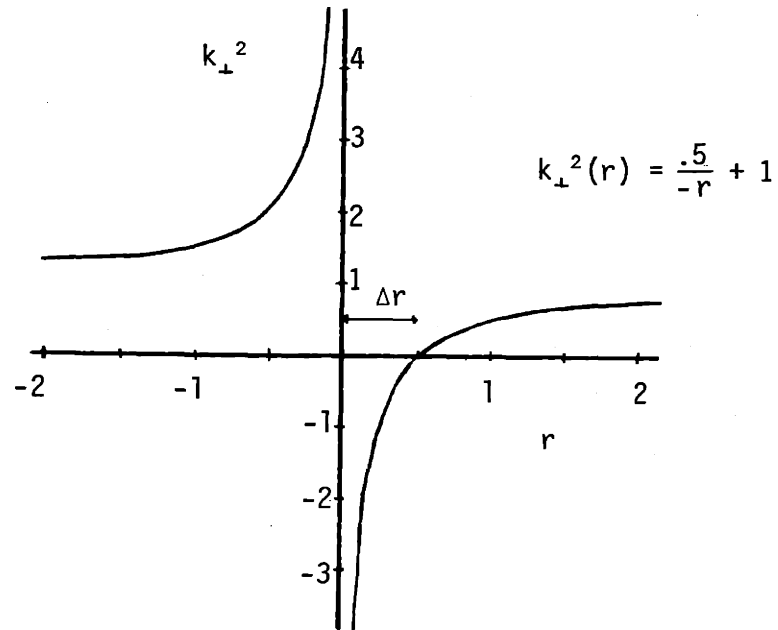


Figure 2

# Power absorption from high and low field side

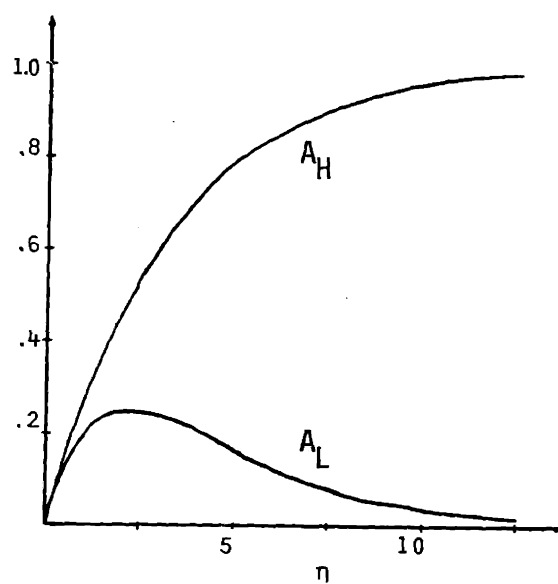


Figure 3

## $k_z$ to $k_{||}$ transformation

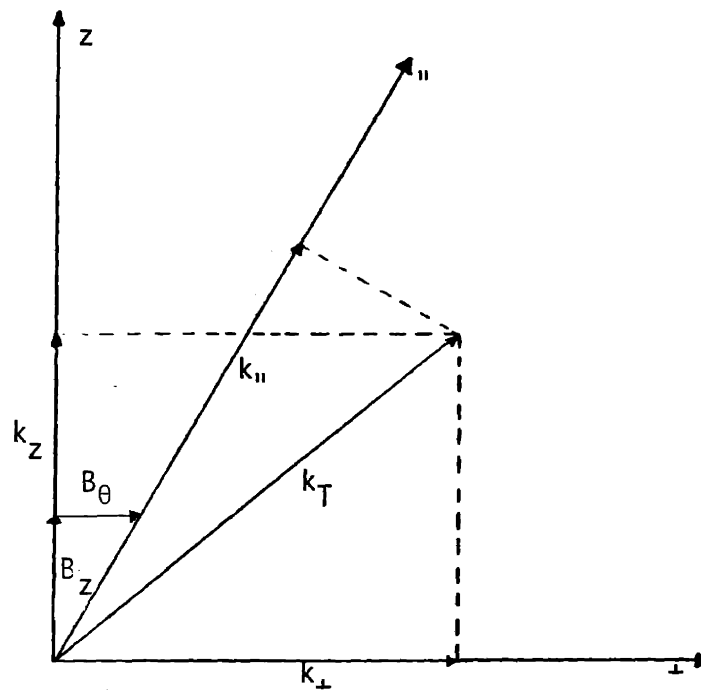


Figure 4



## V. Synthesis, Recommendations for Further Work, and Conclusions

### V-1. Synthesis

The Alcator A ICRF experiment can be schematically summarized by the block diagram of Figure 1. Some of the main controllable and uncontrollable input variables are shown with the most important ICRF propagation and damping processes. A certain number of wave measurements, products, and by-products are also shown.

From a theoretical point of view, one is interested in understanding the processes, so that an efficient heating scheme can be devised. From a more near-sighted, practical point of view, we need only to produce heating (product) with as little plasma deterioration as possible (by-products). Unfortunately, neither point of view has seemed encouraging up until now for ICRF at high density. Although significant amounts of RF power were coupled to the plasma without any significant plasma deterioration, no evidence of any bulk heating was observed.

Although unpromising (unrealistically high goals were set), many important new results were observed at high density. Before the Alcator experiment, one could not have made a single statement about the effect of any input variable on any of the output measurements, products or by-products of Figure 1. This work has experimentally mapped out most of the different parameter correlations in conjunction with plausible, simple theories.

The most important new scaling law is

$$(1) \quad R_R \propto d^2 \ell^{1.5} \phi^0 F^0 n_e^{-1} \omega_0^2 P^{0.5} z^{-2} I_p^0 B_0^0$$

where we have successively listed the effect of center conductor to wall

spacing, antenna length, antenna phasing, Faraday shield, plasma density, transmitter frequency, P power, ion mass, ion charge, plasma current, and toroidal field. Most other results, on the other hand, cannot be cast in such condensed form.

In the fundamental regime, cyclotron or cyclotron harmonic damping were obviously important processes, as shown by the dramatically different probe signal and radiation resistance spikes behavior during a toroidal field scan. At second and fourth harmonic, no such behavior was observed, and some other strong damping mechanism must be accounted for. Collisional damping is a possibility since edge neutral density is unknown, but similar results were observed with gas puffed at the antenna port and 90° away.

A likely source of problems are the trapped particles in the port ripple. Trapped particles were the major concern of the Alcator lower hybrid experiment. Figures 2 and 3 show typical lower hybrid heating shots (data taken by the author in October, 1976). Essentially the same symptoms as in ICRF were observed; energetic ion tail, fast decay time, no plasma deterioration, and no bulk heating. After extensive parameter scanning, a very narrow density window was found where thermonuclear neutron production was dramatically increased, but again, without bulk heating.

Most of the ICRF wave measurements were made far away from the antenna (except for the near-field, which is too close), and it is possible (and even likely) that the observed probe signals are not representative of the bulk power leaving the antenna. There could be, of course, near-field absorption, but also single perpendicular and parallel pass absorption. In all three cases,  $R_R$  would be independent of toroidal field (damping strength) as experimentally observed. This idea is especially supported by the surprising fact that  $k_{\parallel}$ , away from the antenna, is very large, in contradiction with good

mode coupling (small evanescent edge), but consistent with weakly damped ( $2\omega_{ci}$ ) low  $k_{\perp}$  modes. This may explain why the probe eigenmodes are independent of layer position. Put concisely, these eigenmodes are not necessarily representative of bulk power flow and absorption.

All three regimes of damping (single perpendicular pass, single toroidal pass and stochastic mode stacking) at high density can be used to explain the basic behavior of the radiation resistance (equation(1)) and probe signal background. For example,  $R_R$  theoretically scales as  $d^2$  independently of damping,  $\ell^1$  for single perpendicular pass and  $\ell^1$  to  $\ell^4$  for perpendicular eigenmodes, and is independent of antenna phasing, etc.

Comparing available parameter space with the space that was actually carefully investigated immediately reveals a large discrepancy; the low density fundamental regime which has been proven successful on the two largest ICRF experiments (TFR, PLT). Three main factors contributed to the brevity of the investigations of this regime; first, the extremely unfavorable power coupling scaling law ( $R_R \propto n_e \omega^2$ ). Second, Alcator is a high density experiment, and considerably more scientific impact could be obtained by heating a high density plasma (commercial reactors will undoubtedly operate above  $n_e \approx 10^{14}/\text{cm}^3$ ). Third, the tokamak operators have little experience in low density work, the machine is difficult to run, and diagnosticians are usually not interested in this regime. In hindsight, not fully investigating the fundamental regime has been a mistake, although preliminary results with proper antenna phasing (push-pull in single pass regime) low minority concentration, and fully shielded antenna ( $A_4$ ), proved unsuccessful.

We are now left with the dilemma of pinpointing the source of the problem. Antenna design is the most unlikely since  $A_4$  is all metal and fully shielded. Trapped particles clearly need further investigation (even simple up-down charge exchange scans were never done), and could be

the sole problem, explaining everything. Either high density, field or frequency could also be at the heart of the problem, as they are significantly higher than the TFR and PLT experiments (the evanescent edge is many times worse in Alcator). PLT always stays at very low density while TFR has shown good results up to  $10^{14}/\text{cm}^3$ .

Tokamak size has also been suggested, since the large PLT, TFR and Macrotron tokamaks are easily heated, while the smaller Microtron and Alcator have not been heated. The "Magic Touch" can be ruled out since both Microtron and Macrotron are at the same laboratory. A major difference, caused by tokamak size increase at a given central density, is the much lower plasma edge density. In PLT, the plasma density near the Faraday shield is of the order of  $10^{10} - 10^{11}/\text{cm}^3$ , and reaches  $10^{12}/\text{cm}^3$ , only several centimeters into the plasma. In Alcator, on the other hand, the plasma edge density starts immediately at the  $10^{13}/\text{cm}^3$  level, at least two orders of magnitude larger than PLT. We could, of course, suggest the counter-argument that  $A_2$  and  $A_4$  behaved the same. If edge density was the problem, an unshielded antenna would have undoubtedly been much worse. Antenna current shorted out by plasma is also ruled out by the same argument. It was most unfortunate that no time was available to use the Langmuir probe installed in the  $A_4$  Faraday shield after so much work went into implementing the intricate design.

# Schematic Diagram of the Alcator ICRF Experiment

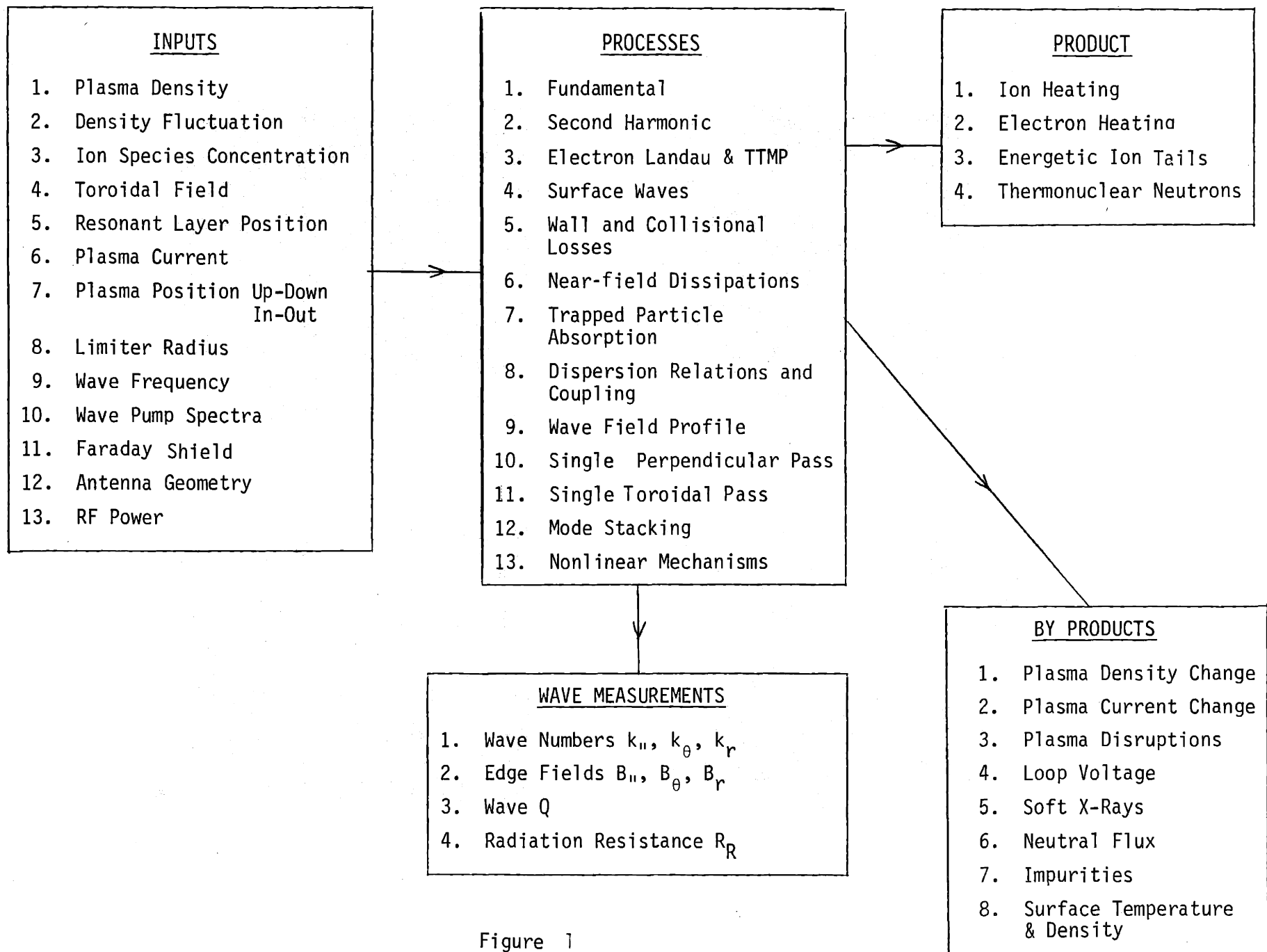
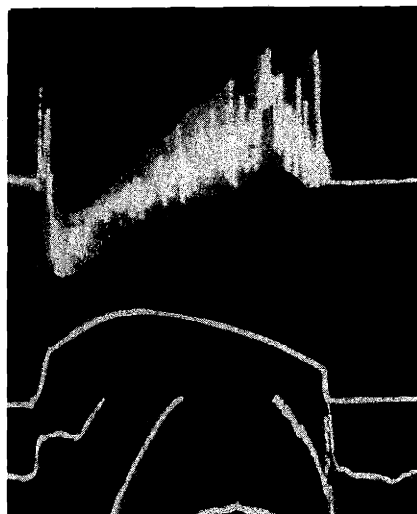
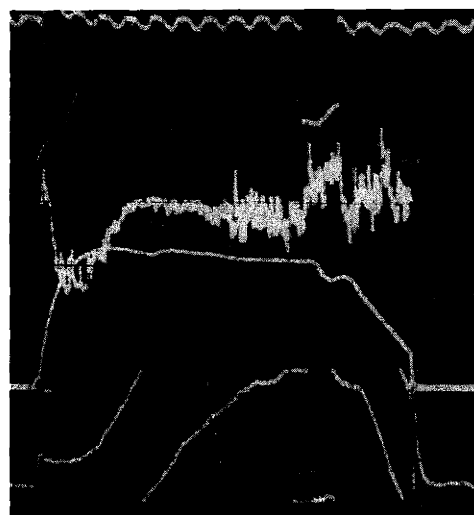


Figure 1

Lower Hybrid Data  
10/7/76



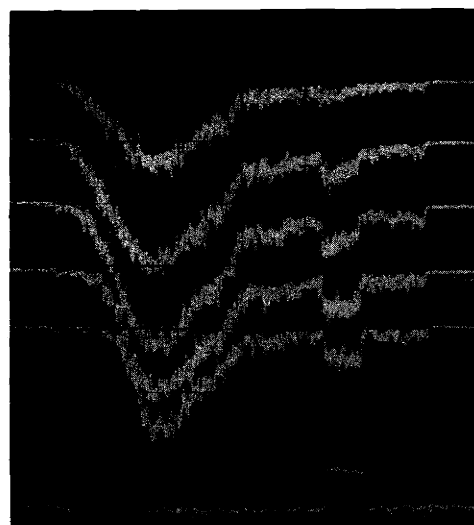
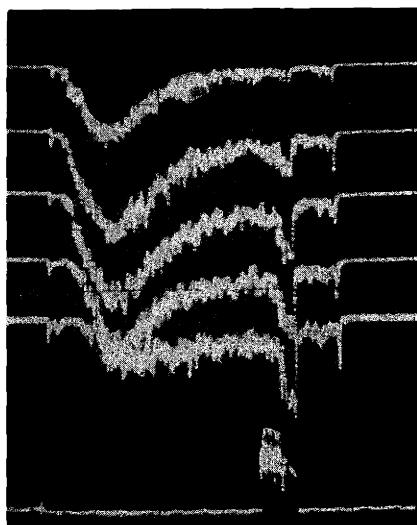
Lower Hybrid Data  
10/28/76



$T_i$

$I_p$

$n_e$



1

2

cx

3

4

5

RF

Figure 2

Figure 3

## V-2. Recommendations for Further Work

The next theoretical investigations should include an in depth study of hot plasma effects in the TIIH regime at high density and a realistic description of the antenna near-fields, including density gradient and a very strong, localized absorption mechanism similar to those modeled by Y. Lapierre.<sup>120,121</sup> The most important, urgent, and possibly difficult problem is producing a good model of the trapped particle power absorption.

The next stage of experiments on Alcator "A" or "C" should definitely be done with shielded antennas until some evidence of bulk heating has been achieved, since there is no reason to believe that Faraday shields have any detrimental effects (except mechanical, which is of secondary importance at this point), and may in fact be necessary (PLT and TFR groups claim they are).  $A_6$  is an excellent overall design, and only the vacuum breaks (and multipactor) need further investigation.

In the area of diagnostics, much initial work, discussed in Chapter III, needs to be finished, particularly the  $A_4$  Langmuir probe, capacitive probe radial scans, trapped particle detector, high speed stainless steel foil bolometer, antenna breakdown detectors, and most importantly, the antenna viewer. More complete  $k_{\parallel}$  array and electron heating (soft X-ray and electron cyclotron emission) diagnostics, as well as less cumbersome data acquisition, are also in order.

The next generation of experiments should concentrate on the more well-known and proven minority regime at low density (Note again that Microtor has not yet been heated successfully). Second harmonic experiments should also concentrate on the lower density with close attention given to power balance and wave activity at the antenna port instead of far away from it.

Of course, any promising results at higher density on Alcator C would completely change these recommendations.

The most important parameter scans are minority concentration, in-out position and up-down charge exchange and, of course, resonant layer position. Of more academic importance are the  $d^2$ ,  $l^{1.5}$ ,  $\phi^0$  and  $\omega^2$  scaling laws, which would be very easily investigated (except for  $d^2$ ) with the  $A_6$  antenna system.



### V-3. Conclusions

With a nearly complete comprehensive study of the theoretical foundation of ICRF at high density, and through comparison with experimental data over a wide range of parameters, a more enlightened understanding of the physics of launching, propagation, and absorption has resulted. Even with apparently large discrepancies between theory and experiment, important new scaling laws have been formulated, which can be compared with other experiments and more sophisticated theories.

In any case, many technological advances have been made in the field of antenna design, matching, and diagnostics through careful study of the  $A_1$ ,  $A_2$ ,  $A_4$  and  $A_6$  antenna designs and experiments.

In short, this work has tried to substantiate a choice of ICRF schemes for future high field, high density, compact tokamaks, through a broad comparison of different parameter regimes. The outcome is still unclear, and much more experimental and theoretical work is still necessary, continuing, and planned.<sup>143</sup>

## APPENDIX 1

### Alfvén Regime Approximations

For the purpose of this work, we shall use the usual perturbation technique, and linearize most of the basic equations.<sup>1,45</sup>

$$(1) \quad n = n_0 + n_1 + \dots$$

$$\vec{E} = \vec{E}_0 + \vec{E}_1 + \dots$$

The Fourier transformed first order quantities in a wave are of the form

$$(2) \quad A_1 = \bar{A}_1 e^{i(k \cdot r - \omega t)}$$

where we will use only the real part of  $A_1$  as measurable quantity. Also,

$$(3) \quad \vec{k} \cdot \vec{r} = k_x x + k_y y + k_z z$$

$$k_{\perp}^2 = k_x^2 + k_y^2$$

$$k_{\parallel}^2 = k_z^2$$

$$\vec{B}_0 = \hat{z} B_0$$

For each species in the plasma, we can write the full fluid equation, i.e., for ions<sup>1</sup>

$$(4) \quad M n \frac{d\vec{v}_i}{dt} = en(\vec{E} + \vec{v}_i \times \frac{\vec{B}}{c}) - \nabla P_i - \nabla \cdot \pi_i + P_{ie}$$

where  $M n dv/dt$  represents the change in momentum,  $en(E + v_i \times B)$  the electromagnetic forces,  $\nabla P_{ie}$  the pressure gradients,  $\nabla \cdot \pi_i$  the off diagonal shear, and  $P_{ie}$  the collisions. For the plasma wave regimes in the near collisionless regime we will be analyzing, the shear and collision terms represent small resistive losses that will be evaluated in Section IV-4.2-5. The pressure term is reactive<sup>1</sup> and would only be important if its corresponding phase velocity, the speed of sound in a plasma ( $\approx V_{thi}$  for  $T_i \approx T_e$ ), was of the order of, or greater than, our electromagnetically induced wave phase velocity ( $V_p \approx V_A \gg V_{thi}$ , Appendix 2).

Cancelling the density term on each side, we are then left with the equation of motion of the single particle

$$(5) \quad M \frac{d\vec{v}}{dt} = e \left( \vec{E} + \vec{v} \times \frac{\vec{B}}{c} \right)$$

with

$$(6) \quad \frac{d\vec{v}}{dt} = \frac{\partial \vec{v}}{\partial t} + (\vec{v}_0 \cdot \nabla) \vec{v}_1$$

The term on the right is usually called the connective derivative and may be neglected if there is no constant drift. Thus,

$$(7) \quad \vec{v} = \vec{v}_0 + \vec{v}_1 \approx \vec{v}_1$$

and our equation of motion is then simply

$$(8) \quad M \frac{\partial \vec{v}_1}{\partial t} = e \left( \vec{E} + \vec{v}_1 \times \frac{\vec{B}}{c} \right)$$

which has orthogonal components

$$(9) \quad -i\omega M v_x = e \left( E_x + v_y \frac{B_0}{c} \right)$$

$$(10) \quad -i\omega M v_y = e \left( E_y - v_x \frac{B_0}{c} \right)$$

For simplicity, we assume for now that  $E_y = 0$ , and solve for the ion velocities

$$(11) \quad v_{ix} = \frac{-i\Omega}{1-\Omega^2} \frac{E_c}{B_0} \quad \text{polarization current}$$

$$(12) \quad v_{iy} = \frac{-1}{1-\Omega^2} \frac{E_c}{B_0} \quad \text{EXB current}$$

where we defined

$$(13) \quad \Omega = \frac{\omega}{\omega_{ci}}$$

Substituting for electrons in the relatively low ICRF, where  $\omega \ll \omega_{ce}$

$$(14) \quad M \rightarrow m$$

$$e \rightarrow -e$$

$$\omega_{ci} \rightarrow -\omega_{ce}$$

$$\Omega_i \rightarrow -\Omega_e \ll 1$$

we thus have for electron velocities

$$(15) \quad v_{ex} = i \Omega_e \frac{E_c}{B_0} \ll v_{ix}, v_{iy}, v_{ey}$$

$$(16) \quad v_{ey} = -\frac{E_c}{B_0}$$

and the electron current along  $E_x$  (polarization current) may be neglected.

Combining Ampère's and Faraday's laws with the definition of current density,

$$(17) \quad \nabla \times \vec{E} = -\frac{\vec{B}}{c}$$

$$(18) \quad c \nabla \times \vec{B} = 4\pi \vec{J} + \vec{E}$$

$$(19) \quad \vec{J} = ne(\vec{v}_i - \vec{v}_e)$$

we have the wave equation<sup>1</sup>

$$(20) \quad \nabla \times (\nabla \times \vec{E}) = -k(\vec{k} \cdot \vec{E}) + k^2 \vec{E} = \frac{4\pi i \omega}{c^2} \vec{J} + \frac{\omega^2}{c^2} \vec{E}$$

We assume for now that  $k = k_z$ , (an electromagnetic wave since  $\vec{k} \cdot \vec{E} = 0$  (Figure 1)), so that

$$(21) \quad k^2 E = \frac{\omega^2}{c^2} (1 + \epsilon) E$$

where we defined an effective plasma dielectric constant

$$(22) \quad \epsilon = \frac{4\pi i}{\omega} \frac{J}{E} = \frac{\pi i^2}{\omega^2 c_i} = \epsilon_{\text{Alfvén}} \text{ for } \omega \ll \omega_{ci}$$

In Section IV -2.1. this will correspond to the total dielectric tensor element

$$(23) \quad K_{xx} = S = \epsilon_{\text{vacuum}} + \epsilon_{\text{plasma}} = 1 + \epsilon$$

In the very low frequency approximation  $\omega \ll \omega_{ci}$

$$(24) \quad v_{iy} = -\frac{Ec}{B_0} = v_{ey}$$

$J_y = 0$ , and  $E_x$  linear polarization is justified, since the other orthogonal linear polarization,  $E_y$ , would not couple power to the original wave

$$(25) \quad J_{y1} E_{y2}^* = 0$$

Also, since

$$(26) \quad \frac{v_{ix}}{v_{iy}} = i\Omega$$

the bulk of the particle wave kinetic energy is oscillating in motion perpendicular to both  $\vec{B}_0$  and  $\vec{E}$ , ( $E_x \times B_{0z}$  drift). The phase velocity is simply

$$(27) \quad \frac{\omega}{k} = \frac{1}{\sqrt{\mu_0 \epsilon_A}} = c \frac{\omega_{ci}}{\pi_i} = v_A$$

From Faraday's law (17)

$$(28) \quad B_y = E_x \frac{ck_z}{\omega}$$

and we note that this  $B_y$ , when added to  $B_{0z}$ , gives rise to a small ( $\vec{B}_1 \ll \vec{B}_0$ ) shear sinusoidal ripple on the confining field (Figure 1). The y component of the velocity of the field line is simply

$$(29) \quad v_{By} = -\frac{\omega}{k} \frac{B_y}{B_0} = -\frac{E_x c}{B_0}$$

which is the same velocity as the fluid  $\vec{E} \times \vec{B}$  drift motion

$$(30) \quad v_{\text{fluid}} = -\frac{E_x c}{B_0}$$

For all these reasons, this mode is usually called the electromagnetic shear Alfvén wave.

In the frequency regime above the ion cyclotron frequency  $\omega_{ci} < \omega \ll \omega_{ce}$

$$(31) \quad \frac{-v_{iy}}{v_{ey}} \approx \frac{1}{\Omega^2} \ll 1$$

and our assumption that  $E_y$  was zero is not valid anymore, since  $E_{y1}$  couples with  $J_{y2} \neq 0$ . To treat the problem correctly, we must allow  $E_y$  to be arbitrary and solve the coupled set of equations (this will be done in Section IV-2.1). In the ion cyclotron range of frequencies, and in particular, for  $\Omega \approx 2$ ,

$$(32) \quad |v_{ix}| \approx |v_{iy}| \approx |v_{ey}| \gg |v_{ex}|$$

For  $\omega = \omega_{ci}$ , we have singularities in the ion currents. These singularities will be removed in hot plasma theory, and give rise to collisionless damping, (Section IV-42).

If we now look at the case where  $\vec{E}$ ,  $\vec{k}$ , and  $\vec{B}_0$  are all perpendicular to each other, i.e.,  $E_x, k_y, B_{0z}$ , our previous derivations remain unchanged, except that now the  $\vec{E} \times \vec{B}_0$  drift is along  $\vec{k}$ , and the plasma will be compressed (instead of sheared), as in a compressional acoustic wave (Figure 2). One could have also kept the pressure term in the fluid equation and achieved a slightly modified result<sup>1</sup>.

$$(33) \quad \frac{\omega^2}{k^2} = c^2 \frac{v_s^2 + v_A^2}{c^2 + v_A^2} \approx v_A^2 \quad \text{for } v_s \ll v_A$$

where  $v_s$  is the sound velocity.

We note also that around the cyclotron frequency (ie  $\Omega \approx 2$ ), the magnitude of the dielectric constant  $\epsilon$  (S) is smaller than  $\epsilon_A$ , and the wave phase velocity is slightly faster than the Alfvén speed. For all these reasons, this mode is usually called the fast wave, fast compressional wave, fast compressional Alfvén wave, or even fast magnetosonic compressional

Alfvén wave. The fact that  $S$  is negative for  $\Omega > 1$  should be of some concern, but the overall dielectric constant will be positive when  $E_y$  will be allowed to be finite. The basic physical picture of the problem is nevertheless correct. A more powerful mathematical formulation will be necessary to get a complete picture at the cost of physical simplicity. This formulation will be the subject of Section IV-2.

The last possibility is  $\vec{E}$  parallel to  $\vec{B}_0$ . This can be simulated by simply setting  $B_0 = 0$  in (9) or (15)

$$(34) \quad v_{ez} = \frac{-ie}{m\omega} E_z \gg v_{iz}$$

$$(35) \quad \epsilon_{||} = \frac{-\omega_{pe}^2}{\omega^2}$$

which corresponds to an enormous dielectric constant. For many practical purposes, this can be considered a short circuit and  $E_z = 0$ . This approximation is often called the zero electron mass approximation ( $m_e/m_i \rightarrow 0$ )

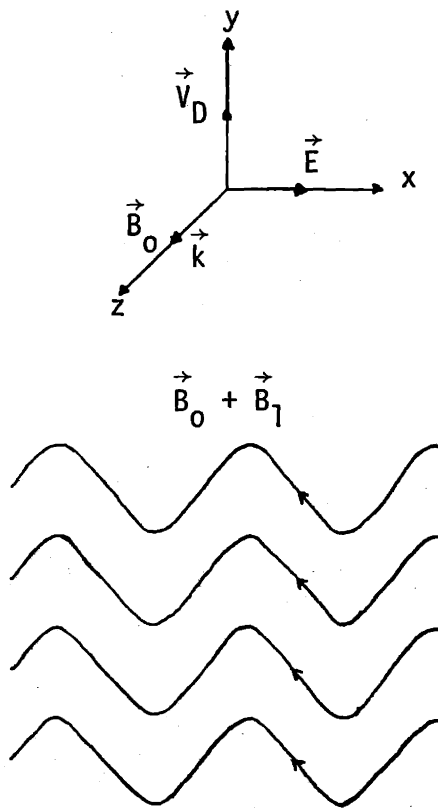
We can now qualitatively conclude that, except for the cyclotron frequency singularity, one has a non-isotropic dielectric constant with a perpendicular component of order  $\pi_i^2/\omega_{ci}^2$ , and a much larger parallel component  $\omega_{pe}^2/\omega^2$ . In particular, for ICRF

$$(36) \quad \frac{\epsilon_{||}}{\epsilon_{\perp}} \approx \frac{m_i}{m_e} \gg 1$$

The dispersive character of the wave is essentially controlled by the ion's frequency dependent polarization and  $\vec{V} \times \vec{B}$  drifts, with respect to the electron's  $\vec{V} \times \vec{B}$  drift (Figure 3)

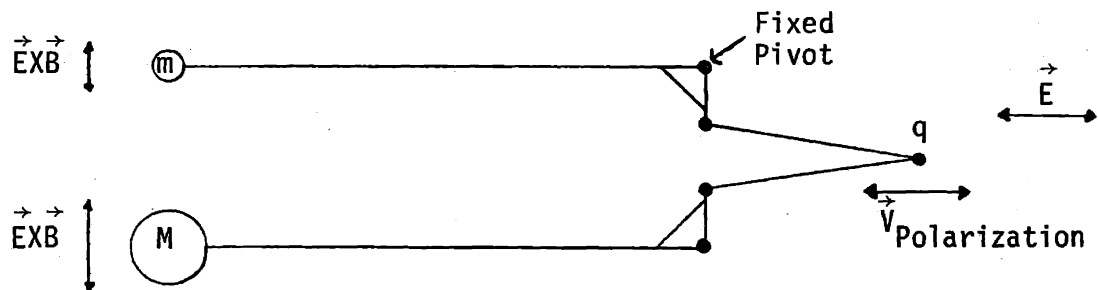
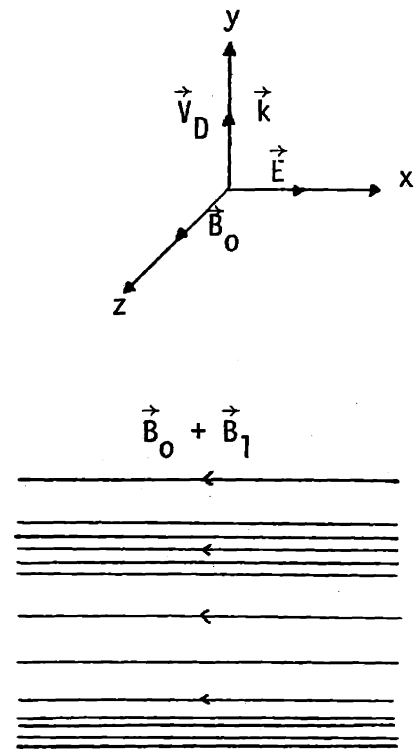
# Shear Wave

Figure 1



# Compressional Wave

Figure 2



Simple mechanical analogy to  $\vec{E} \times \vec{B}$  and polarization drift, and energy storage

Figure 3



## APPENDIX 2

### Table of Formulas and Typical Values<sup>82</sup>

Numerical values are for  $T_e$  and  $T_i$  in eV,  $B$  in gauss,  $\mu = m_i/m_p$ ,  $n$  in  $\text{cm}^{-3}$ .  
 Typical values are for  $T_i = T_e = 1 \text{ keV}$ ,  $\mu = 1$ ,  $n = 3 \times 10^{14}/\text{cm}^3$ ,  $B = 60 \text{ kG}$ ,  $\lambda_c = 15$ ,  $f_o = 200 \text{ MHz}$ ,  $k_{||} = .5$ ,  $k_{\perp} = 1/\text{cm}$ .

Name	Symbol	CGS Formula	Numerical Value	Typical Value
<b><u>FREQUENCIES</u></b>				
electron gyrofrequency	$\omega_{ce}$	$\frac{eB}{m_e c}$	$1.76 \times 10^7 B$	$\approx 10^{12}/\text{sec}$
ion gyrofrequency	$\omega_{ci}$	$\frac{eB}{m_i c}$	$\frac{9.58 \times 10^3 B}{\mu}$	$\approx 5.7 \times 10^8/\text{sec}$
electron plasma frequency	$\pi_e$	$\sqrt{\frac{4\pi n e^2}{m_e}}$	$5.64 \times 10^4 \sqrt{n_e}$	$\approx 9.8 \times 10^{11}/\text{sec}$
ion plasma frequency	$\pi_i$	$\sqrt{\frac{4\pi n e^2}{m_i}}$	$1.32 \times 10^3 \sqrt{\frac{n}{\mu}}$	$\approx 2.3 \times 10^{10}/\text{sec}$
electron collision frequency	$\gamma_e$	$2.9 \times 10^{-6} n \lambda_c T_e^{-3/2}$		$\approx 4.1 \times 10^5/\text{sec}$
Ion collision frequency	$\gamma_i$	$4.8 \times 10^{-8} n \lambda_c T_i^{-3/2}$		$\approx 6.8 \times 10^3/\text{sec}$
<b><u>VELOCITIES</u></b>				
speed of light	$c$		$3 \times 10^{10} \text{ cm/sec}$	
electron thermal velocity	$v_{te}$	$\sqrt{\frac{KT_e}{m_e}}$	$4.19 \times 10^7 \sqrt{T_e}$	$\approx 1.3 \times 10^9 \text{ cm/sec}$
ion thermal velocity	$v_{ti}$	$\sqrt{\frac{KT_i}{m_i}}$	$9.79 \times 10^5 \sqrt{\frac{T_i}{\mu}}$	$\approx 3.0 \times 10^7 \text{ cm/sec}$
ion sound velocity	$v_s$	$\sqrt{\frac{KT_e}{m_i}}$	$9.79 \times 10^5 \sqrt{\frac{T_e}{\mu}}$	$\approx 3.1 \times 10^7 \text{ cm/sec}$
Alfvén velocity	$v_A$	$\frac{B}{\sqrt{4\pi n_i m_i}}$	$\frac{2.18 \times 10^{11} B}{\sqrt{\mu n_i}}$	$\approx 7.5 \times 10^8 \text{ cm/sec}$

# Table of Formulas (cont.)

Name	Symbol	CGS Formula	Numerical Value	Typical Value
<u>LENGTH</u>				
electron gyroradius	$\rho_e =$	$v_{te}/\omega_{ce} = \frac{2.38 \sqrt{T_e}}{B}$	$\approx 1.2 \times 10^{-3}$	cm
ion gyroradius	$\rho_i =$	$v_{ti}/\omega_{ci} = \frac{1.02 \times 10 \sqrt{\mu T_i}}{B}$	$\approx 5.4 \times 10^{-2}$	cm
free space wavelength	$\lambda_o =$	$c/f_o =$		150 cm
Alfvén wavelength	$\lambda_A =$	$V_A/f_o =$		3.8 cm
debye length	$\lambda_D =$	$\sqrt{\frac{KT}{4\pi n e^2}} = 7.43 \times 10^2 \sqrt{\frac{T}{n}}$	$\approx 1.4 \times 10^{-3}$	cm
<u>DIMENSIONLESS</u>				
proton/electron mass ratio	$=$	$\frac{m_p}{m_e} =$	1837	
Alfvén refractive index	$\frac{c}{V_A} =$	$\frac{\pi i}{\omega_{ci}} = \frac{.137 \sqrt{\mu n}}{B}$	$\approx$	40
thermal/magnetic energy ratio	$\beta =$	$\frac{8\pi nKT}{B^2} = \frac{4.03 \times 10^{-11} nT}{B^2}$	$\approx 3.4 \times 10^{-3}$	

## Appendix 3

### 2 x 2 Dispersion Relation Code

disper.fortran 04/03/81 1654.8 est Thu

```

C NORMALIZED 2X2.
C
C Calculates and plots the 2x2 dispersion relation
C nperp**2=(a-npar**2)+a*(1-npar**2)/(a-npar**2)
C where a =1/(1-omega). Nperp vs Npar is also plotted.
C
      dimension npar(500),nperp(500),npars(500),nperps(500),omeg(10)
      real npar,nperp,npars,nperps,ndel,nmax,nmin,nmin2,nmax2,ndel2
      external plot_setup(descriptors),plot_scale(descriptors),
      & plot_(descriptors)
C
C Data.
      omega=2.
      nmin=-3.
      nmax=3.
      nstep=200
      ome(1)=4.
      ome(2)=3.
      ome(3)=2.
      ome(4)=1.5
      ome(5)=1.25
      ome(6)=.75
      ome(7)=.5
      ome(8)=.25
      ome(9)=0.
      nmin2=0.
      nmax2=1.
      write(6,25)
25      format(1x,'!wor 30 h !gra 1,35 !shr b')
C
C Initial calculations
      a=1./(1.-omega**2)
      ndel=(nmax-nmin)/float(nstep)
      ndel2=(nmax2-nmin2)/float(nstep)
      write(6,30)omega
30      format(5x,'omega=',f5.3)
C Beginning of the loop.
      do 100 j=1,nstep
        npars(j)=nmin+float(j-1)*ndel
        nperps(j)=(a-npars(j)**2)+(a*(1.-a)/(a-npars(j)**2))
        if(nperps(j) .gt. nmax) nperps(j)=nmax
        if(nperps(j) .lt. nmin) nperps(j)=nmin
100      continue
      call plot_setup("DISPERSION RELATION","NPAR**2",
      & "NPERP**2",1,0e0,1,0)
      call plot_scale(-3.,3.,-3.,3.)

```

```

      call plot_(npars,nperps,nstep,1," ")
      write(6,25)
      call plot_setup("DISPERSION RELATION","NPAR","NPERP",1,0e0,1,0)
      call plot_scale(nmin2,nmax2,nmin2,nmax2)
      write(6,44)
44      format(4x,'omega=8., 4., 3., 2., 1.5, 1.25 .75, .5, .25 ')
      do 210 i=1,9
        do 200 j=1,nstep
          a=1./(1.-omeg(i)**2)
          npar(j)=nmin2+float(j-1)*ndel2
          nperps(j)=(a-npar(j)**2)+(a*(1.-a)/(a-npar(j)**2))
          if(nperps(j) .gt. 1.) nperps(j)=1.
          if(nperps(j) .le. 0.) nperps(j)=0.
          nperp(j)=sqrt(nperps(j))
200          continue
210          call plot_(npar,nperp,nstep,1," ")
          continue
        call exit
      end

```

r 16:55 0.474 9 level 2

# Appendix 4

## Inhomogeneous Plasma Heuristic Code

hdamp.fortran 04/09/81 1852.3 est Thu

```

C
C HEURISTIC
C
C Heuristic profile calculation of Kperp, Ne, Te, Etheta, and WKB
C in inhomogeneous cartesian plasma.
C
  dimension r(500),n(500),kperp(500),kperps(500),t(500),
& fint(500),sint(500),etheta(500),kint(500),
& ewkb0(500),ewkb1(500),reukb0(500),reukb1(500),ewkb10(500),
& ewkb11(500),ewkbm0(500),ewkbm1(500),kperi(10)
  real n,npar,nperp,nhat,kpar,kperps,mu,npars,kpeps,kperi
  complex kperp,ewkb0,ewkb1,kint
  external plot_setup(descriptors)
  external plot_scale(descriptors)
  external plot_(descriptors)
C
C Data.
  f=2.e8
  omega=2.
  mu=1.
  nstep=500
  istep=0
  kperi(1)=.6728
  kperi(2)=0.
  kperi(3)=.2
  kperi(4)=.2
  kperi(5)=.4
  kperi(6)=.6
  kperi(7)=.8
  kperi(8)=1.
  nhat=5.e14
  rmax=12.5
  rmin=0.
  pp=9.
  q0=.9
  q1=.5.
  t0=1000.
  fint(1)=-1.
  sint(1)=1.
  etheta(1)=1.
  kint(1)=(0.,0.)
C Nprof=1 for TFR and 2 for Alcator.
  nprof=2
C
C Initial calculations.

```

```

  c=3.e10
6  rdel=(rmax-rmin)/float(nstep)
  w=6.28*f
  atsq=(3./2.)*pp**2*q0/q1
C Beginning of the loop.
  do 500 i=1,istep
    kpar=kperi(i)
    do 100 j=1,nstep
      r(j)=rmin+float(j)*rdel
C Temperature profile calculation.
      tnorm=exp(-r(j)**2/atsq)
      t(j)=tnorm*t0
      if(nprof .eq. 1) go to 7
C Alcator density profile calculation.
      rlim1=9.
      rlim2=9.7
      xpon=12.
      if(r(j) .ge. rlim1) go to 90
      n(j)=nhat*(1.-(r(j)**2./rlim2**2.))
      go to 91
90  n(j)=nhat*.1389*exp(-1.*xpon*(r(j)/9.-1.))
      go to 91
C TFR density profile calculation.
7  if(r(j) .ge. .9*pp) go to 96
      n(j)=nhat*(1.-(r(j)**2./pp**2.))
      go to 91
96  n(j)=nhat*(.04+.15*exp(-12.*(r(j)/pp-.9)))
C Dispersion relation.
91  npars=(kpar*c/w)**2
      up1=1.32e3*sqrt(n(j))/sqrt(mu)
      ad=(up1/w/omega)**2
      s=1.+ad/(1.-omega**2)
      d=-omega*s
      kperps(j)=(w/c)**2*((s-npars)**2-d**2)/(s-npars)
      if(j .eq. 1) go to 100
C First order integration.
92  fint(j)=fint(j-1)+kperps(j)*etheta(j-1)*rdel
      sint(j)=sint(j-1)+fint(j)*rdel
      etheta(j)=1.-sint(j)
C 0th and 1st order WKB profile.
      kperp(j)=csqrt(cmplx(kperps(j),0.))
      kint(j)=kint(j-1)+kperp(j)*rdel
      ewkb0(j)=cexp((0.,1.)*kint(j))
      ewkb1(j)=ewkb0(j)/csqrt(kperp(j))
      reukb0(j)=real(ewkb0(j))
      ewkb10(j)=aimag(ewkb0(j))
      ewkbm0(j)=cabs(ewkb0(j))
      reukb1(j)=real(ewkb1(j))
      ewkb11(j)=aimag(ewkb1(j))
      ewkbm1(j)=cabs(ewkb1(j))
100 continue
C Plotting routines.
  if(i .eq. 1 .or. i .eq. 2) then

```

```

16      write(6,16)
      format(1x,'!uor 30 h !gra 1,35 !shr b')
      write(6,101)nprof,t0,nhat,f,kpar1(1),mu,omega
101     format(/4x,'nprof=',i3,3x,'t0=',f8.2,3x,'nhat=',e10.5 /
&      4x,'f=',e10.5,3x,'kpar(1)=' ,f8.5,3x,'mu=' ,f5.2,
&      3x,'omega=' ,f5.2)
      end if
      if(1 .eq. 2 .or. 1 .eq. 1) write(6,102)
102     format(/4x,'Kpar= 0., .2, .4, .6, .8, 1.0, 1.2')
      if(1 .gt. 1) go to 450
      call plot_setup("DENSITY PROFILE", "RAD(cm)", "N", 1, 0e0, 1, 0)
      call plot_(r, n, nstep, 1, " ")
      call plot_setup("TEMPERATURE PROFILE", "RAD(cm)", "T", 1, 0e0, 1, 0)
      call plot_(r, t, nstep, 1, " ")
240     call plot_setup("ETHETA vs RADIUS", "RAD(cm)", "ETHETA", 1, 0e0, 1, 0)
      call plot_(r, etheta, nstep, 1, " ")
490     call plot_setup("EKB0 vs RADIUS", "RAD(cm)", "EKB0", 1, 0e0, 1, 0)
      call plot_scale(0., 14., -3., 3.)
      call plot_(r, eukb0, nstep, 1, " ")
      call plot_(r, reukb0, nstep, 1, " ")
      call plot_(r, eukb10, nstep, 1, " ")
      call plot_setup("EKB1 vs RADIUS", "RAD(cm)", "EKB1", 1, 0e0, 1, 0)
      call plot_scale(0., 14., -3., 3.)
      call plot_(r, eukb1, nstep, 1, " ")
      call plot_(r, reukb1, nstep, 1, " ")
      call plot_(r, eukb11, nstep, 1, " ")
      go to 500
450     if (1 .eq. 2) then
      call plot_setup("KPERP**2", "RAD(cm)", "KPERP**2", 1, 0e0, 1, 0)
      call plot_scale(0., 14., -2., 4.)
      end if
      call plot_(r, kperps, nstep, 1, " ")
500     continue
501     call exit
      end

```

r 10:53 0.920 52

## Appendix 5

### 3 x 3 Analytic Dispersion Relations Code

kperp.fortran

04/09/81 1221.7 est Thu

```

C
C ANALYTIC 3X3.
C
C Calculates kperp for the fast and the slow waves as
C Kpeps=(w/c)**2*(s-npar**2)**2-d**2/(s-npar**2) and
C Kpems=(w/c)**2*(p/s)*(s-npar**2) using one ion species,
C electrons, no magnetic field dependence, and the TFR
C density profile.
C
      dimension r(500),n(500),kpeps(500),kpems(500)
      real n,kpar,kpeps,kpems,nhat,npars,mu
      external plot_setup(descriptors)
      external plot_scale(descriptors)
      external plot_(descriptors)
C
C Data.
      f=2.e8
      oma=2.
      mu=1.
      kpar=.5
      nhat=5.e14
      pp=9.
      rmax=12.5
      rmin=0.
      nstep=200
C
C Initial calculations.
      c0=3.e10
      rdel=(rmax-rmin)/float(nstep)
      w=2.*3.1416*f
      b0=(w/oma)*(mu/9.58e3)
      npars=(kpar*c0/w)**2
C Beginning of the loop.
      do 100 j=1,nstep
      r(j)=rmin+float(j-1)*rdel
C Density profile.
      if(r(j)/pp .gt. .9) go to 10
      n(j)=nhat*(1.-(r(j)**2/pp**2))
      go to 20
10     n(j)=nhat*(.04+.15*exp(-12.*(r(j)/pp-.9)))
C Tensor elements.
20     upi=1.32e3*sqrt(n(j))/sqrt(mu)
      upe=5.64e4*sqrt(n(j))
      ad=(upi/w*oma)**2
      s=1.+ad/(1.-oma**2)
      d=-ad*oma/(1.-oma**2)

```

```

      p=1.-(wpe/w)**2
C Dispersion relations.
      kpeps(j)=(w/c0)**2*((s-npars)**2-d**2)/(s-npars)
      kpems(j)=(w/c0)**2*wp*(s-npars)/s
100 continue
C Plot Kperp**2.
      write(6,49)
49      format(1x,'!wor 30 h !gra 1,35 !shr b')
      write(6,50) f,oma,b0,kpar,mu
50      format(//1x,'f=',e10.5,3x,'oma=',f6.3,3x,'b0=',
& e10.5,3x,'kpar=',f6.4,3x,'mu=',f4.2)
      write(6,51) nhath,pp,rmax
51      format(//1x,'nhath=',e10.5,3x,'pp=',f6.3,3x,'rmax=',f6.3//)
      call plot_setup("KPEPS vs RADIUS","RAD(cm)","KPEPS",1,0e0,1,0)
      call plot_(r,kpeps,nstep,1," ")
      call plot_setup("KPEMS vs RADIUS","RAD(cm)","KPEMS",1,0e0,1,0)
      call plot_(r,kpems,nstep,1," ")
      call exit
      end

r 12:22 0.432 25

```

## Appendix 6

### 3 x 3, 3 Species Computational B<sub>0</sub>/R Dispersion Relations Code

kperp8.fortran 04/09/81 1633.3 est Thu

```

C
C TIIM 3G.
C
C Plots kperp squared for the fast and slow waves using the cold
C plasma 3G tensor with two ion species, electrons, and 1/R
C magnetic field.
C
C Dimension statements.
      dimension rad(500),na(500),nb(500),kpeps(500),kpems(500),
& b0z(500),bsq(500),dens(500),mrad(500)
      real mua,mub,nahat,nbhat,na,nb,kpeps,kpems,1,nz,kz,npar,
& nparn,kpar,mue,mrad
      external plot_setup(descriptors)
      external plot_scale(descriptors)
      external plot_(descriptors)
C
C Data.
      nstep=200
      nprof=1
C (Nprof=1 for TFR and 2 for Alcator.)
      f=.9e8
      b0=6.08e4
      mua=2.
      mub=1.
      kpar=.3
      nahat=5.e14
      nbhat=2.5e13
      pp=9.
      rmax=15.
      rmaj=54.
C
      write(6,60)
60      format(1x,'!wor 30 h !gra 1,35 !shr b')
      write(6,7) kpar,f,mua,mub,nprof
7      format(//1x,'kpar=',f6.3,2x,'f=',e10.5,2x,'mua=',f6.3,
& 2x,'mub=',f6.3,2x,'nprof=',i2)
      write(6,8) nahat,nbhat,rmax,rmaj
8      format(//1x,'nahat=',e10.5,2x,'nbhat=',e10.5,2x,'rmax=',f6.3,2x,
& 'rmaj=',f6.3)
      write(6,9) nstep,b0,pp
9      format(//1x,'nstep=',i5,2x,'b0=',e10.5,2x,'pp=',f8.2)
C Initial calculations and constants.
      c0=3.e10
      mue=1./1837.
      rmin=rmax
      rdel=(rmax-rmin)/float(nstep)

```

```

      w=2.*3.141593*f
      npar=kpar*c0/w
C Start of the loop.
C Density and magnetic field profiles.
      do 100 j=1,nstep
        rad(j)=rmin+float(j-1)*rdel
        mrad(j)=abs(rad(j))
        if(nprof .eq. 2) go to 14
C TFR profile.
        if(mrad(j) .gt. .9*pp) go to 18
        dnorm=1.-(mrad(j)**2/pp**2)
        go to 20
10      dnorm=.04+.15*exp(-12.*(mrad(j)/pp-.9))
        go to 20
C Alcator profile.
14      if(mrad(j) .gt. .9) go to 15
        dnorm=1.-(mrad(j)**2/.9**2)
        go to 20
15      dnorm=.1389*exp(-12.*(mrad(j)/.9-.1))
20      na(j)=nahat*dnorm
        nb(j)=nbhat*dnorm
        dens(j)=dnorm
        b0z(j)=b0*rma/(rma+j*rad(j))
C Tensor elements.
        pia=1.32e3*sqrt(na(j))/sqrt(mua)
        pib=1.32e3*sqrt(nb(j))/sqrt(mub)
        pie=1.32e3*sqrt(na(j)+nb(j))/sqrt(mue)
        uca=9.58e3*b0z(j)/mua
        ucb=9.58e3*b0z(j)/mub
        uce=9.58e3*b0z(j)/mue
        r=1.-(pie**2/(w*(w-uce)))-(pia**2/(w*(w-uca)))
        & -(pib**2/(w*(w-ucb)))
        l=1.-(pie**2/(w*(w-uce)))-(pia**2/(w*(w-uca)))
        & -(pib**2/(w*(w-ucb)))
        s=.5*(r+l)
        d=.5*(r-l)
        p=1.-(pie**2+pia**2+pib**2)/w**2
C Dispersion relations.
        a=1.
        b=-(p/s+1)*(s-npar**2)+d**2/s
        c=(p/s)*((s-npar**2)**2-d**2)
99      bsq(j)=b**2-4.*a*c
        if(bsq(j) .lt. 0.)bsq(j)=0.
        kpeps(j)=(w/c0)**2*(-b+sqrt(bsq(j)))/(2.*a)
        kpems(j)=(w/c0)**2*(-b-sqrt(bsq(j)))/(2.*a)
C Plot the dispersion relations.
        if(kpeps(j) .gt. 2.)kpeps(j)=2.
        if(kpeps(j) .lt. -.5)kpeps(j)=-.5
        if(kpems(j) .lt. -3000.) kpems(j)=-3000.
100     continue
      call plot_setup("KPEPS vs RADIUS","RAD(cm)","KPEPS",1,0e0,1,0)
      call plot_scale(rmin,rmax,-.5,2.)
      call plot_(rad,kpeps,nstep,1," ")

```

```

      call plot_setup("KPEMS vs RADIUS","RAD(cm)","KPEMS",1,0e0,1,0)
      call plot_scale(rmin,rmax,-3000.,0.)
      call plot_(rad,kpems,nstep,1," ")
      call exit
      end

```

r 16:33 0.685 21

## TIIH,2 Dimensional Code

```

C C TIH 2D
C C
C C Calculates the two ion ion hybrid layer position
C C in two dimensions for a simple parabolic
C C density profile, 2 ion species, and a 1/R
C C magnetic field.
C
  dimension rad(200),x(200),y(200),dn(200),bz(200)
  dimension rad2(200),x2(200),y2(200),alpi(10)
  real nehat,nehat,mua,mub,npar,kpar
  external plot_setup(descriptors)
  external plot_scale(descriptors)
  external plot_(descriptors)

C C Data
C C
  nstep=100
  nehat=5.e14
  f0=.97e8
  b0=.7e5
  mua=2.
  mub=1.
  kpar=.1
  rmax=12.5
  rmaj=54.

C C Write statements.
C C
50  write(6,50)
    format(1x,' !wor 30 h !gra 1,35 !shr b')
    write(6,7) f0,mua,mub,b0,nehat
7   format(1x,'f0=',e10.5,2x,'mua=',f6.3,2x,'mub=',f6.3,2x,
    & 'b0=',e10.5,2x,'nehat=',e10.5)
    write(6,8) rmax,rmaj,kpar,nstep
    write(6,51)
51  format(1x,'alpha=.05, .1, .2, .3, .45, 1.0')
8   format(1x,'rmax=',f6.3,2x,'rmaj=',f6.3,2x,
    & 'kpar=',f6.3,2x,'nstep=',i3)

C C Initial calculations and constants.
  c0=3.e10
  rmin=-rmax
  rdel=(rmax-rmin)/float(nstep)
  w=2.*3.1416*f0
  npar=kpar*c0/w
  alpi(1)=.05
  alpi(2)=.1

```

```

alpi(3)=.2
alpi(4)=.3
alpi(5)=.45
alpi(6)=1.
call plot_setup("Y vs RADIUS", "RAD(cm)", "Y", 1, 0e0, 1, 0)
call plot_scale(-rmax, rmax, 0., 2.*rmax)
do 10 i=1,6
  alp=alpi(i)
  nahat=nehat/(1.+alp)
C Start of the loop.
  do 200 j=1, nstep+1
    rad(j)=rmin+float(j-1)*rdel
    b0z=b0*rmax/(rmax+rad(j))
    bz(j)=b0z
    wca=9.58e3*b0z/mua
    c5=(1./(w**2-wca**2))+((alp*mua/mub)/(w**2-wca**2
    & *(mua/mub)**2))
    dnorm=(1.-npar**2*mua)/(c5*nehat*1.32e3**2)
    dn(j)=dnorm
    x(j)=rad(j)
C Calculate r (any density profile can be implemented).
    if(dnorm .le. .99 .and. dnorm .ge. .1e-5) then
      r=rmax*sqrt(1.-dnorm)
      if((r**2-x(j)**2) .le. 0.) then
        y(j)=0.
      else
        y(j)=sqrt(r**2-x(j)**2)
      end if
    else
      y(j)=0.
    end if
  end do
C The following loop calculates the large circle of radius rmax.
  rad2(j)=rmin+float(j-1)*rdel
  x2(j)=rad2(j)
  y2(j)=sqrt(rmax**2-x2(j)**2)
200 continue
C Call the plotting routines.
  call plot_(x2,y2,nstep+1,1," ")
  call plot_(x,y,nstep+1,1," ")
10 continue
call exit
end

r 10:04 0.562 1

```



## Appendix 8

### Inhomogeneous Plasma Cylindrical Eigenmode Fields and Damping Code

damp.fortran 04/15/81 0923.0 est Med

```

C TFR - Ez
C
C Calculates and plots Etheta, Er, Ez, Btheta, Br, Bz,
C E+*2, E'+*2, Ne, Te, Peld/cm**3, P2uci/cm**3, PHI,
C P2uci/cm, and radiation resistance for two ion species,
C inhomogeneous plasma, uniform magnetic field and m from
C -3 to +3.
C
C Dimension and format statements.
  dimension y(3),yp(3),q(3,8),eplus(3),rau(3),cep(3)
  dimension rad(200),etheta(200),err(200),bzz(200),btheta(200),
  & brr(200),phi1(200),epsq(200),depsq(200),dens(200),ez(200),
  & erp(200),tempi(200),tempe(200),peld(200),p2uci(200),
  & peldpl(200),peldpp(200),p2upl(200),p2upp(200)
  real npar,lant,kperps,nperps
  external plot_setup(descriptors),plot_scale(descriptors),
  & plot_(descriptors)
  EXTERNAL SYDEL
  COMMON A,B,C,D,F,G,H,U,R,S,OMAC,OMBC,PM,SOM,ER,IT
  + ,OMCA,OMCB,POM,OMA,OMB,CA
  COMMON /FP/ PP,FA,FB
  common /fix/om,/densty/dnorm
111  format(/12x,'omega a=',f4.1,3x,'omega b=',f4.1,4x,'na=',e10.5,3x,
  & 'nb=',e10.5 / 12x,'ma=',f4.2,8x,'mb=',f4.2,9x,'Ti=',
  & f6.1,7x,'Te=',f6.1 / 12x,'Kpar=',f6.5,4x,'m=',f4.1,10x,
  & 'rmax=',f5.2,5x,'pp=',f5.2 / 12x,'lant=',f5.2,5x,'f0=',e10.5,
  & 3x,'rmaJ=',f5.2,6x,'q0=',f5.2 / 12x,'ql=',f5.2,7x,'rant=',f5.2,
  & 6x,'pas=',f5.3)
  lprnt1=6
1    nt=1
    nimp=1
C
C Data.
C For hot plasma effects set:
C gma=gmb=1.
C oma=omb=2.
C pna= . .
C pnb=0.
C
    oma=2.
    omb=2.
    pna=5.e14
    pnb=0.
    gma=1.
    gmb=1.
    ti=1000.
    te=1000.

```

```

ca=.60291
pm=0.
rm=14.
pp=9.
lant=20.
f0=2.e8
rmaJ=54.
q0=.9
ql=5.
pas=.1
rant=11.

```

C Initial data reduction.

```

44  c0=3.e10
    w=6.28*f0
    pom=w/c0
    FA=2.E-15*PNA/GMA
    FB=2.E-15*PNB/GMB
    DFA=-2.*FA/PP**2
    DFB=-2.*FB/PP**2
    A=CA**2-POM**2
    OMAC=OMAX**2
    OMBC=OMB**2
    OMCA=OMAC-1.
    OMCB=OMBC-1.
    B=OMCA*OMCB
    C=OMAC*OMCB
    D=OMBC*OMCA
    F=A*B
    G=OMAX
    H=OMB*D
    U=PM**2+1.
    R=OMAC*OMBC*(OMA*OMB-1.)
    S=2.*PM**2*B
    DDR=2.*PAS*NIMP
    write(6,71)
71  format(1x,'!wdr 30 h',1x,'!gra 1,35',1x,'!shr b')
    write(lprnt1,111)oma,omb,pna,pnb,gma,gmb,ti,te,ca,pm,
    & rm,pp,lant,f0,rmaJ,q0,ql,rant,pas
C Poloidal mode number initial conditions.
  Y(1)=0.
  IF(PM.EQ.0.)GO T012
  NET=ABS(PM)
  GO TO (11,12,13,14),NET
14  Y(2)=0.
    Y(3)=0.
    YP(3)=0.
    GO TO 3
13  Y(2)=0.
    Y(3)=0.
    YP(3)=1.
    GO TO 3
12  Y(2)=0.

```

```

Y(3)=1.
YP(3)=0.
GO TO 3
11 IF(OMA.EQ.1..OR.OMB.EQ.1.)GO TO 10
Y(2)=1.
Y(3)=0.
DELB=A*B+FA*C+FB*D
DEPB=DFA*C+DFB*D
DELC=FA*G+FB*H
DEPC=DFA*G+DFB*H
DELD=2.*A**2*B+C*(FA*(4.*A**2-2.*FA*OMAC)+2.*DFA)
      +D*(FB*(4.*A**2-2.*FB*OMBC)+2.*DFB)-4.*FA*FB*R
1 CB=(-DELD+4.*DEPB-2.*FA*DEPC+4.*DELB*(FA*DELC-DELB)/B)/DELB
YP(3)=-CB/B.
GO TO 3
10 IF(OMA.EQ.OMB)GO TO 15
IF(OMA.EQ.1.)GO TO 9
Z=OMA
OMA=OMB
OMB=Z
9 IF(PM.EQ.1.)GO TO 8
Y(2)=0.
Y(3)=0.
YP(3)=1.
GO TO 3
8 Y(2)=1.
Y(3)=0.
YP(3)=A-FA/2.-FB*OMBC*(OMA*OMB-1.)/OMCB
GO TO 3
15 continue
GO TO 7
3 IT=4
ER=0.
IF(PM.NE.0.)ER=Y(2)/PM
CALL SYDELC(YP,Y)
I=0
IT=0
SOM=0.
INI=0
T=CA/POM
P=0.25E10*PAS*T
BR=-T*Y(2)
BTE=T*ER
BZ=(Y(2)+Y(1)*Y(3)-P*MER)/POM
PHI=P*SOM
EPLUS(2)=0.
EPLUS(3)=0.7071*(Y(2)-ER)
LIGNE=7
rad(1)=y(1)
etheta(1)=y(2)
JJ=2
C Main radius loop.
4 DO 5 LI=1,NIMP

```

```

CALL ADIRKG(3,Y,1,PAS,INI,YP,Q,SYDELC)
CALL SYDELC(YP,Y)
IT=0
INI=INI+1
5 CONTINUE
C Field components calculation.
BR=-T*Y(2)
BTE=T*ER
BZ=(Y(2)+Y(1)*Y(3)-P*MER)/POM
PHI=P*SOM
EPLUS(1)=EPLUS(2)
EPLUS(2)=EPLUS(3)
EPLUS(3)=0.7071*(Y(2)-ER)
CEPL=EPLUS(3)**2
DEPL=(EPLUS(3)-EPLUS(1))/DDR
CDEP=DEPL**2
RO=Y(1)
rad(JJ)=y(1)
etheta(JJ)=y(2)
err(JJ)=er
bzz(JJ)=bz/y(1)
brr(JJ)=br
btheta(JJ)=bte
epeq(JJ)=cepl
depeq(JJ)=cdep
phi(JJ)=phi
C Ez calculation.
err(1)=err(2)
erp(JJ)=-1.*(err(JJ-1)-err(JJ))/(nimp*pas)
brack=(err(JJ)/rad(JJ))+erp(JJ)-(pm*etheta(JJ)/rad(JJ))
ez(JJ)=-(ca*2.0e11)/((pnatpnb)*dnorm)*brack
C Hot plasma parameters (for 2uci hot plasma case only).
b0=f0/(2.*1.52e3)
dens(JJ)=pna*dnorm
atsq=(3./2.)*pp**2*q0/q1
tnorm=exp(-rad(JJ)**2/atsq)
tempi(JJ)=tnorm*ti
tempe(JJ)=tnorm*te
vti=9.79e5*sqrt(tempi(JJ))
upi=1.32e3*sqrt(dens(JJ))
r0=(ca*vti/w)*maj
rhoi=(1.02e2/b0)*sqrt(tempi(JJ))
vte=4.19e7*sqrt(tempe(JJ))
alpha=w/(ca*vte)
C Dispersion relation.
aa=-1./3.
npar=ca*c0/(2.*upi)
nperps=(aa-npar**2)+(aa*(1.-aa)/(aa-npar**2))
kperps=nperps*(2.*upi/c0)**2
C Power deposition.
be=4.03e-11*dens(JJ)*tempe(JJ)/b0**2
if(alpha.gt.8.)alpha=8.
peld(JJ)=(w*be/(16.*sqrt(3.14159)))*(kperps*c0**2/w**2)

```

```

& *alpha*exp(-alpha**2)*etheta(jj)**2
p2uci(jj)=(up1**2/(16.*3.14159**u))*(rmaJ/(rad(jj)+0))
& *rho1**2*depsq(jj)
peldpl(1)=0.
p2upl(1)=0.
if(jj .eq. 1) go to 216
peldpl(jj)=peldpl(jj-1)+2.*3.14159*rad(jj)*peld(jj)
& *(rad(jj)-rad(jj-1))
p2upl(jj)=p2upl(jj-1)+2.*3.14159*rad(jj)*p2uci(jj)
& *(rad(jj)-rad(jj-1))
if(rad(jj) .gt. .99*rant .and. rad(jj) .lt. rant)then
  radres=etheta(jj)**2/2.*!ant**2/phi1(jj)*9.e11
  elddl=phi1(jj)/peldpl(jj)
  p2wdl=phi1(jj)/p2upl(jj)
  write(6,113)radres,elddl,p2wdl
113 format(12x,'Rr=',e10.5,4x,'elddl=',e10.5,4x,'p2wdl=',e10.5)
end if
216 jj=jj+1
17 if(y(1)+0.01*pas.lt.rm) go to 4
C End of the main radius loop. Output statements.
7 continue
jjm1=jj-1
go to 198
call plot_setup("PELD vs RADIUS","RAD(cm)","PELD",1,0e0,1,0)
call plot_(rad,peld,jjm1,1," ")
call plot_setup("P2UCI vs RADIUS","RAD(cm)","P2UCI",1,0e0,1,0)
call plot_(rad,p2uci,jjm1,1," ")
call plot_setup("PELDPL vs RADIUS","RAD(cm)","PELDPL",1,0e0,1,0)
call plot_(rad,peldpl,jjm1,1," ")
9000 call plot_setup("P2WFL vs RADIUS","RAD(cm)","P2WFL",1,0e0,1,0)
call plot_(rad,p2upl,jjm1,1," ")
go to 44
198 call plot_setup("Ez vs RADIUS","RAD(cm)","Ez",1,0e0,1,0)
call plot_(rad,ez,jjm1,1," ")
199 call plot_setup("Etheta vs RADIUS","RAD(cm)","Etheta",1,0e0,1,0)
call plot_(rad,etheta,jjm1,1," ")
call plot_setup("Er vs RADIUS","RAD(cm)","Er",1,0e0,1,0)
call plot_(rad,err,jjm1,1," ")
call plot_setup("Br vs RADIUS","RAD(cm)","Br",1,0e0,1,0)
call plot_(rad,brr,jjm1,1," ")
call plot_setup("Btheta vs RADIUS","RAD(cm)","Btheta",1,0e0,1,0)
call plot_(rad,btheta,jjm1,1," ")
call plot_setup("Bz vs RADIUS","RAD(cm)","Bz",1,0e0,1,0)
call plot_(rad,bzz,jjm1,1," ")
call plot_setup("PHI vs RADIUS","RAD(cm)","PHI",1,0e0,1,0)
call plot_(rad,phi1,jjm1,1," ")
call plot_setup("EPSQ vs RADIUS","RAD(cm)","EPSQ",1,0e0,1,0)
call plot_(rad,epsq,jjm1,1," ")
call plot_setup("DEPSQ vs RADIUS","RAD(cm)","DEPSQ",1,0e0,1,0)
call plot_(rad,depsq,jjm1,1," ")
go to 44
500 call exit
END

```

sydelc.fortran 04/01/81 1603.1 est Mad

C  
C  
C

Calculates Runge-Kutta coefficients.

```

SUBROUTINE SYDEL(C(YP,Y))
DIMENSION Y(3), YP(3)
COMMON A,B,C,D,F,G,H,U,R,S,OMAC,OMEC,PM,SOM,ER,IT
+ ,OMCA,OMCB,POM,OMA,OMB,CA
common /fix/ om
common /densty/dnorm
IT=IT+1
IF(IT.EQ.1)GO TO 1
IF(Y(1).EQ.0.)GO TO 10
CALL CALPI(Y(1),PIA,PB,DPIA,DPB)
RC=Y(1)**2
APM=A+PM**2/RC
Z=PIA*RC+PIB*D
DELA=APM*B+Z
DELB=F+Z
DELC=PIA*G+PIB*H
deid=b**2*(a+u/rc)+pia**2*(2.*a-pia*omac+u/rc)+pib**2*(2.*a-pib
& *ombct+u/rc)-2.*pia*pib*
DEPB=DPIA*C+DPB*D
DEPA=DEPB-S/(RC*Y(1))
DEPC=DPIA*G+DPB*H
Z=DEPB/DELB-DEPA/DELA
IF(PH.EQ.0.)Z=0.
COA=1./Y(1)+Z
cob=-deid/deib+z/Y(1)+pm*(delc*(2./Y(1)+depa/deia)-depc)
& /(deib*Y(1))
YP(1)=1.
YP(2)=Y(3)
YP(3)=-COA*Y(3)-COB*Y(2)
IF(IT.EQ.4)RETURN
ER=PM*B*(Y(3)+Y(2)/Y(1))/(DELA*Y(1))+Y(2)*DELC/DELA
OM=(ER**2+Y(2)**2)*Y(1)
IF(IT.EQ.5)GO TO 1
SOM=SOM+2.*OM
RETURN
1 SOM=SOM+OM
RETURN
10 YP(1)=1.
YP(2)=Y(3)
OM=0.
RETURN
END

```

10

```

SUBROUTINE ADIRKG(N,Y,KSTEP,H,INI,YP,Q,SYSTEM)
C
C ROUTINE TO EXECUTE 'KSTEP' RUNGE-KUTTA ITERATIONS.
C
  DIMENSION Y(N),YP(N),Q(N,8),F(4),W(4)
  common /densy/dnorm
  DATA F(1),F(2),F(3),F(4)/.0,.5,.5,1./
  DATA W(1),W(2),W(3),W(4)/1.,2.,2.,1./
C
  DO 1000 NSTEP=1,KSTEP
C FOR EACH STEP, FIRST INITIALIZE:
C Q(I,1) DOESN'T NEED TO BE CLEARED, BECAUSE F(1)=0, BUT DO IT ANYWAY.
C Q(I,6) WILL BE AN ACCUMULATOR, AND Q(I,7) SAVES THE STARTING
C OF Y.
    DO 10 I=1,N
      Q(I,1)=0.
      Q(I,6)=0.
    10  Q(I,7)=Y(I)
C NOW DO THE 4 DERIVATIVE EVALUATIONS
    DO 100 J=1,4
      DO 20 I=1,N
        Q(I,8)=Y(I)+Q(I,J)*H*F(J)
        CALL SYSTEM(YP,Q(I,8))
        DO 30 I=1,N
          Q(I,J+1)=YP(I)
          Q(I,6)=Q(I,6)+Q(I,J+1)*W(J)
        30  CONTINUE
      100  CONTINUE
C CALCULATE THE NEW Y
    DO 200 I=1,N
      Y(I)=Y(I) + Q(I,6)*H/6.
    200  CONTINUE
  1000  RETURN
  END

```

```

SUBROUTINE CALPI(R,A,B,AP,BP)
C
C CALCULATES DENSITY AND DENSITY GRADIENT PROFILES.
C
  COMMON /FP/ FP,FA,FB
  common /densy/dnorm
  RAP=R/FP
  IF(RAP.GT.0.9)GO TO 1
  D=1.-RAP**2
  DP=-2.*RAP/FP
  GO TO 2
1  Z=DP*(-12.*(RAP-0.9))
  D=0.84+0.15*Z
  DP=-1.8*Z/FP
2  A=FA*D
  B=FB*D
  AP=FA*DP
  BP=FB*DP
  dnorm=d
  RETURN
  END

```

# List of Symbols used in TFR-EZ

$CA = k_z$   
 $PM = m$   
 $POM = \omega/c$   
 $OMA = \Omega_a$   
 $OMB = \Omega_b$   
 $RM = \text{maximum radius}$   
 $PP = \text{limiter radius}$   
 $PNA = n_a \text{ cm}^{-3}$   
 $PNB = n_b \text{ cm}^{-3}$   
 $GMA = M_a$   
 $GMB = M_b$   
 $E_\theta$  in statvolts  
 $OM = (E_r^2 + E_\theta^2) r$  poynting flux element  
 $PHI = \text{poynting flux } -z, +z, \text{ erg sec}^{-1}$   
 $NIMP = \text{\# steps between printing intervals}$   
 $INI = \text{\# of applications of ADIRKG during a radius scan}$   
 $Y(1), Y(2), Y(3), = \text{variables } r, E_\theta, E'_\theta$   
 $YP(1), YP(2), YP(3) = \text{derivatives } r', E'_\theta, E''_\theta$   
 others are obvious such  $ER = E_r$  etc.

## Appendix 9

### Stochastic Mode Stacking Code

phi6.fortran 05/05/81 1701.6 edt Tue

```

C
C STOCHASTIC MODE STACKING.
C
C Calculates and plots the real, imaginary, magnitude, phase,
C average, and average deviation of the sum of s=1,3,10,20
C modes as a function of linearly increasing kzt or z, where
C kzt is some random number between .5 and .6/cm with random
C or fixed initial phase fluctuations and z is the distance
C from the antenna.
C
      parameter idim=500
      dimension s1(idim),s3(idim),s10(idim),s20(idim),
&      sre(idim),sim(idim),smag(idim),spha(idim),xarray(idim),
&      zs(idim),sd(idim)
      dimension kzt(20),kztmin(20),kztmax(20),kzrde1(20),
&      kzt(20),kz(20),kzm(20),pk(20),f(20),fp(20),fn(20),
&      pha(20)
      integer s
      real kzt,kztmin,kztmax,kzrde1,kzt,kzi,kzfa,kzff,
&      kminf,kmaxf,ms,mrs,kmult,ksubt
      complex f,fn,fp,s1,s3,s10,s20,kz,kzm,sd
      external plot_setup(descriptors),plot_scale(
&      descriptors),plot_(descriptors)
      namelist /input/plot

C
C Data.
C      scank=1 for k and 0 for z.
C      phase=1 for random and 0 for fixed (0).
C
      s=1
      scank=1.
      phase=0.
      kminf=.9
      kmaxf=1.1
      kmult=1.
      ksubt=0.
      z=0.
      zmin=0.
      zmax=60.
      r=54.
      kzi=.1/r
      p=0.
      n=500
      kzfa=0.
      kzff=1.e3

```

```

C Random kzt.
      kzt(1)=.5534
      kzt(2)=.5069
      kzt(3)=.5110
      kzt(4)=.5427
      kzt(5)=.5277
      kzt(6)=.5779
      kzt(7)=.5186
      kzt(8)=.5765
      kzt(9)=.5965
      kzt(10)=.5150
      kzt(11)=.5381
      kzt(12)=.5609
      kzt(13)=.5905
      kzt(14)=.5490
      kzt(15)=.5104
      kzt(16)=.5653
      kzt(17)=.5406
      kzt(18)=.5516
      kzt(19)=.5697
      kzt(20)=.5580

```

```

C Random phase.
      pha(1)=1.048
      pha(2)=4.657
      pha(3)=2.252
      pha(4)=6.162
      pha(5)=.612
      pha(6)=5.349
      pha(7)=5.686
      pha(8)=2.536
      pha(9)=1.256
      pha(10)=.191
      pha(11)=6.259
      pha(12)=4.934
      pha(13)=2.533
      pha(14)=.815
      pha(15)=5.218
      pha(16)=1.805
      pha(17)=2.816
      pha(18)=4.413
      pha(19)=6.160
      pha(20)=.488

```

```

C Initial calculations and initializations.
44      sumr=0.
      sumi=0.
      summ=0.
      sump=0.
      rs=0.
      rrs=0.
      ms=0.
      mrs=0.
      drs=0.
      drrs=0.

```

```

dms=0.
dms=0.
if(scan .eq. 1.) then
  scanz=0.
else
  scanz=1.
end if
l=6.2832*
C J and I do loops. (increase k or z, different k0 and
C initial phase).
do 300 j=1,n
do 200 i=1,s
C K(i) calculation.
kzrmin(i)=kminf*kmult*(kzr0(i)-ksubt)
kzrmax(i)=kmaxf*kmult*(kzr0(i)-ksubt)
kzrdel(i)=(kzrmax(i)-kzrmin(i))/float(n)*scanz
zdel=(zmax-zmin)/float(n)*scanz
zs(j)=zmin+float(j-1)*zdel
if(scan .eq. 1.) zs(j)=z
if(scan .eq. 0.) kzrmin(i)=kzr0(i)
kzr(i)=kzrmin(i)+float(j-1)*kzrdel(i)*kzfa*
& cos(kzff*float(j-1)*kzrdel(i))
pk(i)=p*kzr(i)
kz(i)=cmplx(kzr(i),kzi)
kzm(i)=kz(i)+cmplx(pk(i),0.)
C F+ and F- calculation.
fp(i)=cexp((0.,1.)*kz(i)*zs(j))/(1.-cexp((0.,1.)*kz(i)*l))
fn(i)=cexp((0.,1.)*kzm(i)*(1-zs(j)))/(1.-cexp((0.,1.)*kzm(i)*l))
f(i)=(fp(i)+fn(i))*cexp((0.,1.)*pha(i)*phase)
C End of I loop.
200 continue
C Stochastic sum.
xarray(j)=kzrmin(i)+float(j-1)*kzrdel(i)
s1(j)=f(i)
s3(j)=s1(j)+f(2)+f(3)
s10(j)=s3(j)+f(4)+f(5)+f(6)+f(7)+f(8)+f(9)+f(10)
s20(j)=s10(j)+f(11)+f(12)+f(13)+f(14)+f(15)+f(16)+f(17)+f(18)
& f(19)+f(20)
if(s .eq. 1) sd(j)=s1(j)
if(s .eq. 3) sd(j)=s3(j)
if(s .eq. 10) sd(j)=s10(j)
if(s .eq. 20) sd(j)=s20(j)
sre(j)=real(sd(j))
sim(j)=aimag(sd(j))
smag(j)=sqrt(sre(j)**2+sim(j)**2)
spha(j)=atan2(sim(j),sre(j))
C Sum and deviation calculation.
sumr=sumr+sre(j)
sumi=sumi+sim(j)
summ=summ+smag(j)
sump=sump+spha(j)
rs=rs+abs(sre(j)-s)
rrs=rrs+abs(sre(j)-sqrt(s))

```

```

ms=ms+abs(smag(j)-s)
mrs=mrs+abs(smag(j)-sqrt(s))
drs=drs+(sre(j)-s)**2
drrs=drrs+(sre(j)-sqrt(s))**2
dms=dms+(smag(j)-s)**2
dmrs=dmrs+(smag(j)-sqrt(s))**2
C End of J loop.
300 continue
C Average calculation.
ar=sumr/n
ai=sumi/n
am=summ/n
ap=sump/n
adrs=rs/n
adrrs=rrs/n
adms=ms/n
admrs=mrs/n
sdrs=sqrt(drs/n)
sdrrs=sqrt(drrs/n)
sdms=sqrt(dms/n)
sdmrs=sqrt(dmrs/n)
C Write input and output data.
110 format(1x,' !jum !hco u')
write(6,110)
119 format(1x,' !wor 30 h !gra 1,35 !shr b')
write(6,120)scanz,s,phase,n,r
120 format(/1x,'INPUT',5x,'scanz=',f3.1,2x,'s=',f2.2x,
& 'phase=',f3.1,2x,'n=',f4.2x,'r=',f4.1)
write(6,121) kzi,p,z,zmin,zmax
121 format(1x,'DATA',6x,'kzi=',f8.6,2x,'p=',f5.2,2x,
& 'z=',f7.2,2x,'zmin=',f7.2,2x,'zmax=',f7.2)
write(6,122)kminf,kmaxf,kmult,ksubt
122 format(11x,'kminf=',f5.3,2x,'kmaxf=',f5.3,2x,
& 'kmult=',f5.2,2x,'ksubt=',f6.3)
write(6,223)kzfa,kzff,kzr0(1)
223 format(11x,'kzfa=',f4.3,2x,'kzff=',e10.5,2x,
& 'kzr0(1)=' ,f7.5/)
write(6,123)ar,ai,am,ap
123 format(1x,'AVERAGE',3x,'ar=',f8.4,3x,'ai=',f8.4,
& 'am=',f8.4,3x,'ap=',f8.4)
write(6,124)adrs,adrrs,adms,admrs
124 format(1x,'AVE DEV',3x,'adrs=',f8.4,2x,'adrrs=',
& f8.4,2x,'adms=',f8.4,2x,'admrs=',f8.4)
write(6,225)sdrs,sdrrs,sdms,sdmrs
225 format(1x,'STAND DEV',1x,'sdrs=',f8.4,2x,'sdrrs=',
& f8.4,2x,'sdms=',f8.4,2x,'sdmrs=',f8.4/)
C Plotting routines.
write(6,125)
125 format(1x,'iplot=1 to plot real')
read(5,input)
if(iplot .eq. 1) then
  if(scan .eq. 1.) then
    call plot_setup("REAL vs KZR","KZR","RE",1,0)

```

```

        call plot_(xarray,sre,n,1," ")
    else
        call plot_setup("REAL vs Z","Z","RE",1,0e0,1,0)
        call plot_(zs,sre,n,1," ")
    end if
end if
write(6,126)
126 format(1x,'iplot=1 to plot imaginary')
read(5,input)
if(iplot .eq. 1) then
    if(scanf .eq. 1.) then
        call plot_setup("IMAGINARY vs KZR","KZR","IM",1,0e0,1,0)
        call plot_(xarray,sim,n,1," ")
    else
        call plot_setup("IMAGINARY vs Z","Z","IM",1,0e0,1,0)
        call plot_(zs,sim,n,1," ")
    end if
end if
write(6,127)
127 format(1x,'iplot=1 to plot magnitude')
read(5,input)
if(iplot .eq. 1) then
    if(scanf .eq. 1.) then
        call plot_setup("MAGNITUDE vs KZR","KZR","MAG",1,0e0,1,0)
        call plot_(xarray,smag,n,1," ")
    else
        call plot_setup("MAGNITUDE vs Z","Z","MAG",1,0e0,1,0)
        call plot_(zs,smag,n,1," ")
    end if
end if
write(6,128)
128 format(1x,'iplot=1 to plot phase')
read(5,input)
if(iplot .eq. 1) then
    if(scanf .eq. 1.) then
        call plot_setup("PHASE vs KZR","KZR","PHA",1,0e0,1,0)
        call plot_(xarray,spha,n,1," ")
    else
        call plot_setup("PHASE vs Z","Z","PHA",1,0e0,1,0)
        call plot_(zs,spha,n,1," ")
    end if
end if
go to 44
call exit
end

```

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## Appendix 10

### Single Perpendicular Pass Radiation Resistance k<sub>u</sub> Spectra Code

imped2.fortran 04/09/81 1108.8 est Thu

```

C
C ANTENNA SPECTRUM.
C
C Calculates the real and imaginary parts of the antenna
C Kpar spectrum.
C
    dimension zk(500),k(500),rzk(500),zki(500)
    real mu,n0,kmin,kmax,k,npar,nreq,nc,np,kdel,kr
    complex c2,c3,zk
    logical first/.true./
    character cch
    external plot_setup(descriptors)
    external plot_scale(descriptors)
    external plot_(descriptors)
    cch=char(33)
C
C Data.
    zmax=.9
    omega=2.
    nu=1.
    n0=1.e14
    rw=12.5
    ra=11.
    rp=9.
    nstep=500
    kmin=0.
    kmax=2.
    f=2.e0
    ww=3.
C ispec=1 for real and 2 for imaginary.
    ispec=1
C Constants.
    pai=3.1415936
    c=3.e10
    nline=36
    write(6,20)cch,cch,cch
    format(1x,a1,'wor 30 h',a1,'gra 1,35',a1,'shr b')
    if(ispec .eq. 1) then
        call plot_setup("REAL Z vs K","K","Z",1,0e0,1,0)
    else
        call plot_setup("IMAG Z vs K","K","Z",1,0e0,1,0)
    end if
    call plot_scale(0.,1.,0.,zmax)
    20 del0=ra-rp
    30 del2=rw-ra
    w=2.*pai*f

```

# Appendix 11

## Inhomogeneous Plasma Cylindrical Eigenmode Dispersion Relation Code

eigen.fortran 05/05/81 1553.6 edt Tue

```

zintr=0.
zinti=0.
kdel=(kmax-kmin)/float(nstep)
do 100 j=1,nstep
k(j)=kmin+float(j)*kdel
nc=(1.+omega)/omega**2*5.15e14*mu*k(j)**2
if(nc.lt..98*n0) go to 50
del1=rw
kr=0.
go to 60
C Dispersion relation.
50 np=(nc+n0)/2.
uplsq=1.32e3**2*np/mu
s=(uplsq/omega**2)*(omega**2/(1.-omega**2))
d=-omega*s
npar=k(j)*c/w
nraq=((s-npar**2)**2-d**2)/(s-npar**2)
kr=w/c*sqrt(nraq)
del1=del0+rp*(1.-sqrt(1.-nc/n0))
del3=del1+del2
60 c1=exp(k(j)*del2)-exp(-k(j)*del2)
c2=k(j)*(exp(k(j)*del1)+exp(-k(j)*del1))+(0.,1.)*kr*(exp(k(j)*
& del1)-exp(-k(j)*del1))
c3=k(j)*(exp(k(j)*del3)+exp(-k(j)*del3))+(0.,1.)*kr*(exp(k(j)*
& del3)-exp(-k(j)*del3))
c4=1./(2.*pai)*sin(k(j)*uu/2.)/(k(j)*uu/2.)
c5=9.e11
zk(j)=2.*pai*(0.,1.)*w/(k(j)*c**2)*c4*c1*c2*c5/c3
rzk(j)=real(zk(j))
zki(j)=aimag(zk(j))
zintr=zintr+(rzk(j)*kdel)*2.
zinti=zinti+(zki(j)*kdel)*2.
100 continue
if(ispec.eq.1) then
call plot_(k,rzk,nstep/2,1," ")
else
call plot_(k,zki,nstep/2,1," ")
end if
write(6,21)cch,nline,cch
21 format(1x,a1,'jum ',i3,1x,a1,'dli 4')
if (first) write(6,22)mu,omega,f,rw,ra,rp,uu
22 format(1x,'mu=',f6.3,2x,'omega=',f6.3,1x,'f=',e10.5,1x,
& 'rw=',f6.3,1x,'ra=',f6.3,1x,'rp=',f6.3,1x,'uu=',f6.3)
if (first) nline=nline+2
first=.false.
write(6,23)n0,zintr,zinti
23 format(1x,'n0=',e10.5,1x,'zintr=',f6.3,1x,'zinti=',f6.3)
nline=nline+1
go to 30
call exit
end

```

```

C
C INHOMOGENEOUS EIGENMODE DISPERSION RELATIONS
C
C Calculates and plots the fast wave eigenmode dispersion
C relations for mu=1,2,...,n, two ion species, inhomogeneous
C plasma, uniform magnetic field, and m from -3 to +3.
C
dimension y(3),yp(3),q(3,8),eplus(3),rau(3),cep(3),etheta(500),
& nesol(500),kpsol(500)
real ne,nesol,nemax,nedel,kpsol,kp,kpar,kpmax,
& kpdel
external plot_$setup(descriptors)
external plot_$scale(descriptors)
external plot_(descriptors)
EXTERNAL SYDEL C
COMMON A,B,C,D,F,G,H,U,R,S,OMAC,OMBC,PM,SOM,ER,IT
+ ,OMCA,OMCB,POM,OMA,OMB,CA
COMMON /FP/ PP,FA,FB
common /fix/ om
common /densty/ dnorm
write(6,991)
991 format(1x,'!wor 30 h !gra 1,35 !shr b')
992 format(/3x,'omega a=',f4.2,2x,'omega b=',f4.2,2x,'nb/na=',
& f5.3 / 3x,'ma=',f4.2,2x,'mb=',f4.2,2x,'pm=',
& f4.2 / 3x,'f0=',e10.5,2x,'rmax=',f5.2,2x,'pp=',f5.2,2x,
& 'rw=',f5.2)
C
C Data.
C
nstep=100
ne=2.e12
kpar=.01
kpmax=1.2
nemax=6.e14
pm=0.
f0=2.e8
pas=.5
oma=2.
omb=2.
rm=14.
rw=12.5
pp=9.
gma=1.
gmb=1.
alpha=1.
C

```



C Initial data reduction.

```
nt=1
jj=1
nimp=1
npunch=0
pom=6.28*f0/3.e10
nedel=nemax/float(nstep)
kpdel=kpmax/float(nstep)
```

C Setup for plot.

```
write(6,992) oma,omb,alpha,gma,gmb,pm,f0,rm,pp,rw
call plot_setup("KPAR vs NE","NE","KPAR",1,0e0,1,0)
call plot_scale(0.,nemax,0.,kpmax)
```

```
ca=kpar
41 do 1000 j=1,nstep
ne=ne+nedel
44 do 7 lt=1,nt
pha=ne/(1.+alpha)
pna=ne*alpha/(1.+alpha)
FA=2.E-15*PNA/GMA
FB=2.E-15*PNB/GMB
DFA=-2.*FA/PP**2
DFB=-2.*FB/PP**2
A=CA**2-POM**2
OMAC=OMPA**2
OMBC=OMB**2
OMCA=OMAC-1.
OMCB=OMBC-1.
B=OMCA*OMCB
C=OMAC*OMCB
D=OMBC*OMCA
F=A*B
G=OMPA*C
H=OMB*D
U=PM**2+1.
R=OMAC*OMBC*(OMPA*OMB-1.)
S=2.*PM**2*B
DDR=2.*PAS*NIMP
```

```
85 y(1)=0.
```

C Poloidal mode number initial conditions.

```
IF(PM.EQ.0.)GO TO 12
NET=ABS(PM)
GO TO (11,12,13,14),NET
14 Y(2)=0.
Y(3)=0.
YP(3)=0.
GO TO 3
13 Y(2)=0.
Y(3)=0.
YP(3)=1.
GO TO 3
12 Y(2)=0.
Y(3)=1.
YP(3)=0.
```

```
GO TO 3
11 IF(OMA.EQ.1..OR.OMB.EQ.1.)GO TO 10
Y(2)=1.
Y(3)=0.
DELB=A*B+FA*C+FB*D
DEPB=DFA*C+DFB*D
DELC=FA*G+FB*H
DEPC=DFA*G+DFB*H
DELD=2.*A**2*B+C*(FA*(4.*A**2-2.*FA*OMAC)+2.*DFA)
1 +D*(FB*(4.*A**2-2.*FB*OMBC)+2.*DFB)-4.*FA*FB*R
CB=(-DELD+4.*DEPB-2.*PM*DEPC+4.*DELB*(PM*DELC-DELB)/B)/DELB
YP(3)=-CB/B.
GO TO 3
10 IF(OMA.EQ.OMB)GO TO 15
IF(OMA.EQ.1.)GO TO 9
Z=OMA
OMA=OMB
OMB=Z
9 IF(PM.EQ.1.)GO TO 8
Y(2)=0.
Y(3)=0.
YP(3)=1.
GO TO 3
8 Y(2)=1.
Y(3)=0.
YP(3)=A-FA/2.-FB*OMBC*(OMA*OMB-1.)/OMCB
GO TO 3
15 continue
GO TO 7
3 IT=4
ER=0.
IF(PM.NE.0.)ER=Y(2)/PM
CALL SYDELC(YP,Y)
I=0
IT=0
SOM=0.
INI=0
T=CA/POM
P=0.25E10*PAS*T
BR=-T*Y(2)
BTE=T*ER
BZ=(Y(2)+Y(1)*Y(3)-PM*ER)/POM
PHI=P*SOM
EPLUS(2)=0.
EPLUS(3)=0.7071*(Y(2)-ER)
LIGNE=7
C Main radius loop.
4 DO 5 LI=1,NIMP
CALL ADIRKG(3,Y,1,PAS,INI,YP,Q,SYDELC)
CALL SYDELC(YP,Y)
IT=0
INI=INI+1
5 CONTINUE
```

```

C Field components calculation.
BR=-T*Y(2)
BTE=T*ER
BZ=(Y(2)+Y(1)*Y(3)-P*ER)/POM
PHI=P*SOM
EPLUS(1)=EPLUS(2)
EPLUS(2)=EPLUS(3)
EPLUS(3)=0.7871*(Y(2)-ER)
CEPL=EPLUS(3)**2
DEPL=(EPLUS(3)-EPLUS(1))/DDR
CDEP=DEPL**2
LIGNE=LIGNE+1
LIGNE=0
IF(NPUNCH.NE.1)GO TO 16
IF(INI.EQ.NIMP)GO TO 16
I=I+1
RAU(I)=RO
CEP(I)=CDEP
IF(I.NE.3)GO TO 16
I=0
16 CONTINUE
RO=Y(1)
17 If(y(1)+0.01*pas.lt.rw) go to 4
7 continue
C Crossover point calculation.
900 etheta(j)=y(2)
If(j .eq. 1) go to 1000
prod=etheta(j)*etheta(j-1)
If(prod .le. 0.) go to 980
nesol(j)=ne
kpsol(j)=ca
If(nesol(j) .gt. nemax) nesol(j)=nemax
If(kpsol(j) .gt. kpmay) kpsol(j)=kpmay
go to 1000
980 ca=ca+kpdel
go to 44
1000 continue
call plot_(nesol,kpsol,nstep,1," ")
go to 41
500 call exit
END

```

r 15:54 1.247 29 level 3

## Engineering Drawings

### First Floor Layout of Transmitter Room

Figure 1

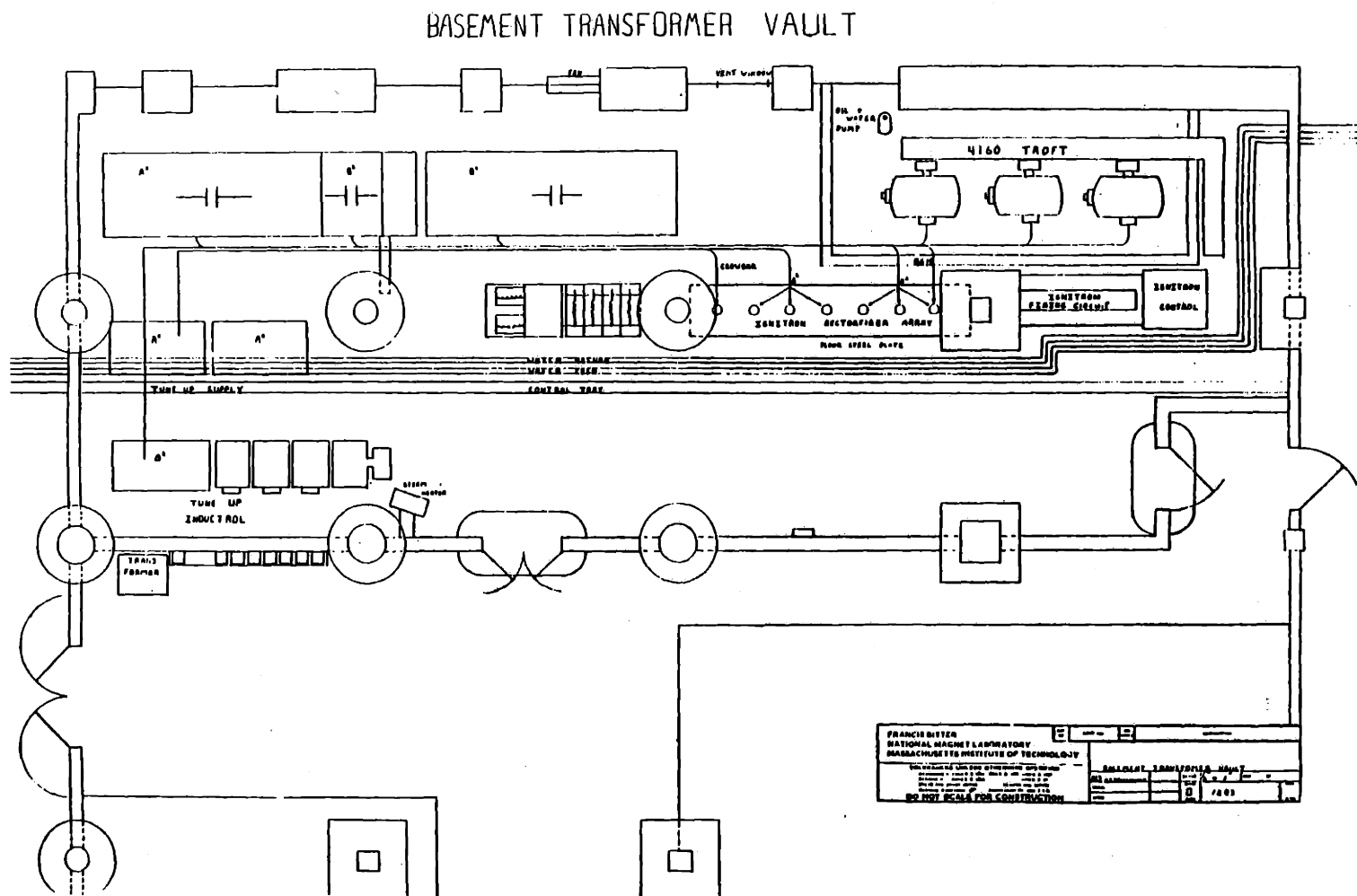
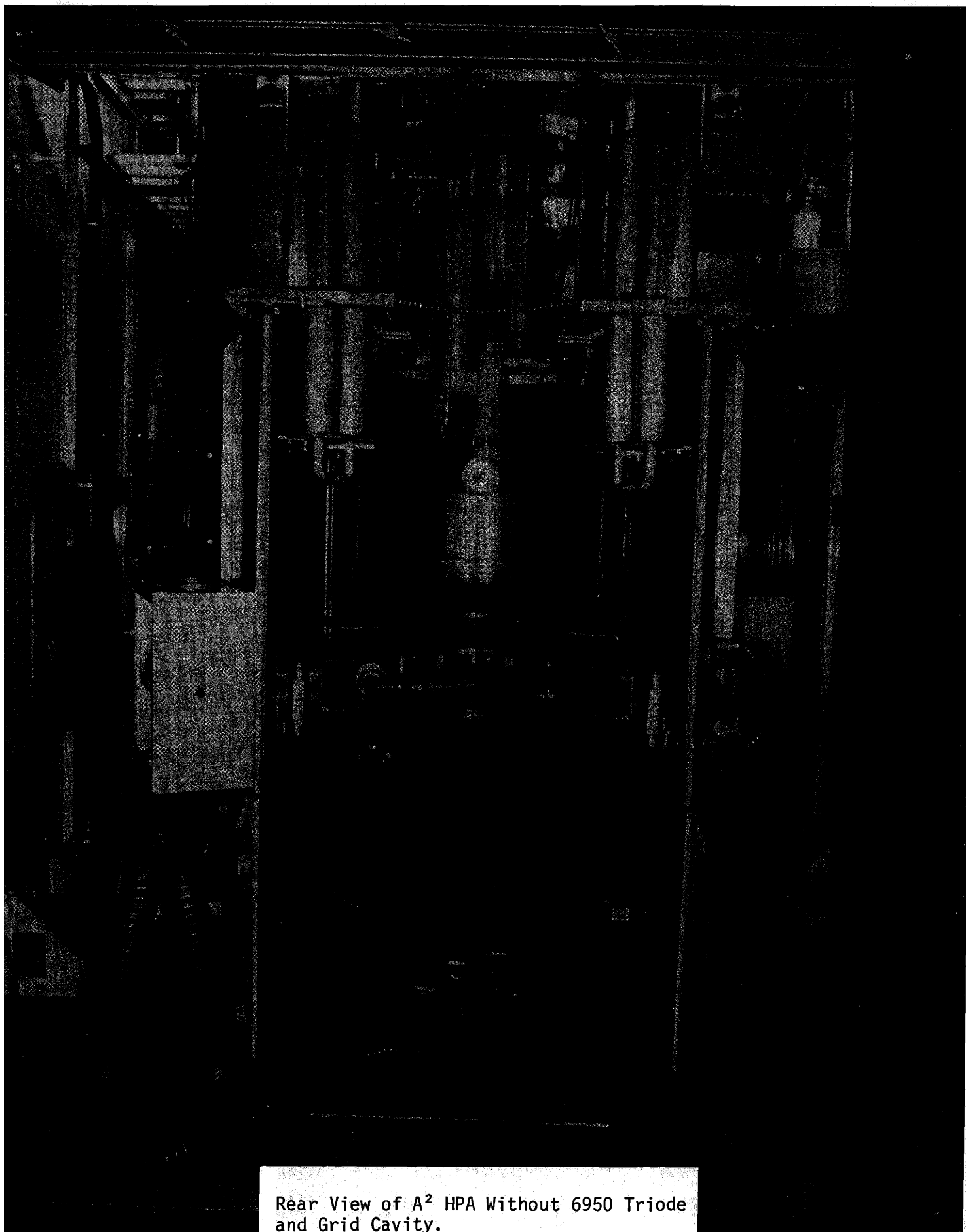
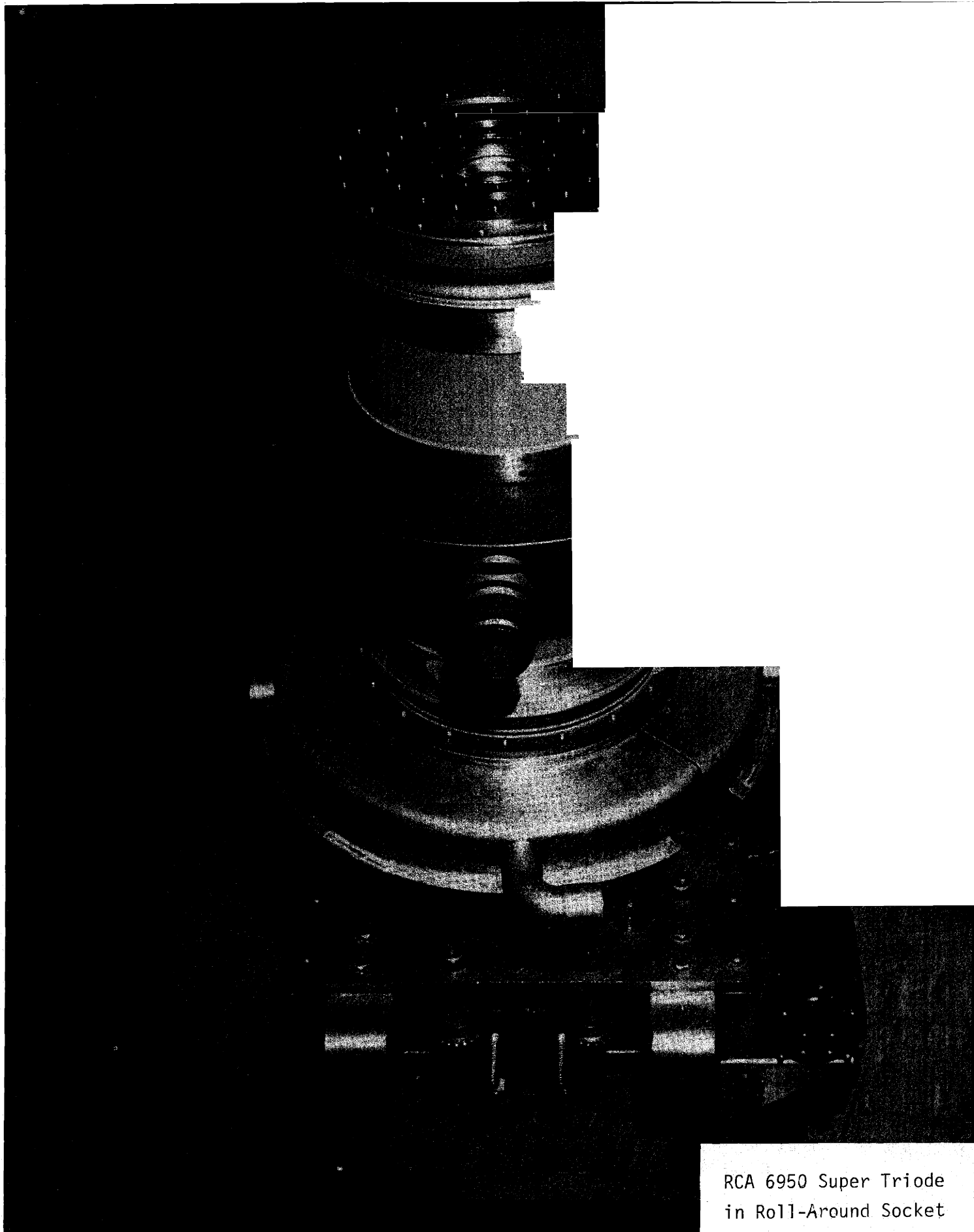


Figure 2

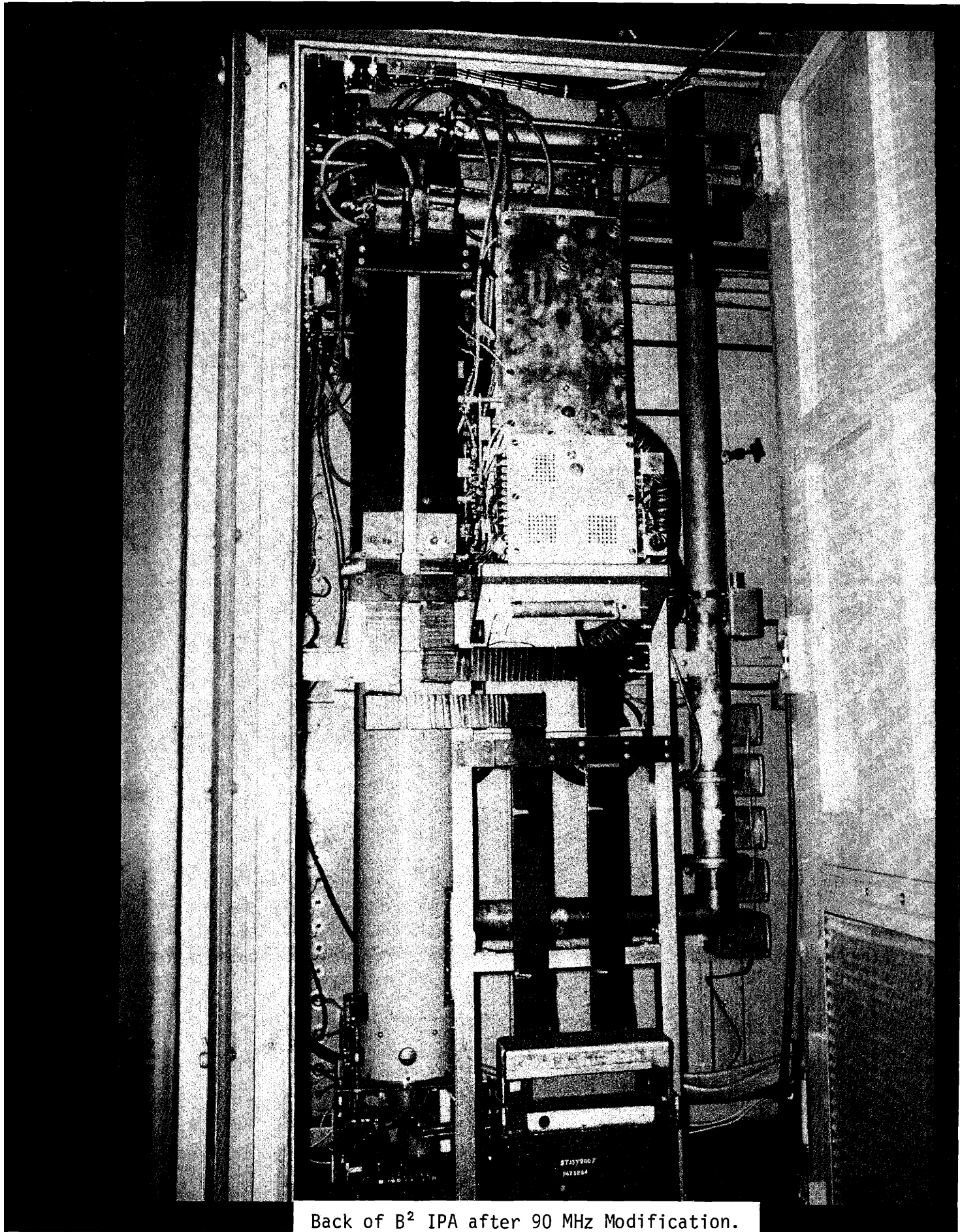


Rear View of A<sup>2</sup> HPA Without 6950 Triode  
and Grid Cavity.

Figure 3



RCA 6950 Super Triode  
in Roll-Around Socket



Back of B<sup>2</sup> IPA after 90 MHz Modification.

Figure 5

Simplified Diagram of the High Power  
Control and Interlock Systems

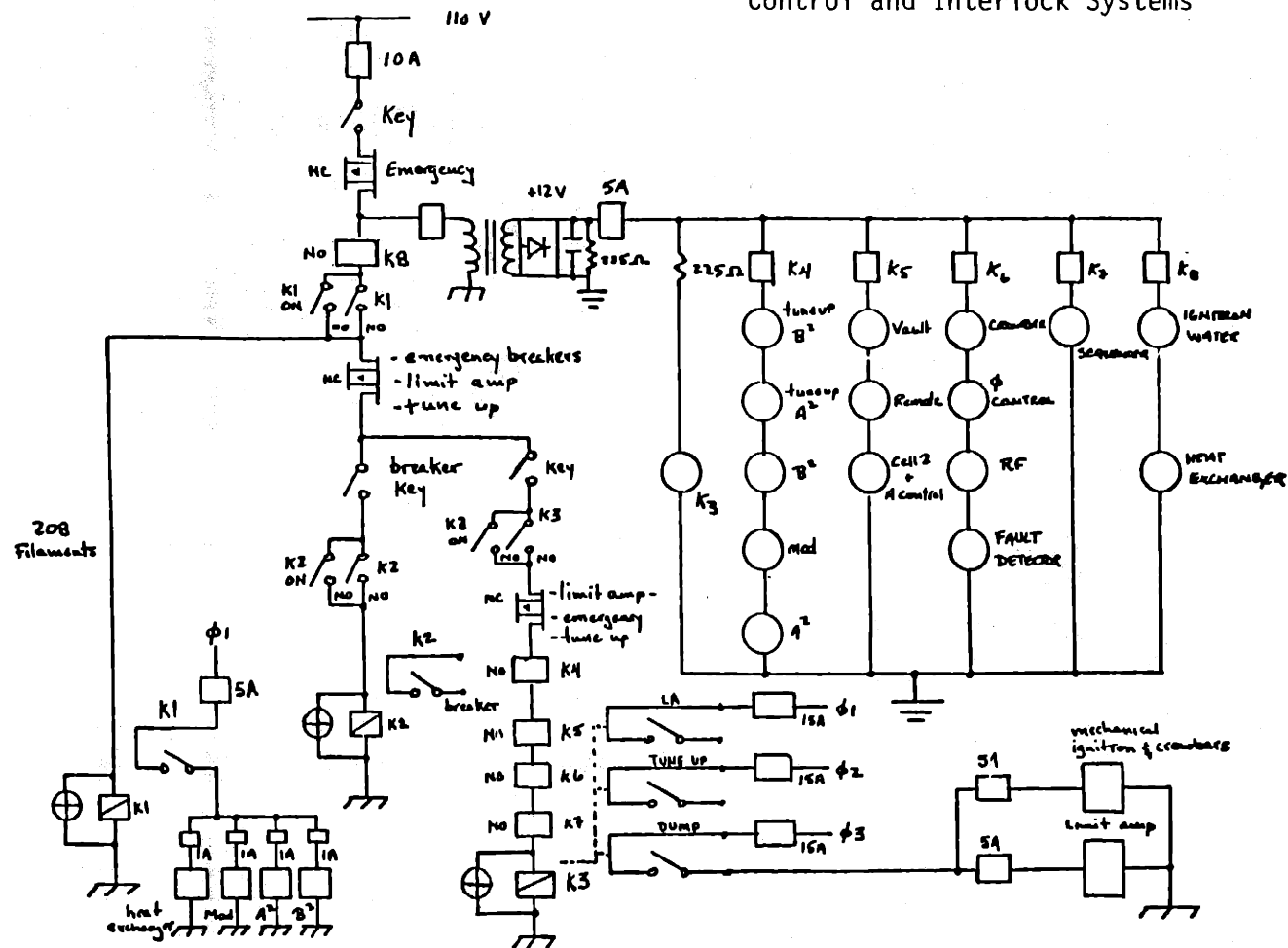
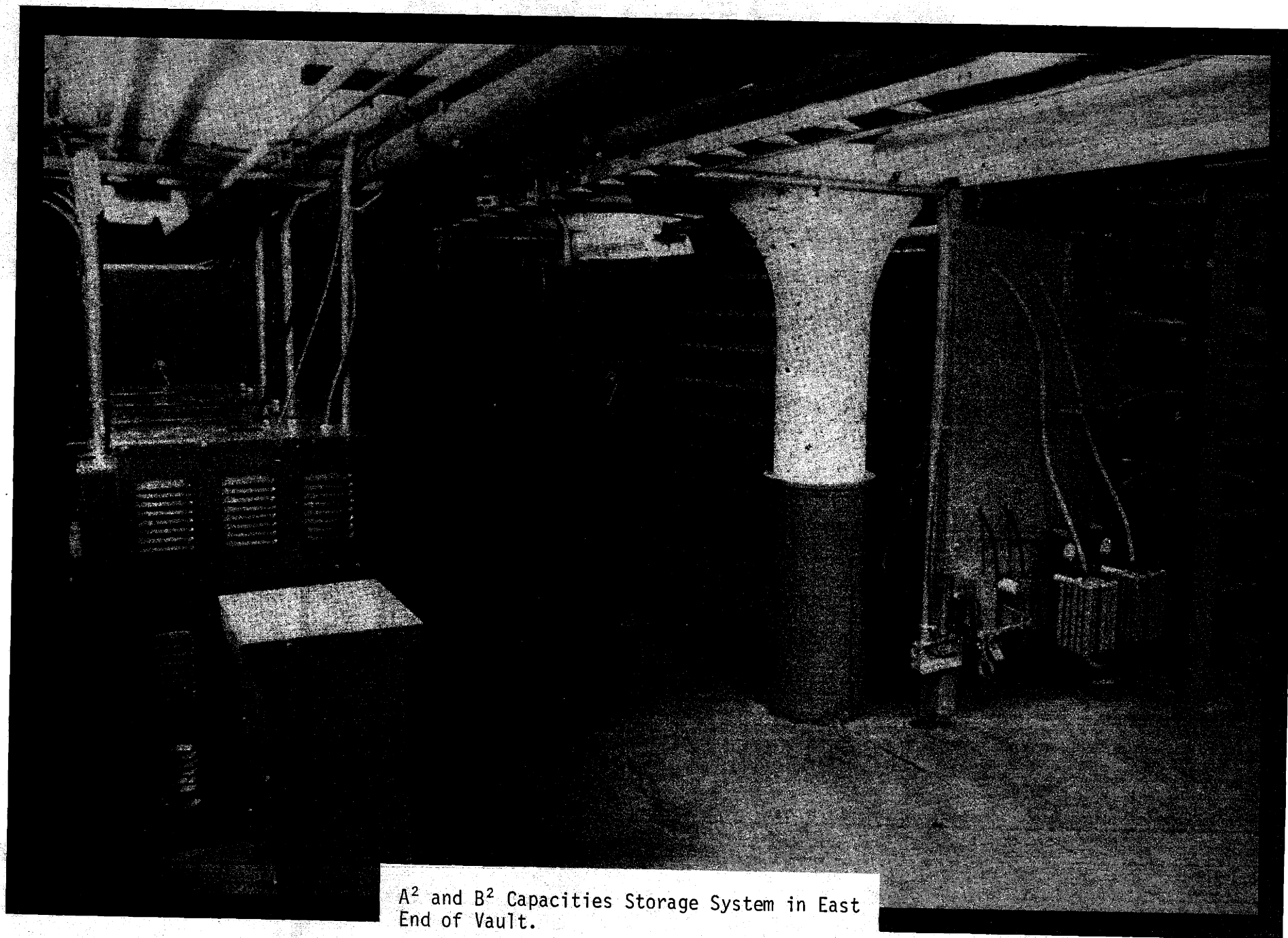


Figure 6





A<sup>2</sup> and B<sup>2</sup> Capacities Storage System in East End of Vault.

Figure 7



Ignition Rectifier and Crowbar Array in Vault.

Figure 8

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The author was born in Black's Harbour, New Brunswick, Canada, in February 1953. He graduated from the private high school (cour classique) of La Pocatière, Province of Quebec, in 1970. In 1972, he received his DEC, (Diplome d'études collegiales), in physical sciences at the CEGEP of La Pocatière.

In the fall of 1972, the author entered the Physics Department at M.I.T. For his bachelor's thesis, he built the high accuracy charge monitoring system for the new M.I.T. 400 Mev accelerator at the Bates National Laboratory. In May 1974, he received his B.S. degree in physics.

In 1975, the author received his Master's degree in aeronautical and astronomical engineering at M.I.T. His Master's thesis, entitled "An On-Line Technique for Tool-Wear Measurements" was done at the Draper Laboratories under contract with Fiat SPA. An outcome of the thesis was an innovative U.S. patent for heavy industrial automation.

Since 1975, he has been a full time research assistant and temporarily a staff member at the Francis Bitter National Magnet Laboratory at M.I.T. The author worked for two years with several teams of Soviet scientists on a new charge exchange diagnostic for Alcator, and has presented several papers on the subject.

In 1977 he was intensively involved in teaching at M.I.T., having helped teach courses in circuit theory, electrostatics, electromagnetics and linear systems analysis. During this year, he was also the principal implementer of the high power DC systems for Alcator C.

In early 1978, he became extensively involved in high power RF work at M.I.T. In particular, he was the project engineer coordinating the

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